Chain rule:

1-If Fx&Fy are contain &w=F(x,y),y&x are function of (t) only.

$$\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial t}$$

2-If W is a function of (x,y&y) in the same time

x=F(r,s),y=F(r,s)&z=F(r,s)then

$$\frac{\partial w}{\partial r}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial r}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial s}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial s}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

Example1: Find the derivative of w=3xy if x=cos t &y=sin t

Solution//

$$\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial t}$$

$$\frac{dw}{dy} = 3x. \qquad \frac{\partial w}{\partial x} = 3y$$

$$\frac{\partial x}{\partial t} = -sin\ t. \qquad \frac{\partial y}{\partial t} = cos\ t$$

$$\frac{\partial w}{\partial \mathrm{t}} = 3y * (-sin\ t) + 3x * cos\ t$$

$$\frac{\partial w}{\partial \mathrm{t}} = -3 * sin^2 t + 3cos^2 t$$

Example2: Find the derivative of w=xy+z if x=cos t ,y=sin t &z=2t^2

Solution//

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}.\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}.\frac{\partial z}{\partial t}$$

=y.(-sin t)+x*cos t+1*4t

= - $sin^2 t$ +$cos^2 t$ +4t

H.W. Find $\frac{\partial w}{\partial t}$ $in\ terms\ of\ w = x^2 + y^2. x = cos\ t. y = sint$

Example 3: Find $\frac{\partial w}{\partial t}$ .w=2ye^x-ln z , x=ln($t^2 + 1$) ,y=$\tan^{-1} t$ &z=e^t

Solution//

$$\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial t}+\frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial t}=2y\,e^{x}\cdot\frac{2t}{t^2+1}+2e^{x}\cdot\frac{1}{t^2+1}+\frac{-1}{z}\cdot e^{t}$$

$$\frac{\partial w}{\partial t}=2\tan^{-1}t\quad e^{\ln(t^2+1)}\cdot\frac{2t}{t^2+1}+2e^{\ln(t^2+1)}\cdot\frac{1}{t^2+1}-\frac{1}{e^{t}}\cdot e^{t}$$

Example 4: Find
$\frac{\partial w}{\partial r}$ & $\frac{\partial w}{\partial s}$ *. in terms of r&s if* **w=x+4y+z^2 , x=$\frac{r}{s}$,y=$2r^2+\ln 2s$ &z=2r**

Solution//

$$\frac{\partial w}{\partial r}=\frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial r}+\frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial r}=\frac{1}{s}+16r+8r=\frac{1}{s}+24\,r$$

$$\frac{\partial w}{\partial s}=\frac{\partial w}{\partial x}\cdot\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}\cdot\frac{\partial y}{\partial s}+\frac{\partial w}{\partial z}\cdot\frac{\partial z}{\partial s}$$

$$\frac{\partial w}{\partial s} = \frac{-1}{s^2} + \frac{4}{s}$$

Example 5: Find $\frac{\partial w}{\partial r}$ & $\frac{\partial w}{\partial s}$ $.\,in\ terms\ of\ r\&s\ if$ w=x^2 +y^2 , x=r + s , y=$r - s$

Solution//

$$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x} . \frac{\partial x}{\partial r} + \frac{\partial w}{\partial y} . \frac{\partial y}{\partial r}$$

$$\frac{\partial w}{\partial r} = 2x.1 + 2y = 4r$$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} . \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} . \frac{\partial y}{\partial s}$$

$$\frac{\partial w}{\partial s} = 2x.1 + 2y.(-1) = 2(r+s) - 2(r-s) = 4s$$