Republic of Iraq

Ministry of Higher Education

and Scientific Research

Al-Mustaqbal University College

Computer Engineering Techniques Department

Subject: Digital Signal Processing

Third Class

Lecture Three

By

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Standard of discrete Time signals (sequences)

a- Unit impulse sequences

The discrete-time counterpart for the continuous-time unit impulse (delta function) is the discrete-time unit impulse. It is defined as:

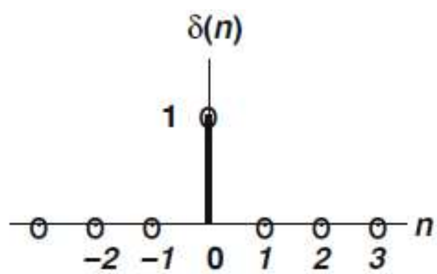
$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

Or

$$\delta(n) = \{ \dots\dots, 0, 0, 0, \underline{1}, 0, 0, 0, \dots. \}$$

Or

$$\delta[n] = u[n] - u[n-1]$$



b- Unit step sequences

The unit step sequence is one that has an amplitude of zero for negative indices and an amplitude of one for non-negative indices. Its definition is:

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Or $$u(n) = \{\dots\dots, 0, 0, \underline{1}, 1, 1, 1, 1, \dots\dots\}$$

Or $$u(n) = \sum_{k=0}^{\infty} \delta(n-k)$$

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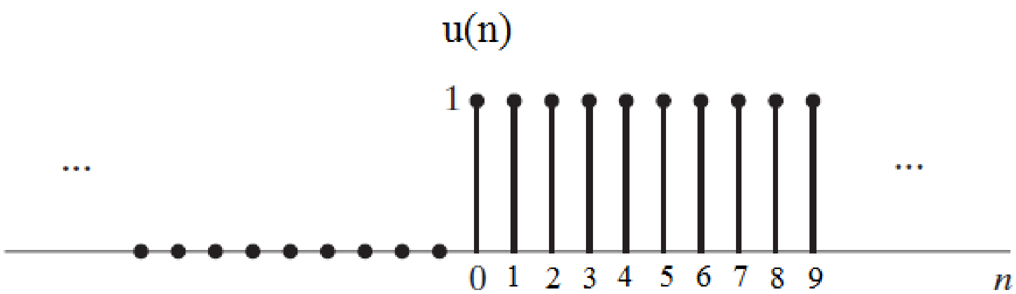
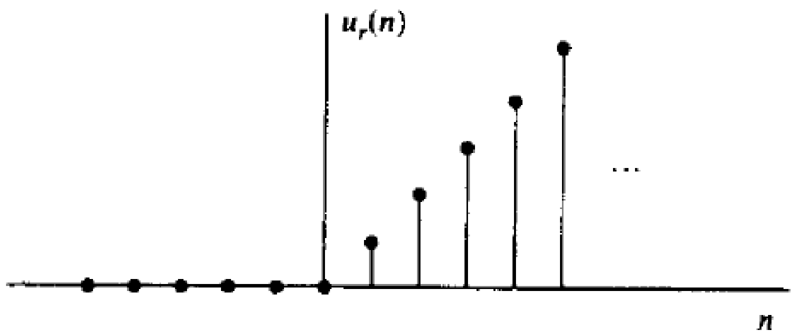


c- Ramp sequences.

It is defined by:

$$u_r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

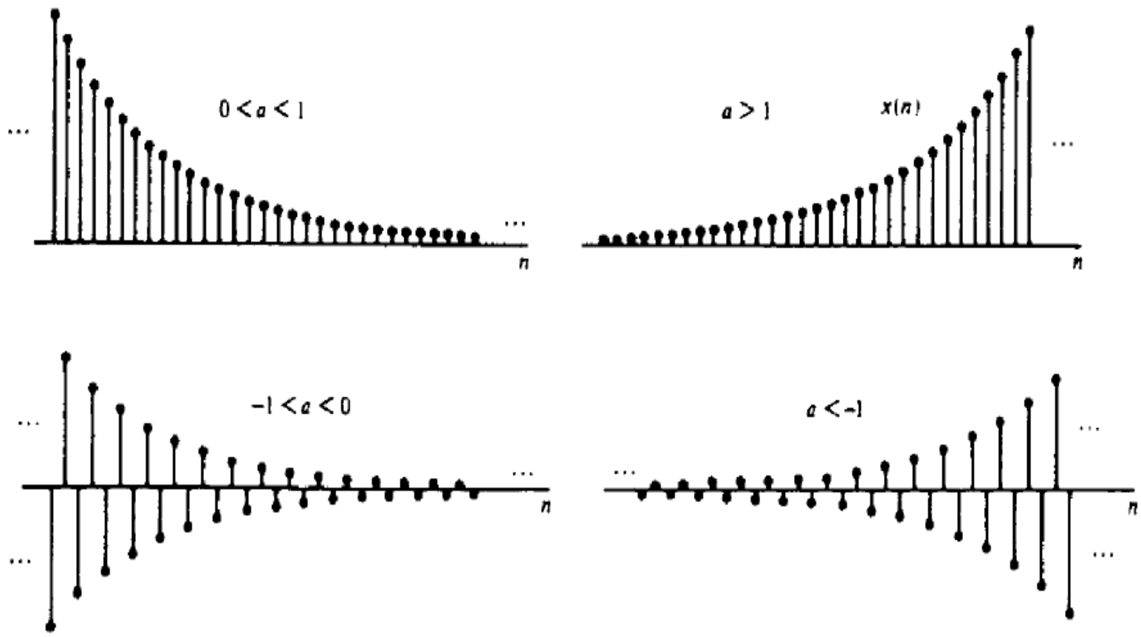


d- Exponential sequences.

Finally, an **exponential sequence** is defined by

$$x[n] = a^n \qquad for\ all\ \ n$$

where ***a*** may be real or complex number.

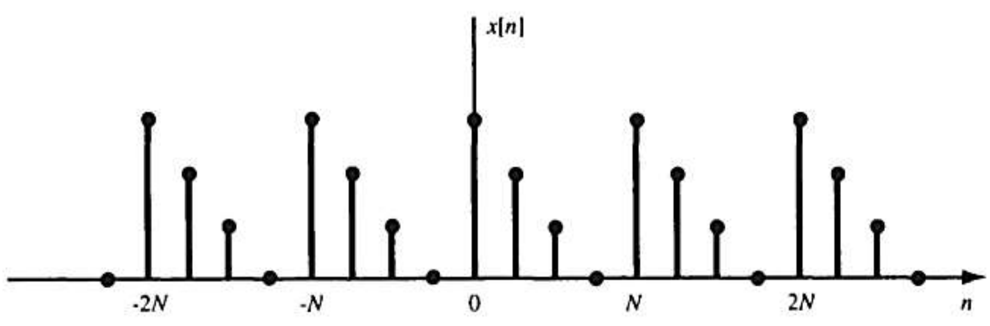


Classification of Discrete time signals

a- Periodic and Aperiodic Sequences

A discrete-time signal may always be classified as either being periodic or aperiodic. A signal x(n) is said to be periodic if, for some positive real integer N,

$$x(n) = x(n + N) \text{ for all } n.$$



This is equivalent to saying that the sequence repeats itself every N samples. If is not satisfied for any integer N, x(n) is said to be an aperiodic signal. The sinusoidal signal of the form

$$x(n) = Asin2\pi f_0 n$$

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Is periodic when f_0 is a rational number, that is, if f_0 can be expressed as

$$f_0 = \frac{k}{N}$$

Where k and N are integers

Example . *Is* $x(n) = \cos(\frac{\pi n}{8})$ *periodic? If so, what is the period?*

Sol: *The sequence can be expressed as*

$$x(n) = \cos(2\pi \frac{1}{16} n)$$

So in this case, $f_0 = 1/16$ *is a rational number and the sinusoidal sequence is periodic with a period* N = 16.

Example 2: Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period.

(a) $x[n] = e^{j(\pi/4)n}$

(b) $x[n] = \cos \frac{\pi}{3}n + \sin \frac{\pi}{4}n$

(c) $x[n] = \cos \frac{1}{4}n$

Sol:

(a) $x[n] = e^{j\left(\frac{\pi}{4}\right)n} = e^{j2\pi f_0 n} \quad \rightarrow \quad 2\pi f_0 = \frac{\pi}{4}$

Since $f_0 = 1/8$ is a rational number, x[n] is periodic, the fundimantail period is N=8

(b) $x[n] = cos\frac{\pi}{3}n + sin\frac{\pi}{4}n = x_1[n] + x_2[n]$

Where

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$$x_1[n] = cos\frac{\pi}{3}n \quad \rightarrow \quad 2\pi f_0 = \frac{\pi}{3}$$

$$x_2[n] = cos\frac{\pi}{4}n \quad \rightarrow \quad 2\pi f_0 = \frac{\pi}{4}$$

Since $f_1 = 1/6$ is a rational number, x_1[n] is periodic, the fundamental period is N_1=6, and Since $f_2 = 1/8$ is a rational number, x[n] is periodic, the fundamental period is N=8. Thus, x[n] is periodic and its fundamental period is given by the least common multiple of 6 and 8, that is N=24

(c) $x[n] = \cos\frac{1}{4}n \quad \rightarrow \quad 2\pi f_0 = \frac{1}{4}$

Since $f_0 = 1/8\pi$ is not a rational number, x[n] is aperiodic.