

**2) Homogenous Equation (first order first degree)**

If  $f(tx, ty) = t^n(f(x, y))$  is Homogenous

$t > 0$  & real number

EX//  $f(x, y) = x^2 + 3xy + 4y^2$  (test)

$$f(tx, ty) = t^n(f(x, y)) \rightarrow f(tx, ty) = t^2x^2 + 3t^2xy + 4t^2y^2$$

$$= t^2(x^2 + 3xy + 4y^2) = t^2 * f(x, y) \text{ is Homogenous}$$

\*  $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$  to be Homogenous

Assume  $u = \frac{y}{x} \rightarrow y = ux \rightarrow \frac{dy}{dx} = u + \frac{du}{dx}$  Sub in \*

$$\frac{dy}{dx} = f(u) \rightarrow u + x \frac{du}{dx} = f(u)$$

$$\left( \int \frac{dx}{x} + \int \frac{du}{u - f(u)} = 0 \right) \text{ Homogenous}$$

Example// Solve  $(x^2 + y^2)dx + 2xydy = 0$

Solution/

test

$$(x^2 + y^2)dx + 2xydy = 0 \rightarrow \frac{dy}{dx} = \frac{-(x^2 + y^2)}{2xy} =$$

$$\frac{-(t^2x^2 + t^2y^2)}{2txty} = \frac{-t^2(x^2 + y^2)}{t^2 2xy} \text{ is Homogenous}$$

Where  $u = \frac{y}{x}$ .  $\frac{dy}{dx} = \frac{-(x^2+y^2)}{2xy} \div x^2$

$$\frac{dy}{dx} = \frac{-(1 + (\frac{y}{x})^2)}{2\frac{y}{x}} = f\left(\frac{y}{x}\right) = f(u)$$

$$f(u) = \frac{-(1 + u^2)}{2u}$$

$$= \left( \int \frac{dx}{x} + \int \frac{du}{u-f(u)} = 0 \right) = \left( \int \frac{dx}{x} + \int \frac{du}{u - \frac{-(1+u^2)}{2u}} = 0 \right)$$

$$= \left( \int \frac{dx}{x} + \int \frac{2udu}{1 + 3u^2} = 0 \right) \rightarrow \ln|x| + \frac{1}{3} \ln|1 + 3u^2| = c$$

$$\rightarrow \ln|x| + \frac{1}{3} \ln \left| 1 + 3\left(\frac{y}{x}\right)^2 \right| = c$$

Example// Solve  $\frac{dy}{dx} = \frac{x-y}{x+y}$

Solution //

test

$$\frac{dy}{dx} = \frac{x-y}{x+y} = \frac{tx-ty}{tx+ty} = \frac{t(x-y)}{t(x+y)} \text{ is Homogenous}$$

Where  $u = \frac{y}{x}$ .  $\frac{dy}{dx} = \frac{x-y}{x+y} \div x$

$$\frac{dy}{dx} = \frac{1 - \frac{y}{x}}{1 + \frac{y}{x}} = f\left(\frac{y}{x}\right) = f(u)$$

$$f(u) = \frac{1 - u}{1 + u}$$

$$= \left( \int \frac{dx}{x} + \int \frac{du}{u - f(u)} = 0 \right) = \left( \int \frac{dx}{x} + \int \frac{du}{u - \frac{1-u}{1+u}} = 0 \right)$$

$$= \left( \int \frac{dx}{x} + \int \frac{(1+u)du}{u^2 + 2u - 1} = 0 \right)$$

$$\rightarrow \ln|x| + \frac{1}{2} \ln|u^2 + 2u - 1| = c$$

$$\rightarrow \ln|x| + \frac{1}{2} \ln \left| \frac{y^2}{x^2} + 2\frac{y}{x} - 1 \right| = c$$

**Example // Solve**  $\left(xe^{\frac{y}{x}} + y\right) dx - x dy = 0$

**Solution//**

**test**

$$\rightarrow \frac{dy}{dx} = \frac{(xe^{\frac{y}{x}} + y)}{x} = \left(xe^{\frac{y}{x}} + y\right) dx - x dy = 0$$

$$\frac{(txe^{\frac{ty}{tx}} + ty)}{tx} = \frac{t(xe^{\frac{y}{x}} + y)}{t(x)} \text{ is Homogenous}$$

$$\text{Where } u = \frac{y}{x}, \quad \frac{dy}{dx} = \frac{(xe^{\frac{y}{x}} + y)}{x} \quad \div x$$

$$\frac{dy}{dx} = e^{\frac{y}{x}} + \frac{y}{x} = f\left(\frac{y}{x}\right) = f(u)$$

$$f(u) = e^u + u$$

$$= \left( \int \frac{dx}{x} + \int \frac{du}{u - f(u)} = 0 \right) = \left( \int \frac{dx}{x} + \int \frac{du}{u - (e^u + u)} = 0 \right)$$

$$= \left( \int \frac{dx}{x} + \int -e^{-u} du = 0 \right) \rightarrow \ln|x| + e^{-u} = c$$

$$\rightarrow \ln|x| + e^{-\frac{y}{x}} = c$$