5.Linear second order Non homogenous equation with constant coefficient

$$a_n(x)\frac{d^n y}{dx^n}+a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}}+\ldots+a_1(x)\frac{dy}{dx}+a_0(x)y=F(x)$$

homogenous of f(x)$\neq$ 0$\therefore$ Non

For n=2 (second order first degree)

$$a_o\frac{d^2y}{dx^2}+a1\frac{dy}{dx}+a2y\neq 0(general\ form)$$

$$y_{total}=y_C+y_p$$

y_c is solution of homogenous of f(x)$=$ 0

y_p -particular soltion, there are two methods to find y_p

1.Variation parameter.(used when f(x)$=$ $variable$)

2.Undertermined coefficent.(used when f(x)$=$ $constant$)

Variation parameter:-

To find solution $a_o\frac{d^2y}{dx^2}+a\frac{dy}{dx}+by=f(x)\quad [(f(x)=variable)]$

by use the following step.

1.solve the equation of homogenous.

$$u_1(x)\ \&u_2(x)$$

2.calculated

$$D=\left|\frac{u1}{u1}\quad\frac{u2}{u2}\right|$$

$\overline{v1}\ \&\overline{v2}$

$\overline{v1}$=-u2.f(x)/D , $\overline{v2}$=u1.f(x)/D

3.Integration $\overline{v1}\ \&\overline{v2}$ *to get* $v1\ \&\ v2$

4.solution y=u1* v1+u2* v2

Example 1// Solve the equation

$$\frac{d^2y}{dx^2}-y=e^x$$

Soltion/

$(D^2-1)y=0 \rightarrow (r^2-1)=0 \rightarrow r1=1\ \ \&\ r2=-1$

1.

$y=(c_1e^{r1x}+c2\ e^{r2x}) \rightarrow y=(c1e^x+c2\ e^{-x})$

$u1=e^x\ \ \&\ \ u2=e^{-x}$

$$\overline{u1} = e^{x} * 1 \quad \& \ \overline{u2} = e^{-x} * (-1) = -e^{-x}$$

$$D = \begin{vmatrix} u1 & u2 \\ \overline{u1} & \overline{u2} \end{vmatrix} = \begin{vmatrix} e^{x} & e^{-x} \\ e^{x} & -e^{-x} \end{vmatrix} = -1-1=-2$$

3. $\overline{V1} = -U2.f(x)/D = -(e^{-x}.e^{x})/(-2) = \frac{1}{2}$

$$V1 = \int \frac{1}{2}dx = \frac{1}{2}x + c1$$

$$\overline{V2} = U1.f(x)/D = (e^{x}.e^{x})/(-2) = -\frac{1}{2}e^{2x}$$

$$V2 = \int -\frac{1}{2}e^{2x}\,dx = -\frac{1}{4}e^{2x} + c2$$

$$y = u1 * v1 + u2 * v2 = e^{x}\left(\frac{1}{2}x + c1\right) + e^{-x}\left(\frac{-1}{4}2x + c2\right)$$

4.

H.W. Solve the equation

$$\frac{d^2y}{dx^2} + y = \tan x$$

<u>6.Higher order linear equation with constant coefficent</u>

***Homogenous**

Example 1// Solve the equation

$$\frac{d^5y}{dx^5} - 3\frac{d^4y}{dx^4} + 3\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} = 0$$

Solution/

$$(D^5 - 3D^4 + 3D^3 - D^2) = 0$$

$$D^2(D^3 - 3D^2 + 3D - 1) = 0$$

$$r^2(r^3 - 3r^2 + 3r - 1) = 0$$

$$\text{either} \quad r^2 = 0 \rightarrow r1 = r2 = 0$$

$$\text{or} \quad (r^3 - 3r^2 + 3r - 1) = 0$$

$$(r-1)^3 = 0 \rightarrow r3 = r4 = r5 = 1$$

$$y = (c_1e^{rx} + c2\,xe^{rx} + c3e^{rx} + c4xe^{rx} + c5x^2e^{rx}) \rightarrow$$

$$y = (c_1e^{0x} + c2\,xe^{0x} + c3e^{1x} + c4xe^{1x} + c5x^2e^{1x})$$

H.W. Solve the equation

$$\frac{d^3y}{dx^3} + 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + y = 0$$