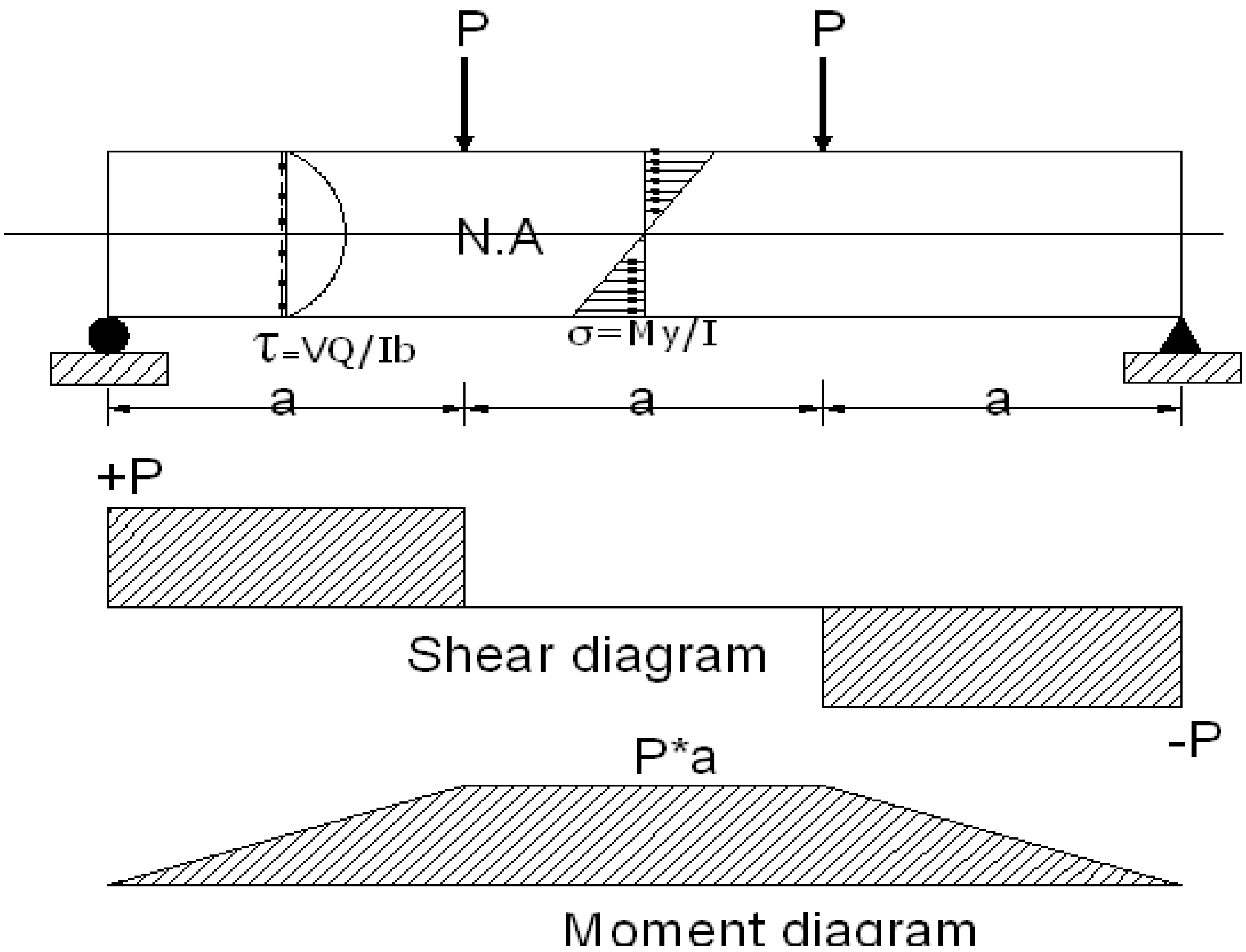
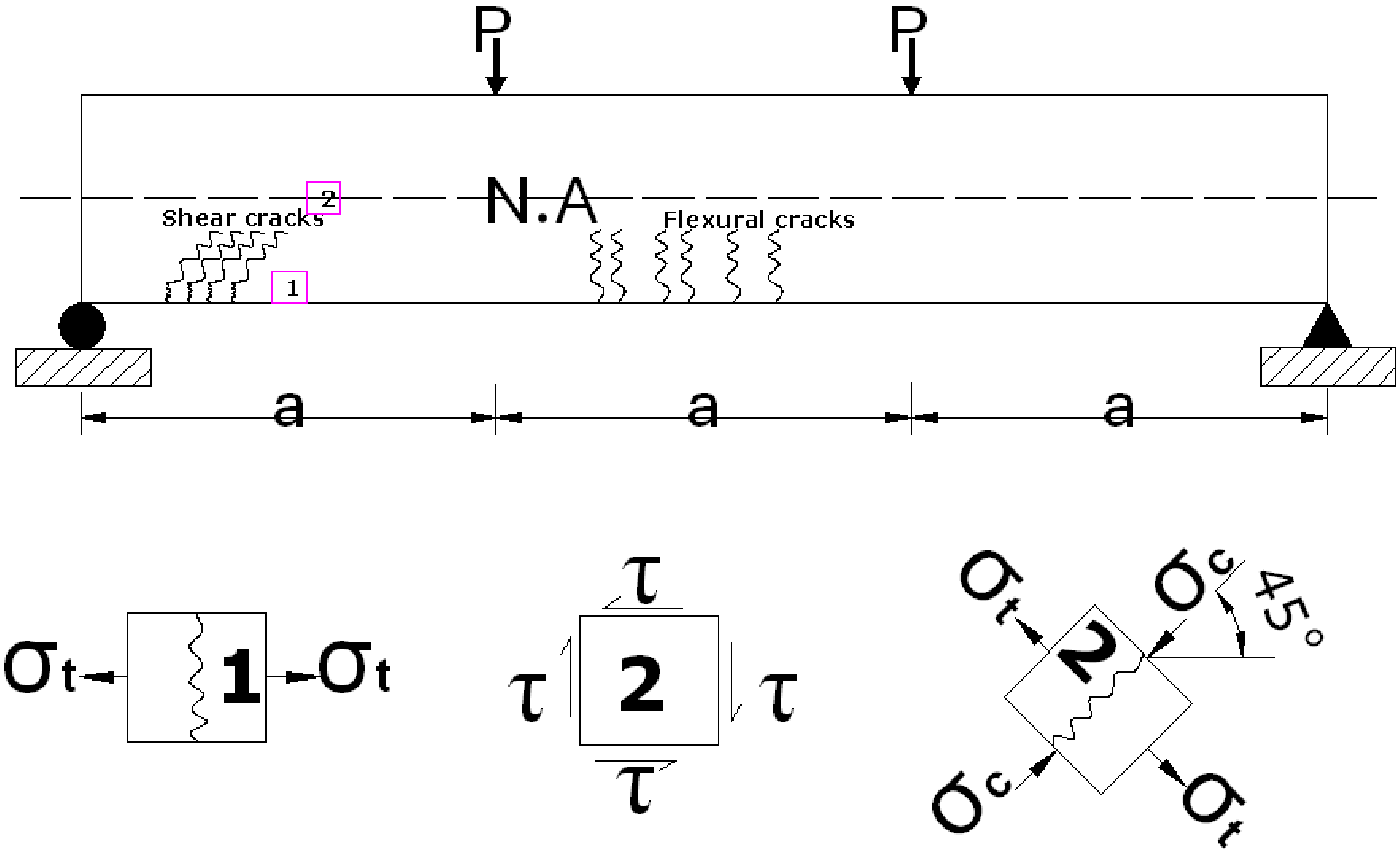
Shear and diagonal tension

Shear in homogenous beams



Shear in reinforced concrete beams



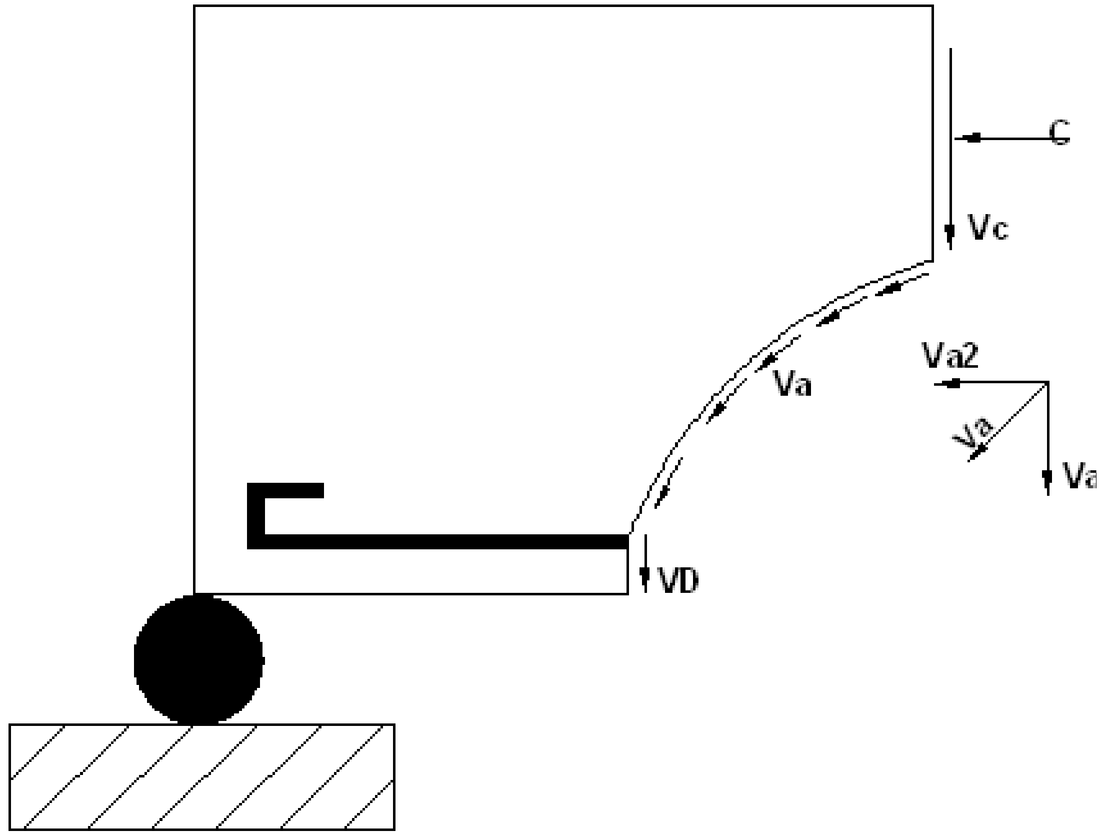
Shear crack(diagonal tension crack):

- Large width
- Sudden formation

Vc: shear force carrying by concrete

VD: shear force carrying by tensile reinforcement

Va1: vertical component of aggregate interlock force



$Vu = Vc + V_D + Va1$ ← To be neglected

Beams with Shear Reinforcement(web reinforcement)

Vn=Vc+Vs

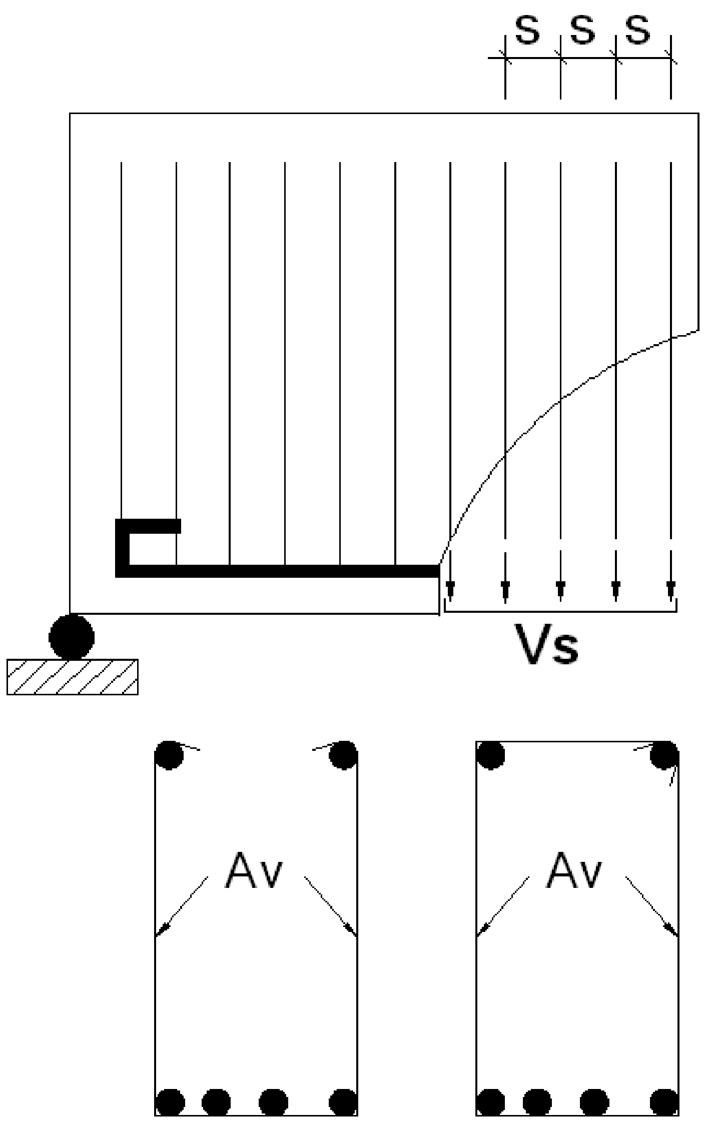
Vn: nominal shear capacity of section

Vs: shear force carrying by stirrups

Vc: shear force carrying by concrete

$Vu = \emptyset Vn \;\; ACI\ 11.1.1$

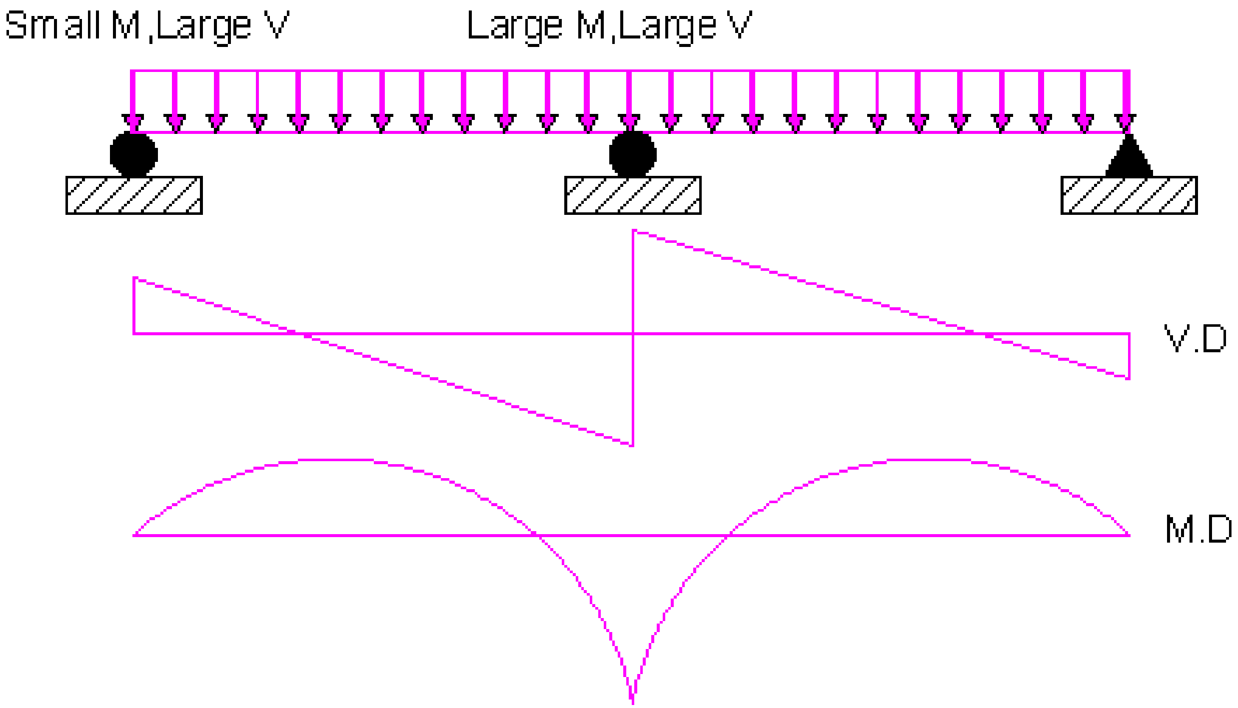
$\emptyset = 0.75 \quad ACI\ 9.32.3$



Shear capacity of concrete Vc:

$$Vc = 0.16\sqrt{fc'}b_w d \quad Large\ M, large\ V$$

$$Vc = 0.29\sqrt{fc'}b_w d \quad Small\ M, large\ V$$



ACI Code equations 11.3.1:

$$Vc = \left(\sqrt{fc'} + 120\rho_w \frac{Vu.d}{Mu}\right)\frac{b_w d}{7} \leq 0.3\sqrt{fc'}\, b_w d \quad \ldots . \text{ACI } 11-5$$

$$\frac{Vu.d}{Mu} \leq 1.0$$

OR

$$Vc = \frac{1}{6}\sqrt{fc'}\, b_w d \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \text{ACI } 11-3$$

Shear strength of shear reinforcement(stirrups) Vs:

S:stirrups spacing

Av: cross sectional area of two legs of stirrup

Vs=nA_v fy

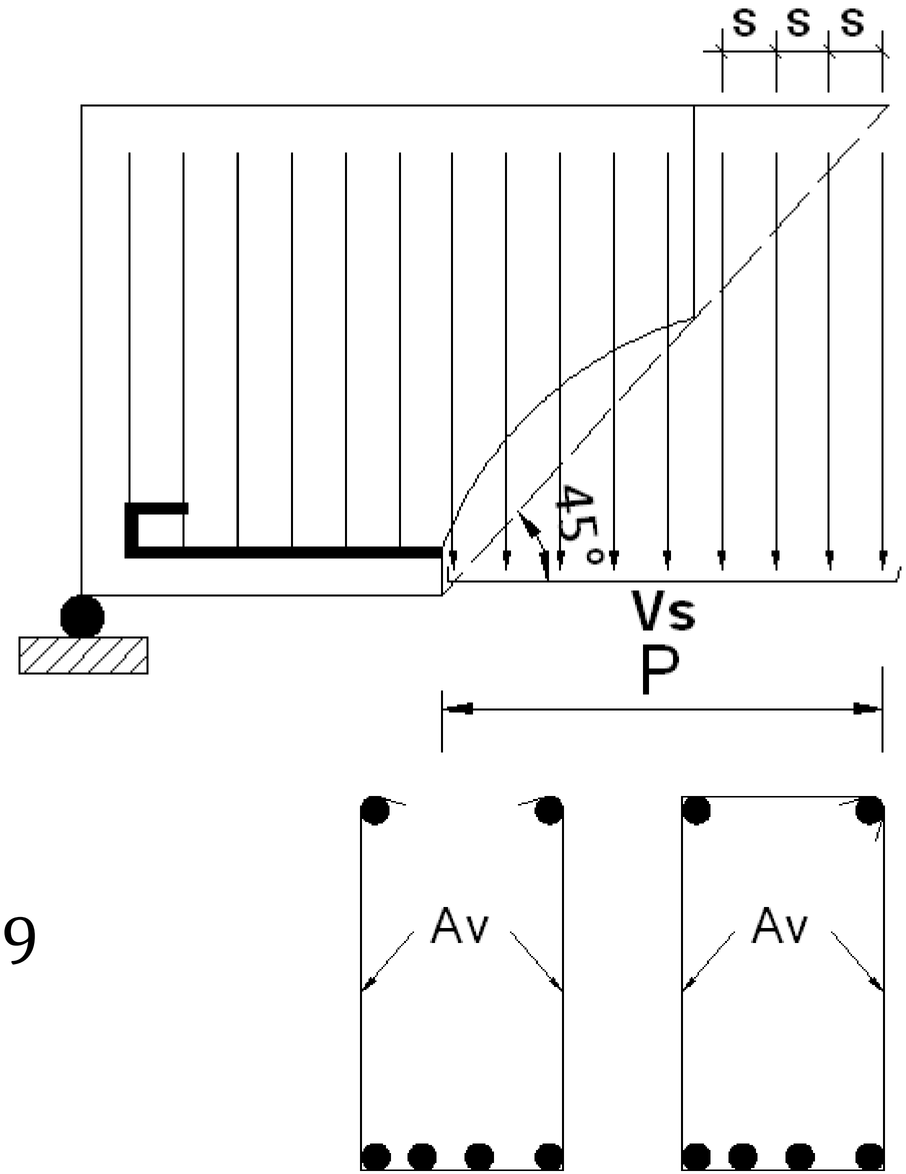
p≈d , since θ=45^o

$$n = \frac{p}{s} = \frac{d}{s}$$

$$Vs = \frac{d}{s}\, A_v fy \qquad ACI\ \ 11.5.6.2$$

$$\leq \frac{2}{3}\sqrt{fc'}\, b_w d = 4Vc \qquad ACI\ \ 11.5.6.9$$

$$fy \leq 420MPa \qquad ACI\ \ 11.5.2$$



Spacing limits for shear reinforcement:

$$S_{max} = \min\left(\frac{d}{2}, 600mm\right) \;\; if \;\; Vs \le \frac{1}{3}\sqrt{fc'}\, b_w d$$

$$= 2Vc \qquad ACI \;\; 11.5.4$$

$$S_{max} = \min\left(\frac{d}{4}, 300mm\right) \;\; if \;\; Vs > \frac{1}{3}\sqrt{fc'}\, b_w d$$

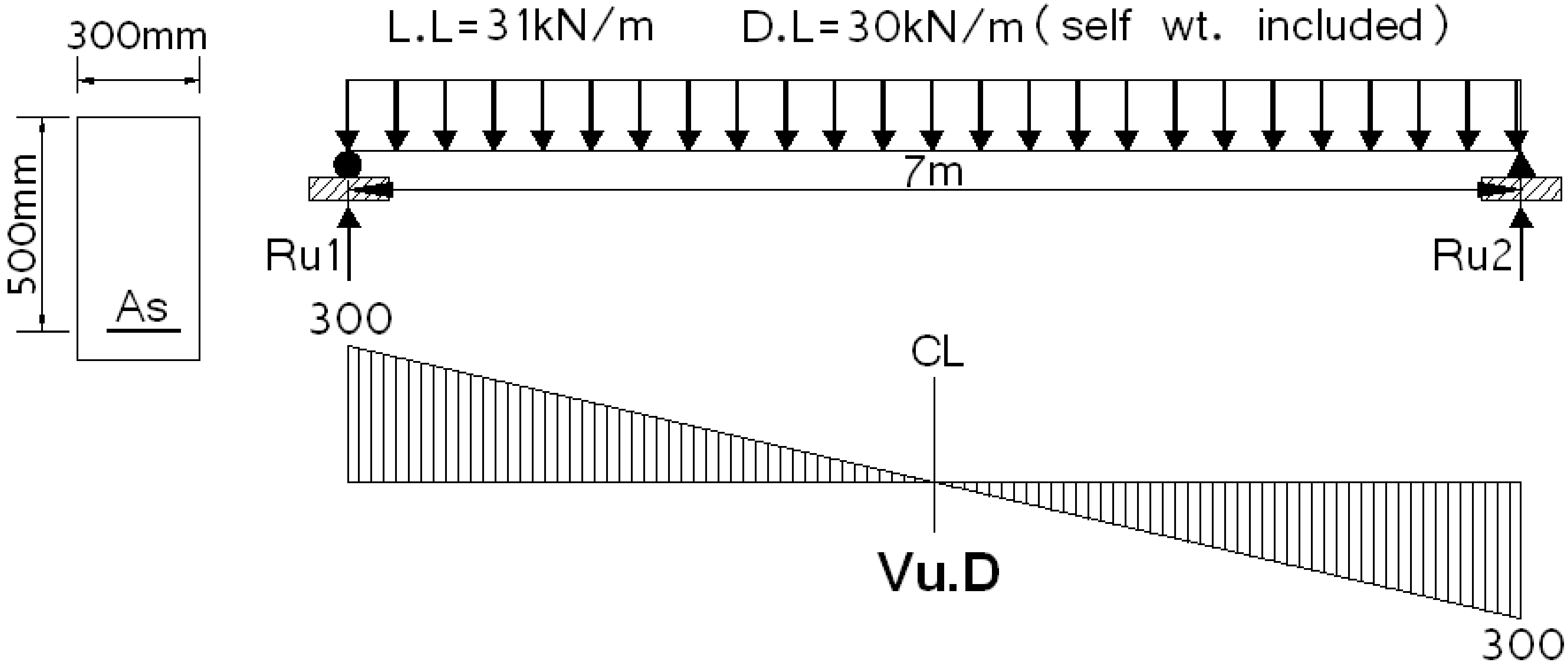
$$= 2Vc \qquad ACI \;\; 11.5.4$$

$$Av_{min} = \max\left(\frac{\sqrt{fc'} b_w s}{16fy}, \frac{b_w s}{3fy}\right) \qquad b_w, s: in\ mm \qquad ACI \;\; 11.5.5.3$$

$$Av_{min} \to S_{max} = \min\left(\frac{16fy\, Av}{\sqrt{fc'} b_w}, \frac{3Av\, fy}{b_w}\right) \qquad ACI \;\; 11.5.5.3$$

EX1:DL=30kN/m(self wt included),LL=31kN/m, $\frac{S}{C}=\frac{300}{30}\,MPa, d=500mm, b=300mm, L=7m, \emptyset_{stir}=10mm,$

design beam for shear, use ACI eq. 11 − 3.



Solution:

Wu=1.2*30+1.6*31=85.6kN/m

Ru1=Ru2=Wu.l/2=85.6*7/2=300kN

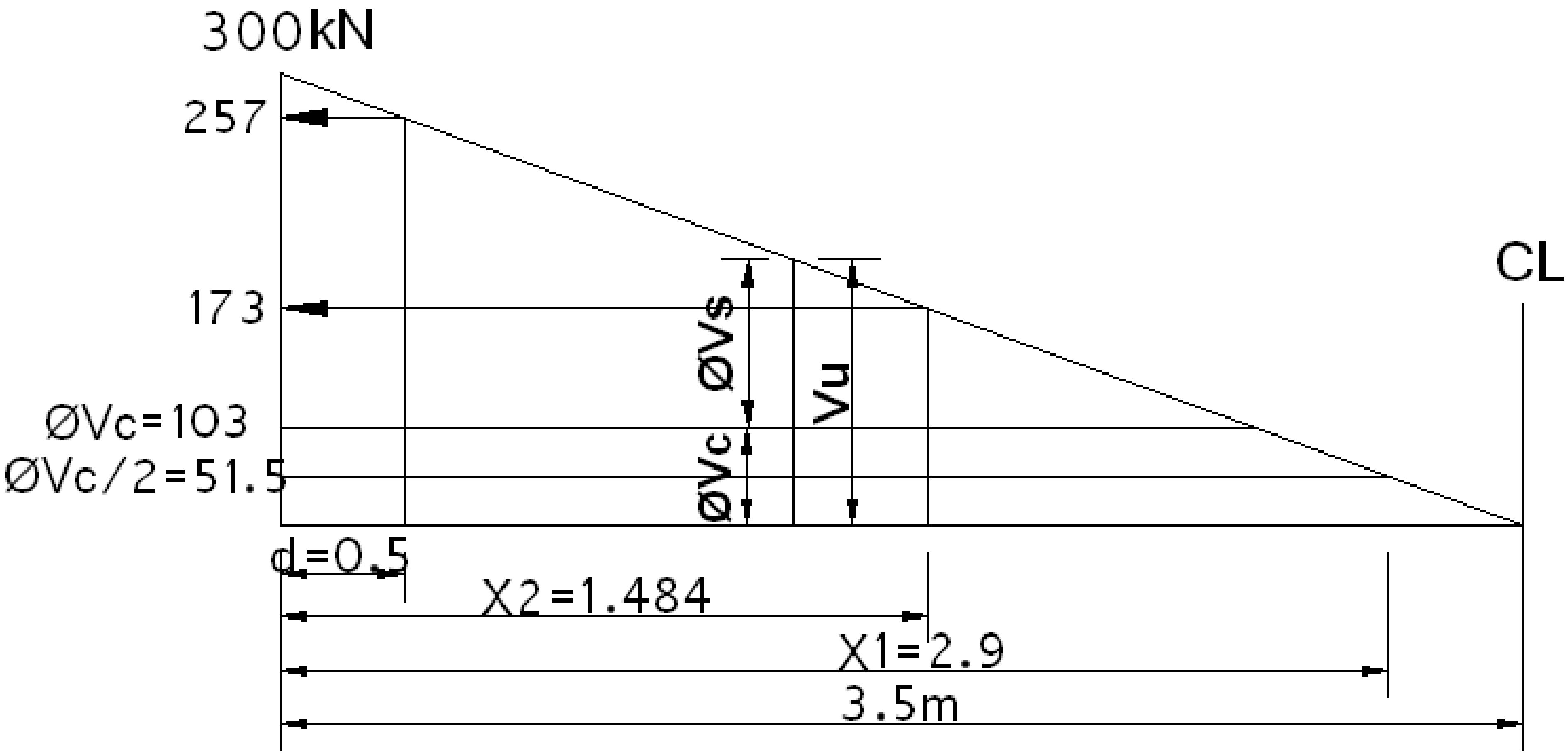
Shear at distance (d) from face of support:

Vud=Ru-Wu*d=300-85.6*0.5=257kN

$$Vc = \frac{1}{6}\sqrt{fc'}b_w d = \frac{1}{6}\sqrt{30} * 0.3 * 0.5 = 0.137MN$$

$$\emptyset Vc = 0.75 * 137 = 103\ kN$$

$$\frac{\emptyset Vc}{2} = \frac{103}{2} = 51.5\ kN$$



$$Vu = \emptyset Vn = \emptyset(Vc + Vs)$$

$$257 = 0.75(137 + Vs) \rightarrow$$

$$Vs = 206kN < 4Vc$$

$$= 547\ kN(else, increase\ cross\ section\ dimensions)$$

$$since\ Vs = 206kN > \frac{\emptyset Vc}{2} = 51.5\ kN$$

$$\rightarrow need\ shear\ reinforcement$$

$$Vs = 206kN < 2Vc = 2 * 137 = 274\ kN \rightarrow$$

$$S_{max} = min\begin{bmatrix} \frac{d}{2} = \frac{500}{2} = 250mm \\ 600mm \\ \frac{16fy\ Av}{\sqrt{fc'}b_w} = \frac{16 * 300 * 2 * 78}{\sqrt{30} * 300} = 455mm \\ \frac{3Av\ fy}{b_w} = \frac{3 * 2 * 78 * 300}{300} = 468mm \end{bmatrix}$$

$$= 250mm$$

$$S_{@d} = \frac{A_v fy\ d}{Vs} = \frac{2 * 78 * 10^{-6} * 300 * 0.5}{0.206}$$

$$= 0.114m \approx 110mm < S_{max} = 250mm\ O.K$$

- **Location at which no stirrups are required**

$$(i.e\ Vu \leq \frac{\emptyset Vc}{2} = 51.5\ kN)$$

$$Ru - Wu * x1 = 51.5kN \rightarrow 300 - 58.6x1 = 51.5kN \rightarrow$$

$$X1 = 2.9m$$

- **Location at which use Av_{nin}(i.e, $S = S_{max} = 250mm$)**

$$S = \frac{A_v fy\, d}{Vs} \rightarrow Vs = \frac{A_v fy\, d}{S_{max}}$$

$$= \frac{2 * 78 * 10^{-6} * 300 * 0.5}{0.25} * 10^3$$

$$= 93.6kN$$

$$Vu = \emptyset(Vc + Vs)$$

$$Vu = 0.75(137 + 93.6) \rightarrow Vu = 173kN$$

$$Ru - Wu * x2 = 173kN \rightarrow X2 = 1.484m$$

$X2 - d = 1.484 - 0.5 = 0.984m$,

- **Section at distance** $= \boldsymbol{d + \frac{0.984}{2} = 0.992m}$

$Vu = Ru - Wu * X = 300 - 85.6 * 0.992 \rightarrow Vu = 215kN$

$Vu = \emptyset(Vc + Vs)$

$215 = 0.75(137 + Vs) \rightarrow$

$Vs = 150kN < 4Vc = 547\ kN\ O.K$

$$S = \frac{A_v fy\ d}{Vs} \rightarrow S = \frac{2 * 78 * 10^{-6} * 300 * 0.5}{0.15} * 10^3$$

$$= 156mm \quad \approx 155mm < S_{max} = 250mm\ O.K$$

- **Distribution of stirrups:**

1. $S_{@d} = 110\text{mm}$

$$S_\circ = \frac{S_{@d}}{2} = \frac{110}{2} = 55\text{mm} \ \ (\text{first stirrup})$$

2. $n = \frac{d - S_\circ}{S} = \frac{500-55}{110} =$

4.04 (use stirrups 5Ø10mm@ 110mm c/c)

$$55 + 5 * 110 = 605\text{mm} > d = 500mm\ O.K$$

3. $n = \frac{992-605}{110} = 3.5$ (use stirrups 4Ø10mm@ 110mm c/c)

$$605 + 4 * 110 = 1045\text{mm} > 992\text{mm O.K}$$

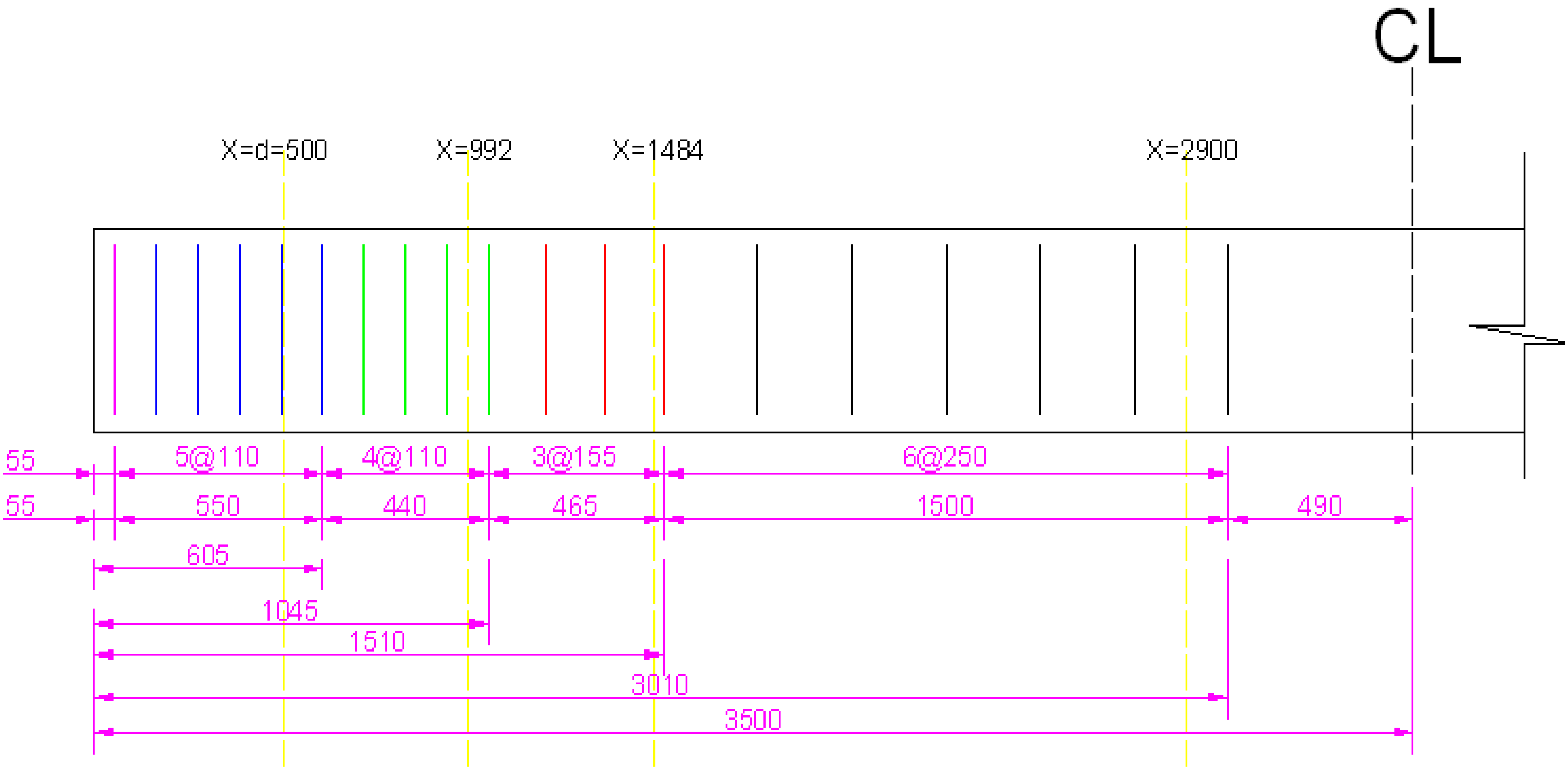
4. $n = \frac{1484-1045}{155} = 2.8$ (use stirrups 3Ø10mm@ 155mm c/c)

$$1045 + 3 * 155 = 1510\text{mm} > 1484\text{mm O.K}$$

5. $n = \frac{2900-1510}{250} = 5.5$ (use stirrups 6Ø10mm@ 250mm c/c)

$$1510 + 6 * 250 = 3010\text{mm} > 2900\text{mm O.K}$$

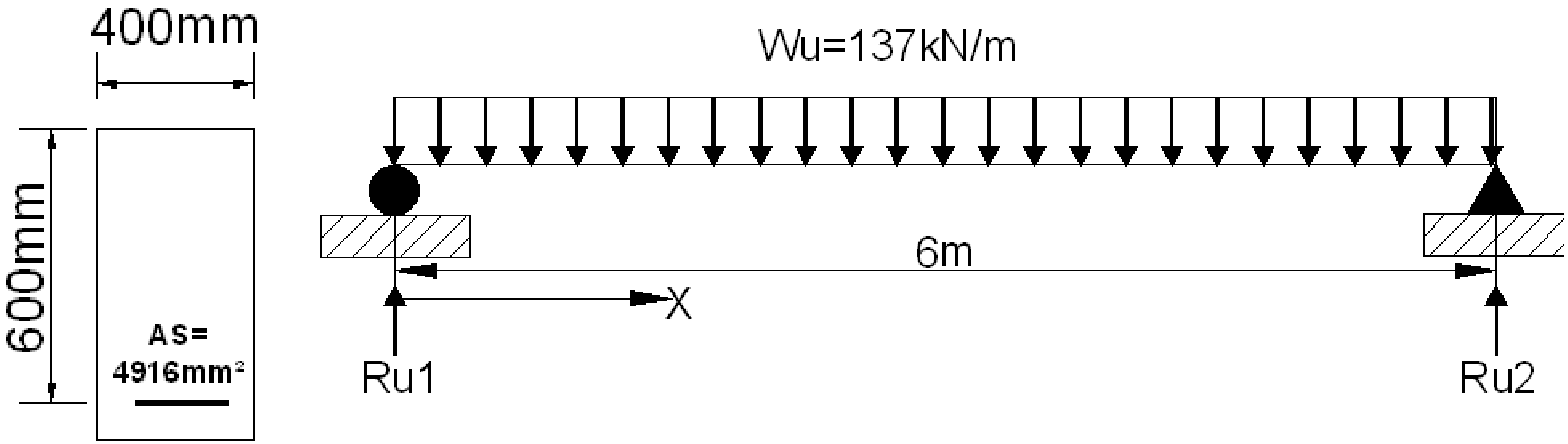
Total No. of stirrups used=19*2=38Ø10mm



Stirrups distribution along the beam

EX2:Wu=137kN/m, $\frac{S}{C} = \frac{414}{28} MPa, d = 600mm, b = 400mm, L = 6m, \emptyset_{stir} = 10mm,$

$design\ beam\ for\ shear, use$ ACI eq. $11 - 5.$



Solution:

$$\rho_w = \frac{4916}{400 * 600} = 0.0205$$

Ru1=Ru2=137*6/2=411kN

Vu(x)=Ru-Wu*X=411-137X

Mu(x)=411X-137*X^2/2=68.5X(6-X)

$$\frac{Vu.d}{Mu} = \frac{(411 - 137X) * 0.6}{68.5X(6 - X)} = \frac{1.2(3 - X)}{X(6 - X)}$$

$$Vc = \left(\sqrt{fc'} + 120\rho_w \frac{Vu.d}{Mu}\right)\frac{b_w d}{7}$$

$$\leq 0.3\sqrt{fc'}\, b_w d \ldots\ldots\ldots \text{ACI } 11-5$$

$$Vc = \left(\sqrt{28} + 120 * 0.0205 * \frac{Vu.d}{Mu}\right)\frac{0.4 * 0.6}{7}$$

$$Vc = 0.181 + 0.0843 * \frac{Vu.d}{Mu}$$

X (m)	$\frac{Vu.d}{Mu} \leq 1.0$	Vc (kN)	Vu (kN)	$\frac{Vu}{\emptyset}$	Vs=(Vu/Ø)-Vc	Vc/2	$S = \frac{A_v fy\, d}{Vs}$ (mm)
d=0.6	0.89	256	329	438	182	128	213≈210
0.9	0.55	227	288	384	157	113	247≈240
$S=S_{max}$ =1.11	0.42	216	259	345	129	108	300= S_{max}
1.2	0.37	212	247	329	117	106	
1.5	0.27	204	205	273	69	102	
1.8	0.19	197	164	218	22	98	
2.1	0.13	192	123	164	-28	96	
2.4	0.08	188	82	109	-78	94	
2.7	0.04	184	41	54	-129	92	
3.0	0	181	0	0	-181	90	
	≤1.0 O.K	$\leq 0.3\sqrt{fc}\ b\ d$ =381 kN O.K			$\leq \frac{2}{3}\sqrt{fc}\ b\ d$ =863kN O.K		

$$Vs_{max} = 182kN < \frac{1}{3}\sqrt{fc'}\, b_w d = 423kN \rightarrow$$

$$S_{max} = min\begin{bmatrix} \frac{d}{2} = \frac{600}{2} = 300mm \\ 600mm \\ \frac{16fy\,Av}{\sqrt{fc'}b_w} = \frac{16 * 414 * 2 * 78}{\sqrt{28} * 400} = 488mm \\ \frac{3Av\,fy}{b_w} = \frac{3 * 2 * 78 * 414}{400} = 484mm \end{bmatrix}$$

$$= 300mm$$

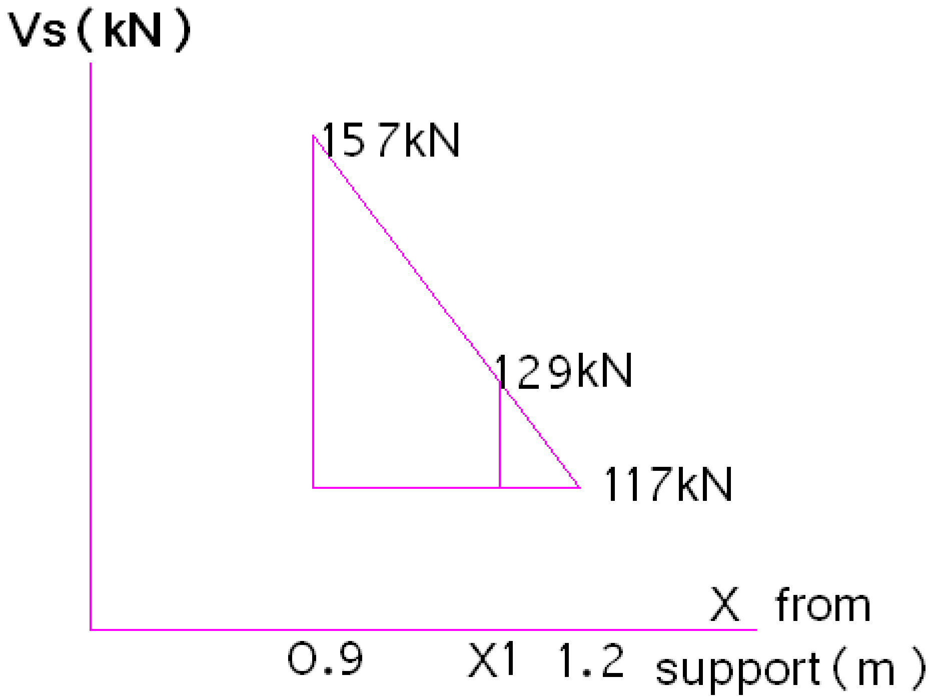
Location at S=Smax:

$$Vs = \frac{A_v fy\ d}{S_{max}} = \frac{2*78*10^{-6}*414*0.6}{0.3}*$$

$$10^3 = 129kN$$

$$\frac{157-117}{1.2-0.9} = \frac{157-129}{X1-0.9} \rightarrow X1$$

$$= 1.11m$$



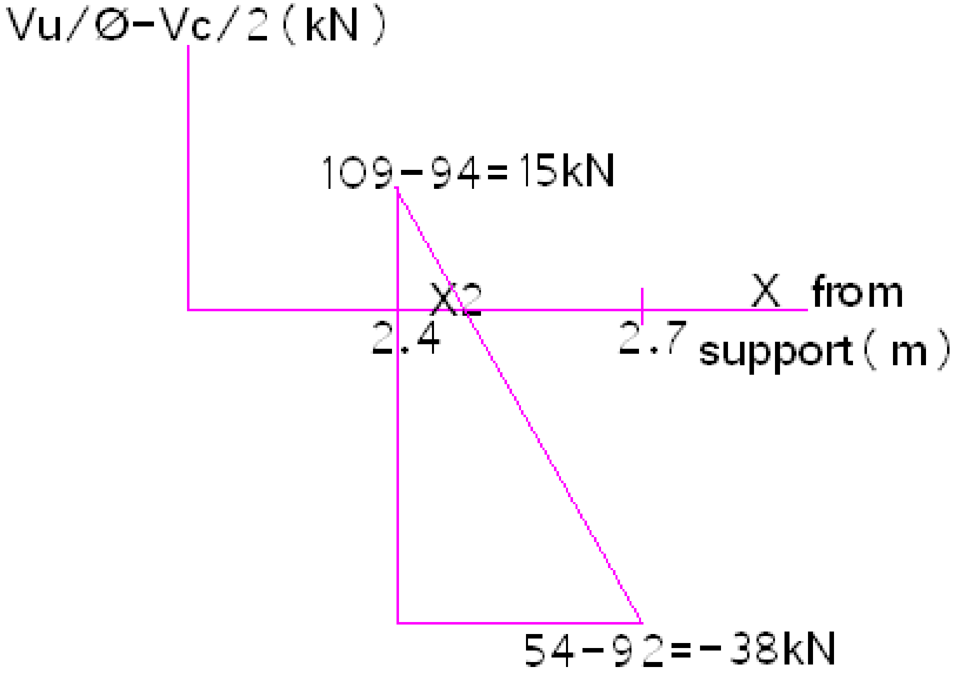
Location at which no stirrups are required:

i.e

$$\frac{\text{Vu}}{\emptyset} \leq \frac{\text{Vc}}{2} \rightarrow \frac{\text{Vu}}{\emptyset} - \frac{\text{Vc}}{2} \leq 0$$

$$\frac{15+38}{2.7-2.4} = \frac{15}{X2-2.4} \rightarrow X2$$

$$= 2.485m$$



- **Distribution of stirrups:**

1. $S_{@d} = 210mm$

$S_{\circ} = \frac{S_{@d}}{2} = \frac{210}{2} = 100mm$ (first stirrup)

2. $n = \frac{0.9-0.1}{0.21} = 3.8$ (use stirrups 4Ø10mm@ 210mm c/c)

$100 + 4 * 210 = 940mm > 900mm$ O.K

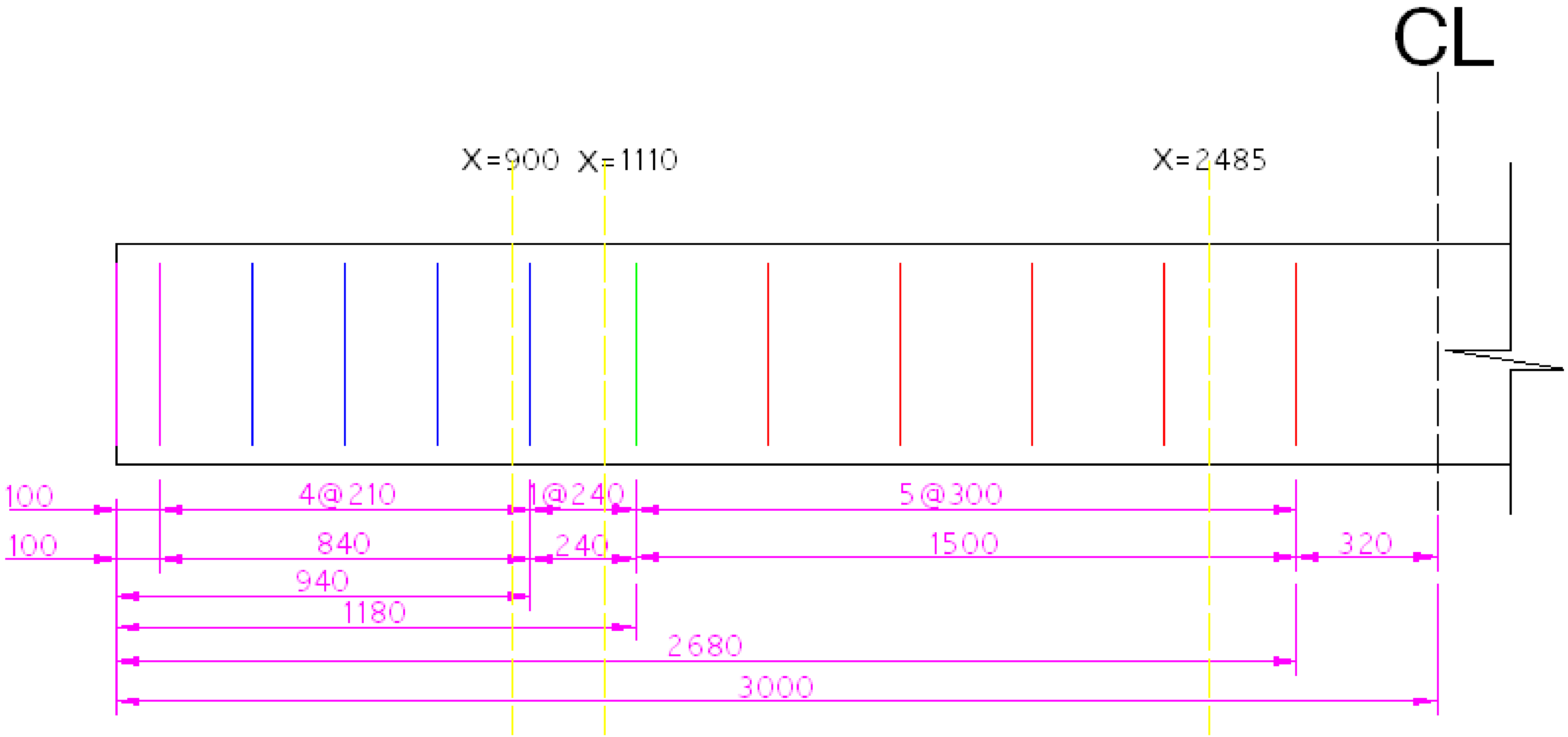
3. $n = \frac{1.11-0.94}{0.24} = 0.7$ (use stirrups 1Ø10mm@ 240mm c/c)

$940 + 1 * 240 = 1180mm > 1100mm$ O.K

4. $n = \frac{2.485-1.18}{0.3} = 4.35$ (use stirrups 5Ø10mm@300mm c/c)

$1180 + 5 * 300 = 2680mm > 2485mm$ O.K

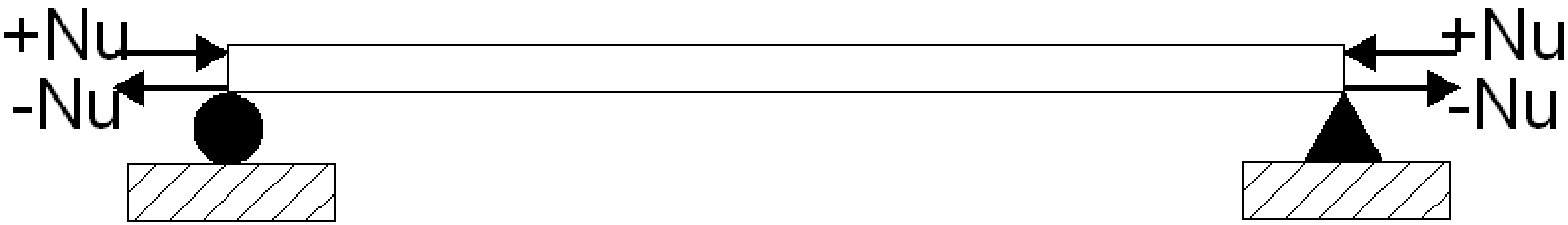
Total No. of stirrups used=11*2=22Ø10mm



Stirrups distribution along the beam

Shear capacity of concrete Vc subject to axial force (ACI 11.3.1):

a. Axial compression force (+Nu):



$$Vc = \left(1 + \frac{Nu}{14Ag}\right)\frac{1}{6}\sqrt{fc'}\, b_w d \text{ ……… ACI } 11 - 14$$

Nu: axial compression force(+ve)

Ag: gross area (bw.h)

$\frac{Nu}{Ag}: expressed\ in\ MPa$ OR

$$Vc = \left(\sqrt{fc'} + 120\rho_w \frac{Vu.d}{Mm}\right)\frac{b_w d}{7} \leq 0.3\sqrt{fc'}\, b_w d\sqrt{1 + \frac{0.3Nu}{Ag}}$$

$$Mm = Mu - Nu\frac{(4h - d)}{8}$$

$$if\ Mm = -ve \rightarrow Vc = 0.3\sqrt{fc'}\, b_w d\sqrt{1 + \frac{0.3Nu}{Ag}}$$

b. Axial tension force (-Nu):

$$Vc = 0 \rightarrow Vu = \emptyset Vs$$

OR

$$Vc = \left(1 + \frac{0.3Nu}{Ag}\right)\frac{1}{6}\sqrt{fc'}\, b_w d \geq 0$$

Nu: axial tension force(-ve)

$\frac{Nu}{Ag}: expressed\ in\ MPa$

Ex: $\frac{S}{C} = \frac{400}{28} MPa, ignore\ beam\ wt,$

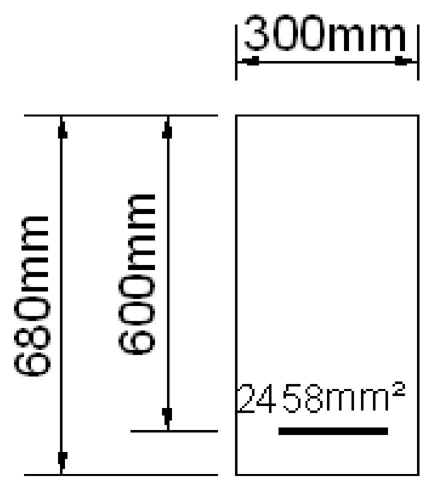
find spacing of Ø10mm stirrups at critical section if:

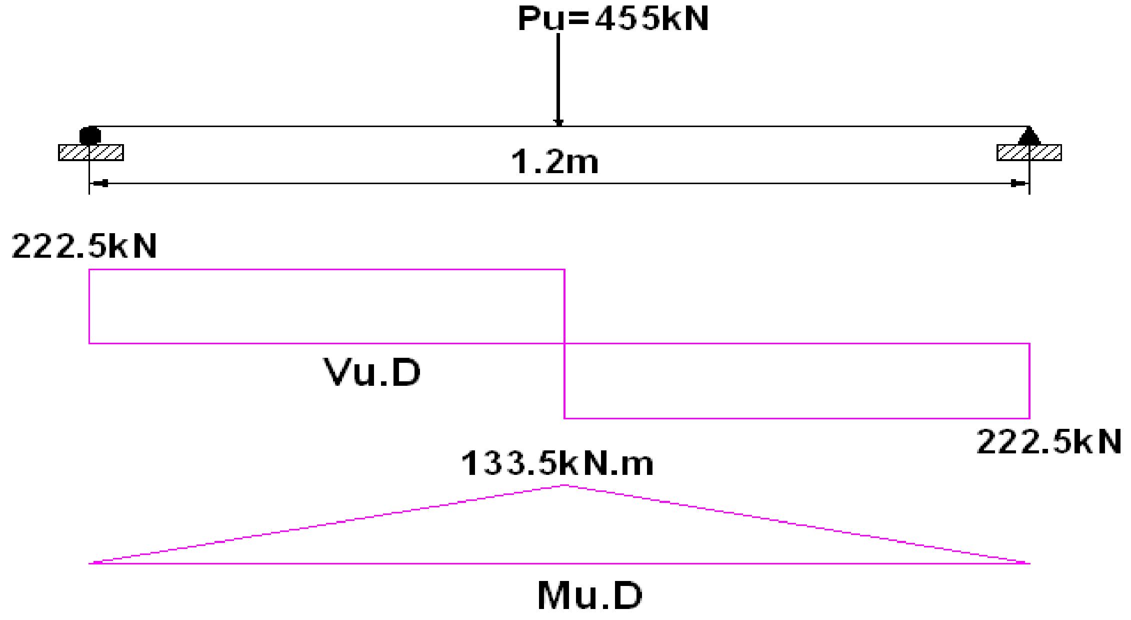
a. No axial force

b. Axial compression force of 267kN

c. Axial tension force of 267kN

Use both simplified and complex equations of the ACI code.





Solution:

a. No axial force

Simplified equation for Vc of the ACI code:

Critical section at distance (d) from face of support,

Vud=222.5kN

$$Vc = \frac{1}{6}\sqrt{fc'}\, b_w d = \frac{1}{6}\sqrt{28} * 0.3 * 0.6 * 1000 = 159kN$$

$$Vu = \emptyset(Vc + Vs)$$

$$222.5 = 0.75(159 + Vs) \rightarrow$$

$$Vs = 138kN < \frac{2}{3}\sqrt{fc'}\, b_w d = \frac{2}{3}\sqrt{28} * 0.3 * 0.6$$

$$= 635\ kN\ O.K$$

$$S = \frac{A_v fy\, d}{Vs} \rightarrow S = \frac{2 * 78 * 10^{-6} * 400 * 0.6}{0.138} * 10^3$$

$$= 271mm$$

$$Vs = 138kN < 2Vc = 2 * 159 = 318\ kN \rightarrow$$

$$S_{max} = min\begin{bmatrix} \frac{d}{2} = \frac{600}{2} = 300mm \\ 600mm \\ \frac{16fy\ Av}{\sqrt{fc'}b_w} = \frac{16*400*2*78}{\sqrt{28}*300} = 629mm \\ \frac{3Av\ fy}{b_w} = \frac{3*2*78*400}{300} = 624mm \end{bmatrix}$$

$$= 300mm$$

$$S = 271mm < S_{max} = 300mm\ O.K$$

$$\rightarrow use\emptyset 10mm@270mm\ c/c$$

Complex equation for Vc of the ACI code:

Critical section at mid span

$$\rho_w = \frac{2458}{300 * 600} = 0.0136$$

$$Vc = \left(\sqrt{fc'} + 120\rho_w \frac{Vu.d}{Mu}\right)\frac{b_w d}{7}$$

$$\leq 0.3\sqrt{fc'}\, b_w d \ldots\ldots\ldots \text{ACI } 11 - 5$$

$$\frac{Vu.d}{Mu} = \frac{222.5 * 0.6}{133.5} = 1.0 \leq 1.0\ O.K$$

$$Vc = \left(\sqrt{28} + 120 * 0.0136 * 1.0\right)\frac{0.3 * 0.6}{7} * 1000 = 178kN$$

$$\leq 0.3\sqrt{28}\, b_w d * 0.3 * 0.6 * 1000 = 286kN \quad O.K$$

$$Vu = \emptyset(Vc + Vs)$$

$$222.5 = 0.75(178 + Vs) \rightarrow$$

$$Vs = 118.7kN < \frac{2}{3}\sqrt{fc'}\, b_w d = \frac{2}{3}\sqrt{28} * 0.3 * 0.6$$

$$= 635 \ kN\ O.K$$

$$S = \frac{A_v fy\, d}{Vs} \rightarrow S = \frac{2 * 78 * 10^{-6} * 400 * 0.6}{0.1187} * 10^3$$

$$= 315mm > S_{max} = 300mm$$

$$\rightarrow use\emptyset 10mm@300mm\ c/c$$

b. Axial compression force of 267kN

Simplified equation for Vc of the ACI code:

Critical section at distance (d) from face of support,

Vud=222.5kN

$$Vc = \left(1 + \frac{Nu}{14Ag}\right)\frac{1}{6}\sqrt{fc'}\, b_w d = \left(1 + \frac{0.267}{14*0.3*0.68}\right)\frac{1}{6}\sqrt{28} * 0.3 * 0.6 * 1000 = 173kN$$

$$Vu = \emptyset(Vc + Vs)$$

$$222.5 = 0.75(173 + Vs) \rightarrow \ Vs = 124kN < \frac{2}{3}\sqrt{fc'}\, b_w d =$$

$$\frac{2}{3}\sqrt{28} * 0.3 * 0.6 = 635\ kN\ O.K$$

$$S = \frac{A_v fy\ d}{Vs} \rightarrow S = \frac{2 * 78 * 10^{-6} * 400 * 0.6}{0.124} * 10^3$$

$$= 302mm > S_{max} = 300mm$$

$$\rightarrow use\text{Ø}10mm@300mm\ c/c$$

Complex equation for Vc of the ACI code:

Critical section at mid span

$$Vc = \left(\sqrt{fc'} + 120\rho_w \frac{Vu.d}{Mm}\right)\frac{b_w d}{7} \leq 0.3\sqrt{fc'}\, b_w d\sqrt{1 + \frac{0.3Nu}{Ag}}$$

$$Mm = Mu - Nu\frac{(4h-d)}{8} = 133.5 - 267 * \frac{(4*0.68-0.6)}{8} =$$

$$62.7kN$$

$$Vc = \left(\sqrt{28} + 120 * 0.0136 \frac{222.5*0.6}{62.7}\right) \frac{0.3*0.6}{7} * 10^3 = 225kN \leq$$

$$0.3\sqrt{fc'}\, b_w d \sqrt{1 + \frac{0.3Nu}{Ag}} =$$

$$0.3\sqrt{28} * 0.3 * 0.6 * \sqrt{1 + \frac{0.3*0.267}{0.3*0.68}} * 10^3 = 337kN$$

$$Vc = 225kN$$

$$Vu = \emptyset(Vc + Vs)$$

$$222.5 = 0.75(225 + Vs) \rightarrow$$

$$Vs = 72kN < \frac{2}{3}\sqrt{fc'}\, b_w d = \frac{2}{3}\sqrt{28} * 0.3 * 0.6 = 635\ kN\ O.K$$

$$S = \frac{A_v f y \, d}{Vs} \rightarrow S = \frac{2 * 78 * 10^{-6} * 400 * 0.6}{0.072} * 10^3$$

$$= 520mm > S_{max} = 300mm$$

$$\rightarrow use \varnothing 10mm@300mm \; c/c$$

c. Axial tension force of 267kN

Simplified equations for Vc of the ACI code:

Critical section at distance (d) from face of support, Vud=222.5kN

$$Vc = 0 \rightarrow Vud = \emptyset Vs \rightarrow Vs = \frac{222.5}{0.75} = 297kN < \frac{2}{3}\sqrt{fc'}\, b_w d = \frac{2}{3}\sqrt{28} * 0.3 * 0.6 = 635\, kN$$

$$S = \frac{A_v fy\, d}{Vs} \rightarrow S = \frac{2*78*10^{-6}*400*0.6}{0.297} * 10^3 = 126mm <$$

$$S_{max} = 300mm\ O.K \rightarrow use\emptyset 10mm@125mm\ c/c$$

OR

$$Vc = \left(1 + \frac{0.3Nu}{Ag}\right)\frac{1}{6}\sqrt{fc'}\,b_w d$$

$$= \left(1 + \frac{0.3(-0.267)}{0.3 * 0.68}\right)\frac{1}{6}\sqrt{28} * 0.3 * 0.6 * 1000 = 96kN$$

$$Vu = \emptyset(Vc + Vs)$$

$$222.5 = 0.75(96 + Vs) \rightarrow\ Vs = 200kN < \frac{2}{3}\sqrt{fc'}\,b_w d$$

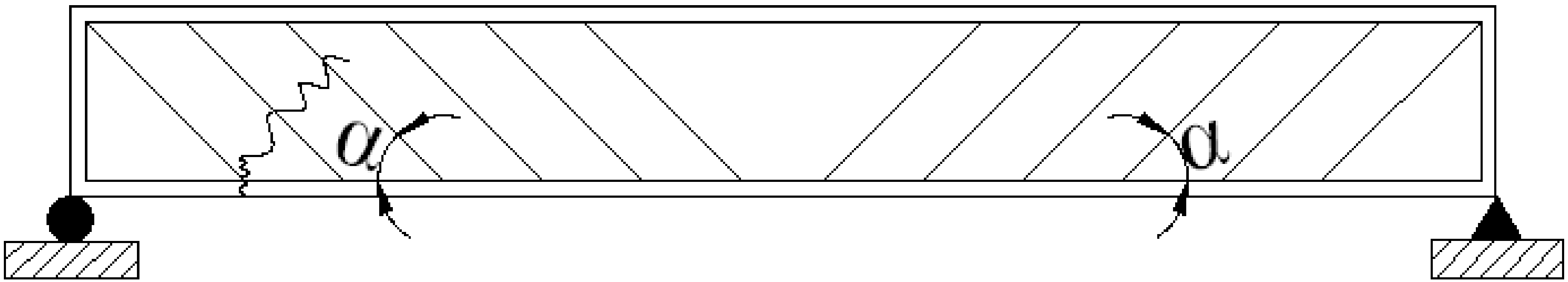
$$= \frac{2}{3}\sqrt{28} * 0.3 * 0.6 = 635\ kN\ O.K$$

$$S = \frac{A_v fy\, d}{Vs} \rightarrow S = \frac{2 * 78 * 10^{-6} * 400 * 0.6}{0.200} * 10^3$$

$$= 187mm < S_{max} = 300mm$$

$$\rightarrow use\emptyset 10mm@185mm\ c/c$$

Shear capacity of inclined stirrups:



Stirrups making an angle of 45^{o} or more with longitudinal tension reinforcement

$\alpha \geq 45° \;\; ACI\ 11.5.1.2$

$$Vs = \frac{Av\, fy\, d(\sin\alpha + \cos\alpha)}{S} \leq \frac{2}{3}\sqrt{fc'}b_w d \quad ACI\ 11 - 16(11.5.6.4)$$

$$S_{max} = \frac{16}{17} d(1 + \cot\alpha) \quad for\ Vs$$

$$\leq \frac{1}{3}\sqrt{fc'} b_w d \quad ACI\ (11.5.4.2)$$

$$S_{max} = \frac{16}{34} d(1 + \cot\alpha) \quad for\ Vs$$

$$> \frac{1}{3}\sqrt{fc'} b_w d \quad ACI\ (11.5.4.3)$$

$$Av_{min} \rightarrow S_{max}$$

$$= \min\left(\frac{16fy\ Av}{\sqrt{fc'} b_w}, \frac{3Av\ fy}{b_w}\right) \qquad ACI\ \ 11.5.5.3$$

Shear capacity of bent bars:

longitudinal reinforcement with bent portion making an angle of 30^{o} or more with longitudinal tension reinforcement

$\alpha \geq 30° \; ACI \; 11.5.1.2$

- Shear reinforcement consists of a <u>single</u> bar or a <u>single group</u> of parallel bars, all bent up <u>*at the same distance*</u> from the support ACI(11.5.7.5):

$$Vs = [Av\, fy\ \sin\alpha \leq \frac{1}{4}\sqrt{fc'}b_w d\ \ which\, ever\ is\ smaller\ ] \leq \frac{2}{3}\sqrt{fc'}b_w d\ \ ACI\ 11-17(11.5.7.5)$$

- Shear reinforcement consists of a <u>series</u> of parallel bent-up bars or <u>groups</u> of parallel bent-up bars at <u>*different distances*</u> from the support ACI(11.5.7.6):

$$Vs = \frac{Av\, fy\, d(\sin\alpha + \cos\alpha)}{S} \leq \frac{2}{3}\sqrt{fc'}b_w d \quad ACI\ 11-16(11.5.6.4)$$

$$S_{max} = \frac{16}{17} d(1 + \cot\alpha) \quad for\ Vs$$

$$\leq \frac{1}{3}\sqrt{fc'} b_w d \quad ACI\ (11.5.4.2)$$

$$S_{max} = \frac{16}{34} d(1 + \cot\alpha) \quad for\ Vs$$

$$> \frac{1}{3}\sqrt{fc'} b_w d \quad ACI\ (11.5.4.3)$$

$$Av_{min} \rightarrow S_{max} = \min\left(\frac{16fy\ Av}{\sqrt{fc'} b_w}, \frac{3Av\ fy}{b_w}\right) \quad ACI\ \ 11.5.5.3$$

EX: The same previous example, use inclined stirrups with angle of 45°

$$Vs = 138kN \leq \frac{2}{3}\sqrt{fc'}b_w d \quad O.K$$

$$Vs = \frac{Av\, fy\, d(\sin\alpha + \cos\alpha)}{S}$$

$$S = \frac{2 * 78 * 10^{-6} * 400 * 0.6(\sin 45 + \cos 45)}{0.138} = 0.383m$$

$$Vs = 138kN \leq \frac{1}{3}\sqrt{fc'}b_w d \quad \rightarrow$$

$$S_{max} = \frac{16}{17} d(1 + \cot\alpha) = \frac{16}{17} * 600(1 + \cot 45) = 423mm$$

$$S_{max} = min \begin{bmatrix} \dfrac{16 fy\ Av}{\sqrt{fc'} b_w} = \dfrac{16*400*2*78}{\sqrt{28}*300} \\ \dfrac{3Av\ fy}{b_w} = \dfrac{3*2*78*400}{300} = 624mm \end{bmatrix}$$

$$S_{max} = 423mm$$

$$S = 383mm < S_{max} = 423mm\ O.K$$

$$use\emptyset 10mm@380mm\ c/c$$

$$compare \quad with\ \emptyset 10mm@270mm\ c/c$$