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. Image Histogram

A histogram is a graph that shows the frequency of anything.

A histogram is an accurate representation of the distribution of numerical data. It is an estimate of the probability distribution of a continuous variable (quantitative variable). A histogram is a graph showing the number of pixels in an image at each different intensity value found in that image. It's a bar chart of the count of pixels of every tone of gray that occurs in the image.

For an 8-bit grayscale image there are 256 different possible intensities, and so the histogram will graphically display 256 numbers showing the distribution of pixels amongst those grayscale values.
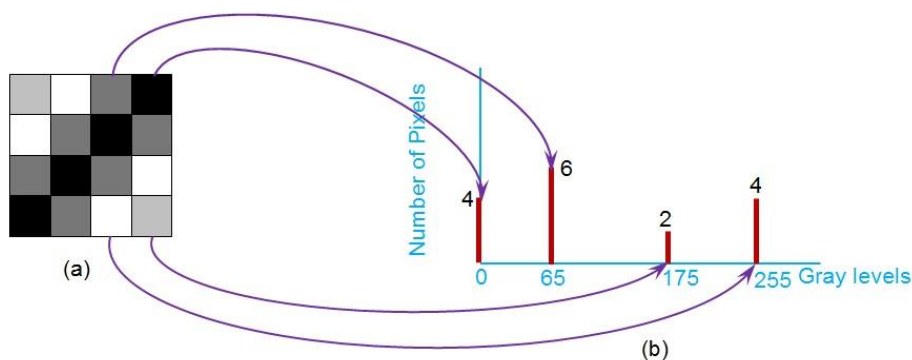
The gray level *histogram* is showing, the gray level, for each pixel in the image.

> The histogram of an image records the frequency distribution of gray levels in the image.

The histogram of an 8-bit image, can be though of as a table with 256 entries, or " bins", indexed from 0 to 255. in bin 0 we record the number of times a gray level of 0 occurs; in bin 1 we record the number

of times a gray level of 1 occurs, and so on, up to bin 255.

An algorithm below shows how we can accumulate in a histogram from an image.

ALGORITHM: for Calculating image Histogram

create an array histogram.

For all gray levels ,I,do

Histogram [I] =0

Endfor

For all pixels coordinates, x and y , do

Increment **Histogram**[f (x,y)] by 1

Endfor\

- The histogram of a digital image with *L* total possible intensity levels in the range [*0, G*] is defined as the discrete function

$$h(r_k) = n_k$$

- Where r_k is the *k*th intensity level in the interval [*0,G*] and n_k is the number of pixels in the image whose intensity level is r_k.

Example: Figure shows an image and its histogram.

Pixel Value	Count
0	2
1	5
2	3
3	4
4	5
5	2
6	4
Total	25
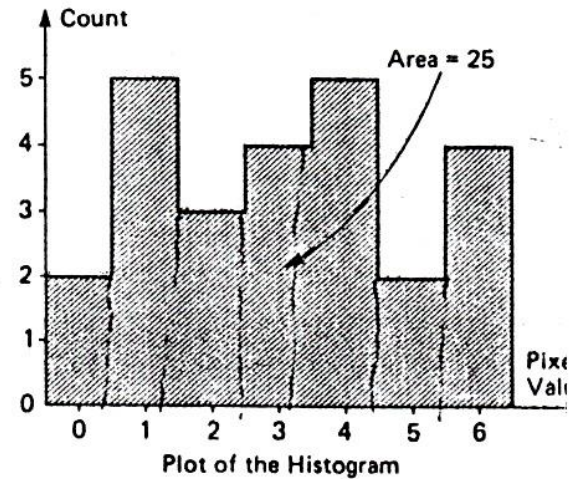
Image (a)   Histogram (b)   Plot of the Histogram (c)

Figure: sub image and its histogram

The shape of the histogram provides us with information about the nature of the image, or sub image if we are considering an object in the image. For example, a **very narrow** histogram implies a low contrast, a histogram **skewed toward the right** implies a bright image, a histogram **skewed toward the left** implies a dark image, and a histogram with **two major peaks**, implies an object that in contrast with the background.

A color histogram counts pixels with a given pixel value in red, green, and blue (RGB). For example, in pseudocode, for images with 8-bit values in each of R, G, B, we can fill a histogram that has 256^3 bins:

```
inthist[256][256][256]; // reset to 0
//image is an appropriate struct
//with byte fields red,green,blue
fori=0..(MAX_Y-1)
for j=0..(MAX_X-1)
{
R = image[i][j].red;
G = image[i][j].green;
B = image[i][j].blue;
hist[R][G][B]++;
}
```

Example : Plot the Histogram of the following example with 4x4 matrix of a 3-bit image.
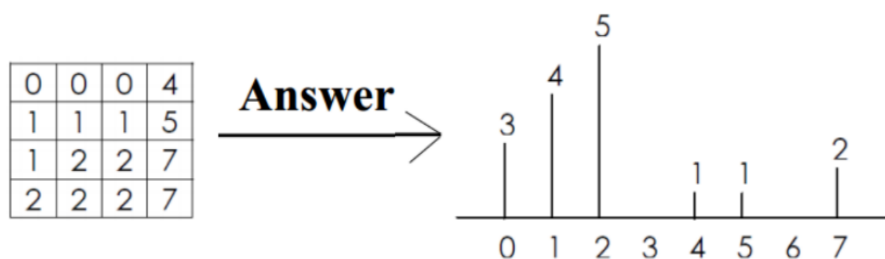

Properties and usage of histogram

One of the ***principle use*** of the histogram is in the selection of ***threshold*** parameter.

The histogram of an image provides a useful indication of the relative importance of different gray levels in an image, indeed, it is sometimes possible to ***determine*** whether ***brightnes***s or ***contras***t ***adjustment*** is necessary merely by ***examining*** the ***histogram*** and ***not the image*** itself.

When an image is condensed into a histogram, *all spatialinformation is discarded.* The histogram specifies the number of pixelshaving each gray level but *gives no hint* as to *where those pixels arelocated* within the image. Thus the histogram is *unique* for any particularimage, but the *reverse is not true*. Vastly different images could have identical histograms. Such operations as moving objects around within an image typically have no effect on the histogram.

Histograms are used in numerous image processing techniques, such as image enhancement, compression and segmentation.
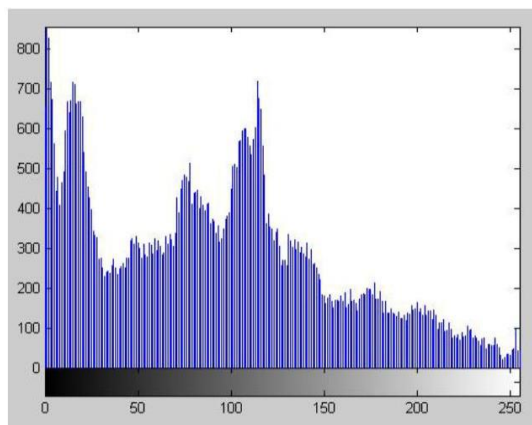
Note that the horizontal axis of the histogram plot (Figure (b) above) represents gray level values, k, from 0 to 255. The vertical axis represents the values of $h(k)$ i.e. the number of pixels which have the gray level k.

It is customary to **"normalize"** a histogram by dividing each of its values by the total number of pixels in the image, i.e. uses the probability distribution (previously stated) as

$$p(k) = \frac{h(k)}{M \times N}$$

Thus, $p(k)$ represents the probability of occurrence of gray level k.

As with any probability distribution:

- ✚ *All the values of a normalized histogram $p(k)$ are less than or equal to 1.*
- ✚ *The sum of all $p(k)$ values is equal to 1*

Another way of getting histogram is to plot pixel intensities vs. pixel probabilities. However, probability histogram should be used when comparing the histograms of images with different sizes.

Example: Suppose that a 3-bit image (L = 8) of size 64 × 64 pixels has the gray level (intensity) distribution shown in the table below.*Perform normalized histogram.*

r_k	n_k
$r_0 = 0$	790
$r_1 = 1$	1023
$r_2 = 2$	850
$r_3 = 3$	656
$r_4 = 4$	329
$r_5 = 5$	245
$r_6 = 6$	122
$r_7 = 7$	81

Solution:*M × N = 4096 .*   We compute the normalized histogram:

r_k	n_k	$p(r_k) = n_k/MN$
$r_0 = 0$	790	0.19
$r_1 = 1$	1023	0.25
$r_2 = 2$	850	0.21
$r_3 = 3$	656	0.16
$r_4 = 4$	329	0.08
$r_5 = 5$	245	0.06
$r_6 = 6$	122	0.03
$r_7 = 7$	81	0.02
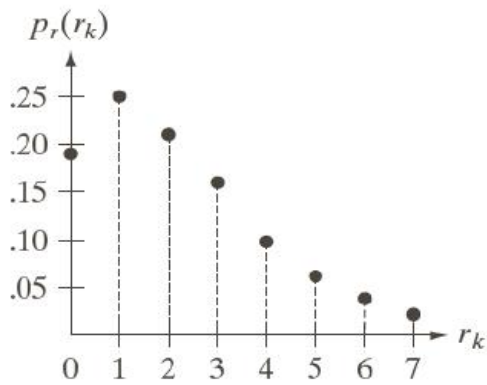
Normalized histogram

An alternate perspective to gray-level modification that performs a similar function is referred to as histogram modification. The gray-level histogram of an image is the distribution of the gray levels in an image. In figure 11 we can see an image and its corresponding histogram. In general a histogram with a **small-spread** has a **low-contrast**. And a histogram with a **wide spread** has a **high contrast**, whereas an image with its histogram clustered at the **low end** of the range is **dark**, and a histogram with the values clustered at the **high end** of the range corresponds to a **bright** image.
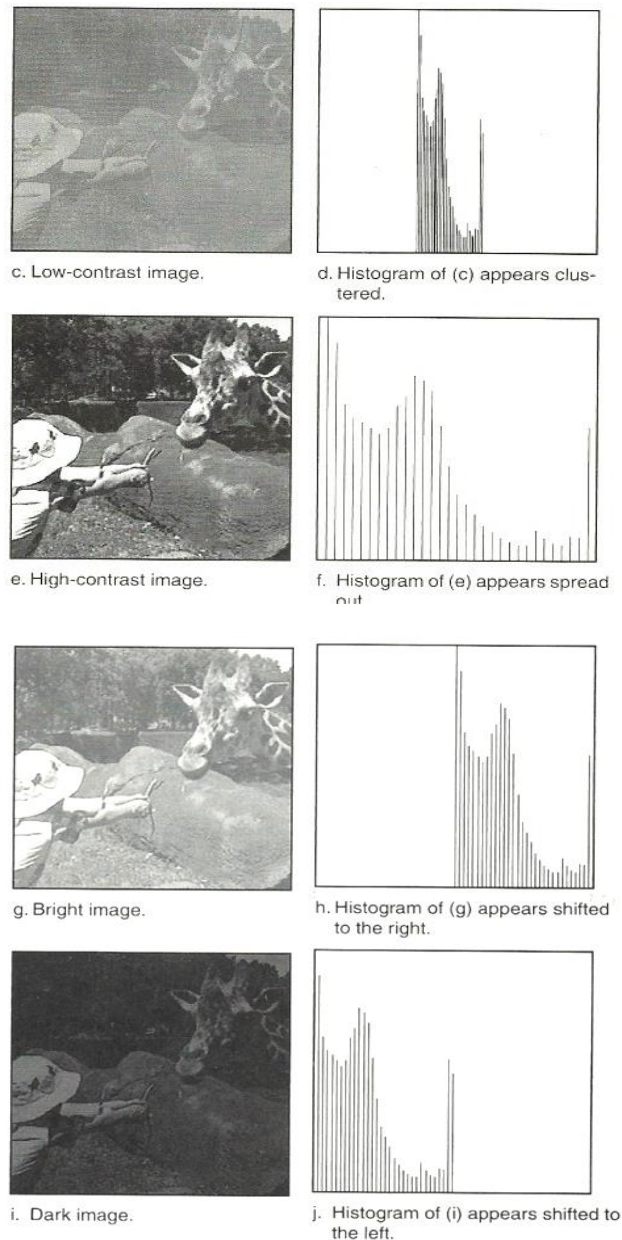
c. Low-contrast image.

d. Histogram of (c) appears clustered.

e. High-contrast image.

f. Histogram of (e) appears spread out

g. Bright image.

h. Histogram of (g) appears shifted to the right.

i. Dark image.

j. Histogram of (i) appears shifted to the left.

Figure 11: a variety types of histograms

The histogram can also be modified by a **mapping function**, which will either **stretch, shrink** (compress), or **slide** the histogram. Histogram stretching and histogram shrinking are forming a gray-level modification, sometimes referred to as histogram scaling. In figure **12** we see a graphical representation of histogram stretch, shrink, and slide.
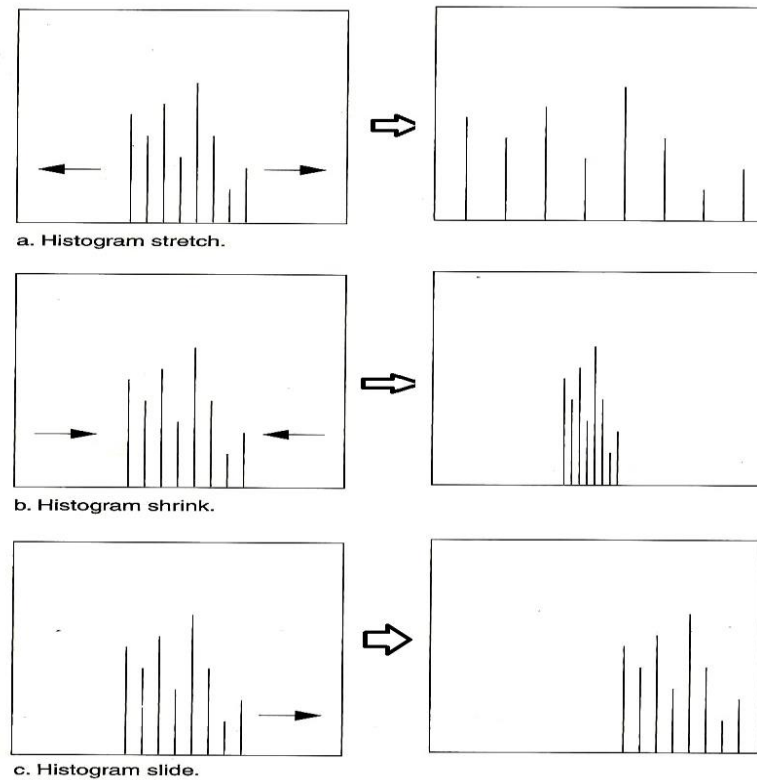
Figure 12: histogram modification

a. Histogram stretch

The mapping function for histogram stretch can be found by the equation :

$$\text{Stretch}(I(r, c)) = \left[\frac{I(r, c) - I(r, c)_{MIN}}{I(r, c)_{MAX} - I(r, c)_{MIN}} \right] [MAX - MIN] + MIN$$

Where :

- *I(r,c)$_{MAX}$ is the largest gray-level value in the image I(r,c)*
- *I(r,c)$_{MIN}$ is the smallest gray-level value in the image I(r,c)*
- *MAX and MIN correspond to the maximum and minimum gray-level values possible (for 8-bitimages these are 0 and 255).*

This equation will take an image and stretch the histogram across

the entire gray-level range, which has the effect of increasing the contrast of a low contrast image . ***If a stretch is desired over a smaller range, differentMAX and MIN values can be specified.***

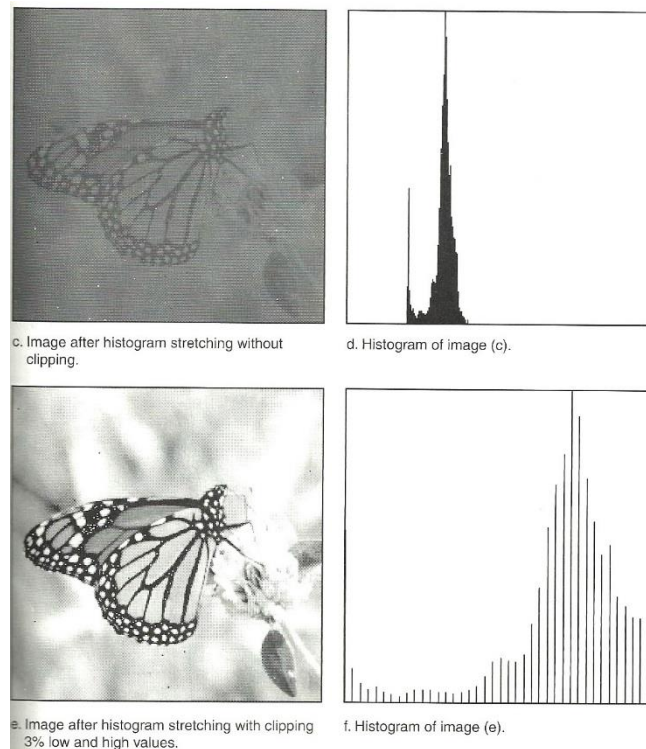In general, histogram stretch will ***increase image contrast***.

c. Image after histogram stretching without clipping.

d. Histogram of image (c).

e. Image after histogram stretching with clipping 3% low and high values.

f. Histogram of image (e).

Figure 13: histogram stretch

b. <u>Histogram shrink</u>

The opposite of a histogram stretch is a histogram shrink, which will decrease image contrast by compressing the gray levels. The mapping function for a histogram shrink can be found by the following equation:

$$\text{Shrink}(I(r, c)) = \left[\frac{\text{Shrink}_{MAX} - \text{Shrink}_{MIN}}{I(r, c)_{MAX} - I(r, c)_{MIN}} \right] \left[I(r, c) - I(r, c)_{MIN} \right] + \text{Shrink}_{MIN}$$

Where :$I(r,c)_{MAX}$is the largest gray-level value in the image I(r,c)

$I(r,c)_{MIN}$ is the smallest gray-level value in the image I(r,c)

Shrink$_{MAX}$ and **Shrink$_{MIN}$** correspond to the maximum and minimum gray-level values *desired* in the

compressed histogram.

a. Original image.

b. Histogram of image (a).

c. Image after shrinking the histogram to the range [75,175].
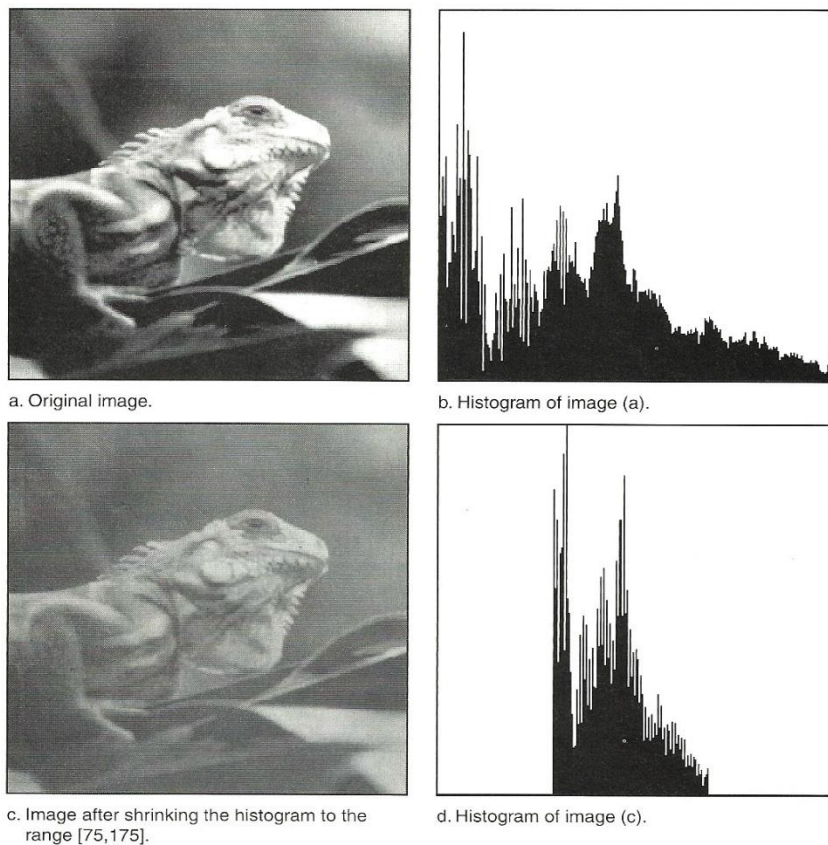
d. Histogram of image (c).

Figure 14: histogram shrink

c.   Histogram slide

The histogram slide technique can be used to make an image either *darker* or *lighter* but retain the relationship between gray-levels values. Thiscan be accomplished by simply **adding** or **subtracting** a fixed number from all the gray level values as follow:

$$\boxed{\text{Slide}(I(r,c)) = I(r,c) + \text{OFFSET}}$$

Where OFFSET value is the amount to slide the histogram**.**

In this equation, we assume that any values slide past the minimum and maximum value will be clipped to the respective minimum or maximum. Aposative OFFSET value will increase the overall brightness, whereas a negative OFFSET will create a darker image. Figure 15 shows a dark image that has been brightened by a histogram slide with a positive OFFSET value.

a. Original image.

b. Histogram of original image.

c. Image after positive-value histogram sliding.

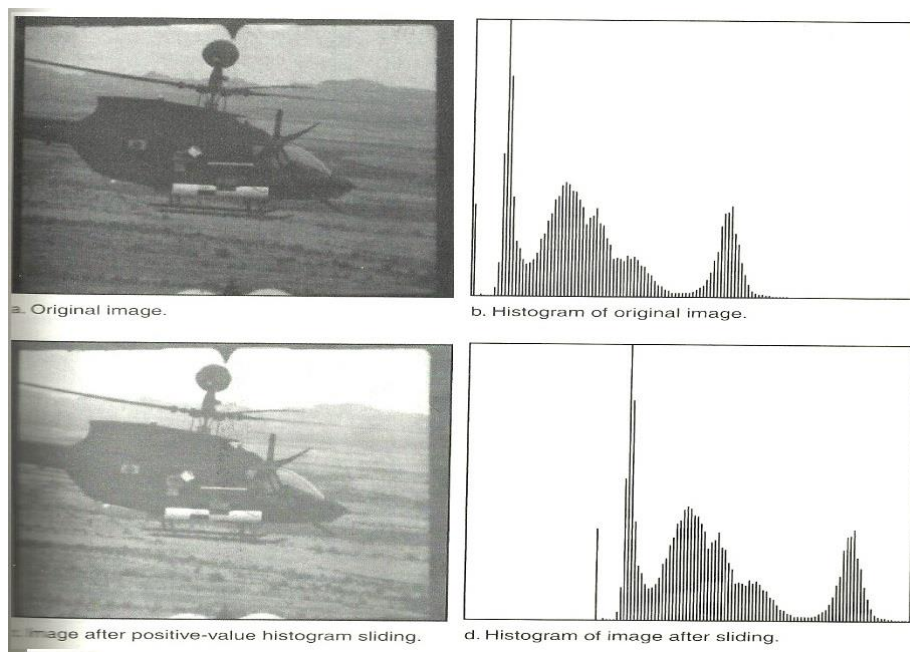d. Histogram of image after sliding.

Figure 15: histogram slide

Example: Apply histogram stretching for the following sub image :

$$\begin{bmatrix} 7 & 12 & 8 \\ 20 & 9 & 6 \\ 10 & 15 & 1 \end{bmatrix}$$

Where: **Max =255 ;**   **Min =0**

Solution:

$$St(r,c) = \left[\frac{I(r,c) - I(r,c)_{min}}{I(r,c)_{max} - I(r,c)_{min}}\right] (Max - Min) + Min$$

I(r,c)$_{min}$ = **1** ;   **I(r,c)**$_{max}$ = **20** ;   **Max** = **255** ;
           Min = **0**

$I_{(0,0)}$ = [7-1 / 20-1] * [255 − 0] + 0 = 80.5
$I_{(0,1)}$ = [12-1 / 20-1] * [255 − 0] + 0 = 147.6
$I_{(0,2)}$ = [8-1 / 20-1] * [255 − 0] + 0 = 93.9
$I_{(1,0)}$ = [20-1 / 20-1] * [255 − 0] + 0 = 255
$I_{(1,1)}$ = [9-1 / 20-1] * [255 − 0] = 107.3
$I_{(1,2)}$ = [6-1 / 20-1] * [255 − 0] + 0 = 67.1
$I_{(2,0)}$ = [10-1 / 20-1] * [255 − 0] + 0 = 120.7
$I_{(2,1)}$ = [15-1 / 20-1] * [255 − 0] + 0 = 187.8
$I_{(2,1)}$ = [1-1 / 20-1] * [255 − 0] + 0 = 0

$$\begin{bmatrix} 80 & 147 & 93 \\ 255 & 107 & 67 \\ 120 & 187 & 0 \end{bmatrix}$$

Example: Apply histogram shrink for the following sub image :

$$\begin{bmatrix} 70 & 120 & 80 \\ 200 & 90 & 60 \\ 100 & 150 & 10 \end{bmatrix}$$

   Where: **Shrink** $_{max}$ = **100** ;   **shrink** $_{min}$ = **20**

Solution:

$$\text{Shrink}(I(r,c)) = \left[\frac{\text{Shrink}_{MAX} - \text{Shrink}_{MIN}}{I(r,c)_{MAX} - I(r,c)_{MIN}}\right]\left[I(r,c) - I(r,c)_{MIN}\right] + \text{Shrink}_{MIN}$$

I(r,c)$_{min}$ = **10** ;   **I(r,c)**$_{max}$ = **200** ;   **Shrink** $_{max}$ = **100** ;   **shrink**
min = **20**

$I_{(0,0)}$ = [100-20 / 200-10] * [**70** − 10] + 20 = 45.2
$I_{(0,1)}$ = [100-20 / 200-10] * [**120** − 10] + 20 = 66.3

$I_{(0,2)}$ = [100-20 / 200-10] * [**80** – 10] + 20= 49.4

$I_{(1,0)}$ = [100-20 / 200-10] * [**200**– 10] + 20= 100

$I_{(1,1)}$ = [100-20 / 200-10] * [**90** – 10] + 20=53.68

$I_{(1,2)}$ = [100-20 / 200-10] * [**60** – 10] + 20= 41.05

$I_{(2,0)}$ = [100-20 / 200-10] * [**100** – 10] + 20= 57.89

$I_{(2,1)}$ = [100-20 / 200-10] * [**150** – 10] + 20= 78.94

$I_{(2,2)}$ = [100-20 / 200-10] * [**10** – 10] + 20= 20

$$\begin{bmatrix} 45 & 66 & 49 \\ 100 & 53 & 41 \\ 57 & 78 & 20 \end{bmatrix}$$

Example: Apply histogram slide for the following sub image, where OFFSET= 10 :

$$\begin{bmatrix} 7 & 12 & 8 \\ 20 & 9 & 6 \\ 10 & 15 & 1 \end{bmatrix}$$

Solution:

$$Slide(\ I(r,c)\ ) = I(r,c) + OFFSET$$

$$\begin{bmatrix} 17 & 22 & 18 \\ 30 & 19 & 16 \\ 20 & 25 & 11 \end{bmatrix}$$

Histogram equalization

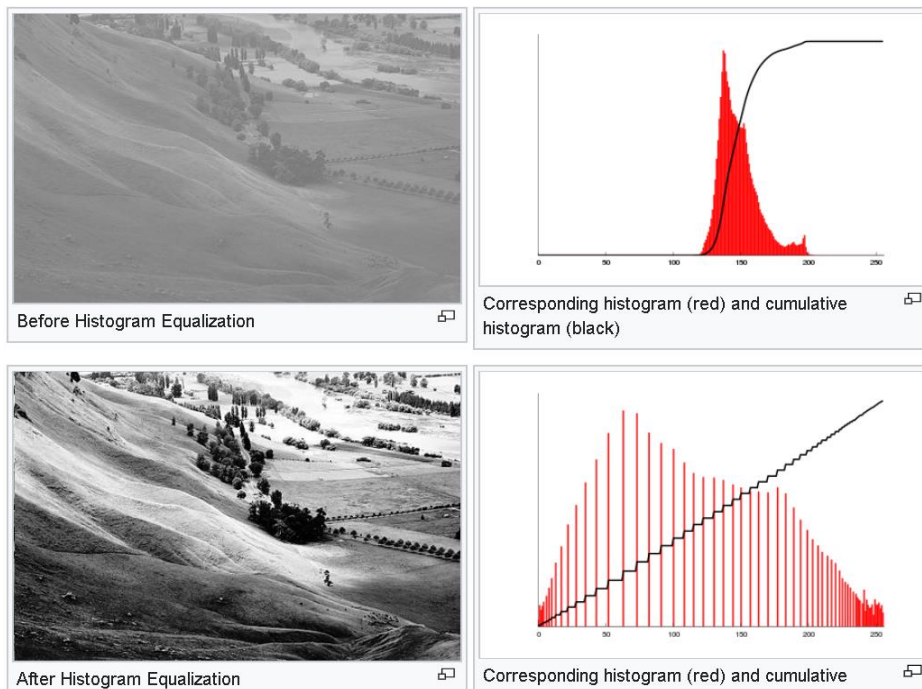**Histogram equalization**: is a technique for adjusting image intensities to enhance **contrast**.

Histogram equalization often produces unrealistic effects in photographs; however, it is very useful for scientific images like thermal, satellite or x-ray images.

To find the histogram equalization must follow:

1- Count the total number of pixels associated with each pixel intensity.

2- Cumulative distribution function (CDF)

3- Calculate as transformation function

$$h(v) = \text{round}\left(\frac{cdf(v) - cdf_{min}}{(M \times N) - 1} \times (L - 1)\right)$$

- cdf_{min} is the minimum non-zero value of the cumulative distribution function.
- $M*N$ gives the image's number of pixels.
- L is the number of grey levels used (in most cases 256)

Before Histogram Equalization

Corresponding histogram (red) and cumulative histogram (black)

After Histogram Equalization

Corresponding histogram (red) and cumulative

Example:

Apply histogram equalization for the following sub image, where image is gray scale :

$$\begin{bmatrix} 50 & 55 & 150 & 150 \\ 51 & 50 & 55 & 55 \\ 70 & 80 & 90 & 100 \\ 50 & 55 & 70 & 80 \end{bmatrix}$$

Solution:

$$h(v) = \text{round}\left(\frac{cdf(v) - cdf_{min}}{(M \times N) - 1} \times (L - 1)\right)$$

M*N = 4 * 4 = **16** **L** = 256 (because its gray scale)

cdf$_{min}$ = **3**

$h(50) = round\left(\frac{3-3}{16-1}\right) * (255 - 1) = 0$

$h(51) = round\left(\frac{4-3}{16-1}\right) * (255 - 1) = 17$

$h(55) = round\left(\frac{8-3}{16-1}\right) * (255 - 1) = 85$

$h(70) = round\left(\frac{10-3}{16-1}\right) * (255 - 1) = 119$

$h(80) = round\left(\frac{12-3}{16-1}\right) * (255 - 1) = 153$

$h(90) = round\left(\frac{13-3}{16-1}\right) * (255 - 1) = 170$

$h(100) = round\left(\frac{14-3}{16-1}\right) * (255 - 1) = 187$

$h(150) = round\left(\frac{16-3}{16-1}\right) * (255 - 1) = 221$

Pixel Intensity	Count	Cdf_r	H(r)
50	3	3	0
51	1	4	17
55	4	8	85
70	2	10	119
80	2	12	153
90	1	13	170
100	1	14	187
150	2	16	221

$$\begin{bmatrix} 0 & 85 & 221 & 221 \\ 17 & 0 & 85 & 85 \\ 119 & 153 & 170 & 187 \\ 0 & 85 & 119 & 153 \end{bmatrix}$$