General Physics

Radiology Techniques Department

1st Class

Al-Mustaqbal University college

Lecture 2: Coordinate Systems and Vectors

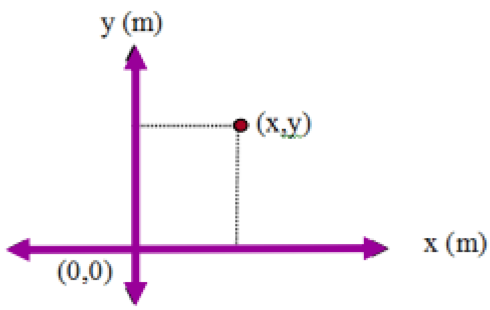
2020/2021

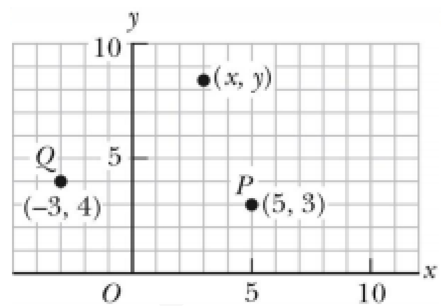
Coordinate Systems

Used to describe the position of a point in space

Cartesian Coordinate System

In Cartesian (Also called rectangular) coordinate system: x and y- axes intersect at the origin Points are labeled (x,y)

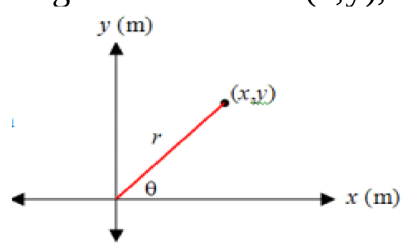




- The x- and y-coordinates may be either positive or negative

Polar Coordinate System

Sometimes it is more convenient to use the polar coordinate system (r,θ), where r is the distance from the origin to the point of rectangular coordinate (x,y), and θ is the angle between r and the x axis.

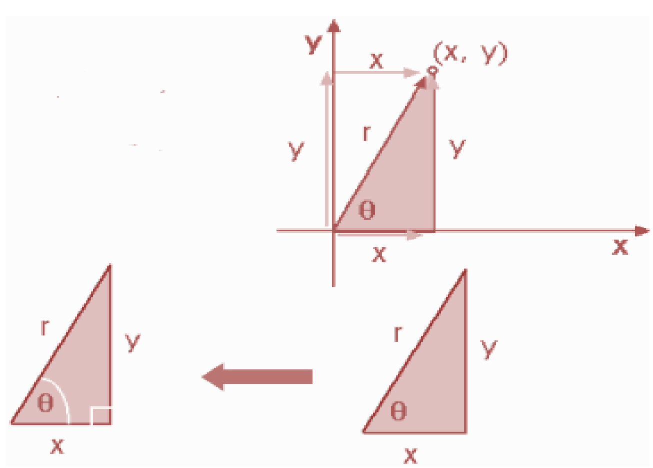


The relation between coordinates

$$\cos\theta = \frac{x}{r}$$

$$\sin\theta = \frac{y}{r}$$

$$\tan\theta = \frac{y}{x}$$



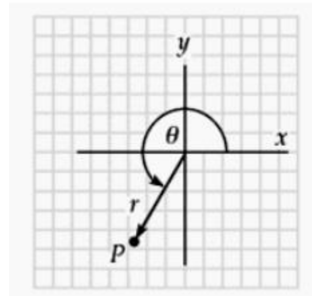
$$r = \sqrt{x^2 + y^2}$$

$$x = r\cos\theta$$

$$y = r\sin\theta$$

Example

The polar coordinates of a point are r = 5.5m and θ = 240^o. What are the Cartesian coordinates of this point?



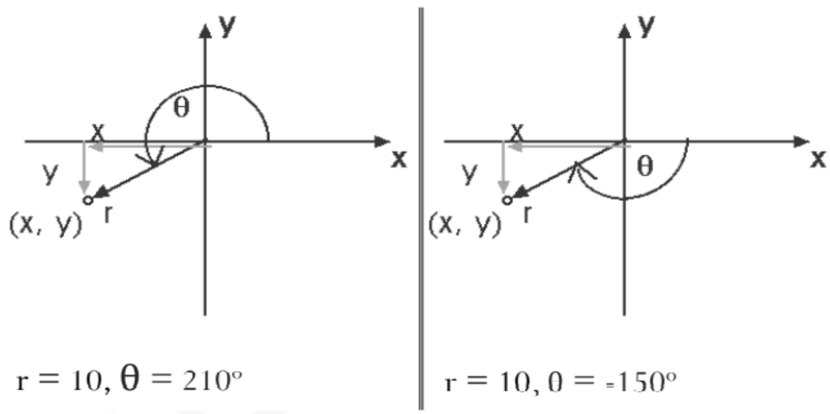
Solution

x = r cos θ = 5.5×cos 240^o = - 2.75 m

y = r sin θ = 5.5×sin 240^o = - 4.76 m

(H.W) The Cartesian coordinates of a point in the xy plane are (x,y) = (-3.50, -2.50) m, as shown in the figure. Find the polar coordinates of this point.

- It is common practice to measure the angle from the positive x-axis and to measure it positive for a counterclockwise direction.



Vectors and Scalars

Scalars: is completely specified by a single value (along with the unit) and has no direction. It may be positive or negative.

- Water freezes at a temperature of 0 o C or 32o F
- The mass of a book is 198.2 g
- The length of room is 5 m
- The car kinetic energy was 0.345 J

Vectors: A quantity that deals with magnitude and direction is called a vector quantity.

- The wind had a velocity of 25 km/h from the North
- The momentum was 1.234 kg m/s to the left

Table 1: Vector and Scalar Quantities

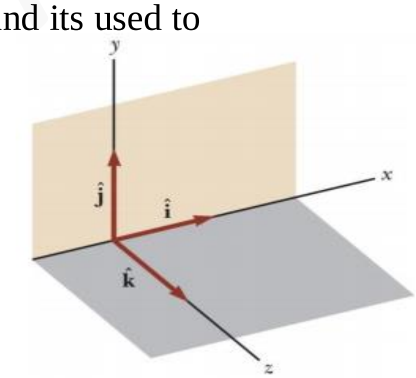
Vector Quantity	Scalar Quantity
Displacement	Length
Velocity	Mass
Force	Speed
Acceleration	Power
Field	Energy
Momentum	Work

- To denote a vector use either **A** or $\vec{A}$

Unit Vectors

A unit vector is a vector having a magnitude of unity and its used to describe a direction in space.

- i = a unit vector along the x-axis
- j = a unit vector along the y-axis
- k= a unit vector along the z-axis



Any vector lying in xy plane can be resolved into two components one in the x-direction and the other in the y-direction as shown in Figure

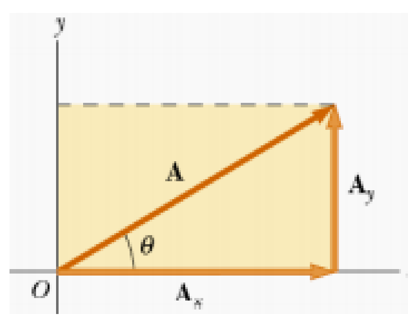
$A_x = |A| \cos\theta$

$A_y = |A| \sin\theta$

$$\mathbf{A} = A_x i + A_y j$$

The magnitude of the vector **A**

$$A \text{ or } |A| = \sqrt{A_x^2 + A_y^2}$$



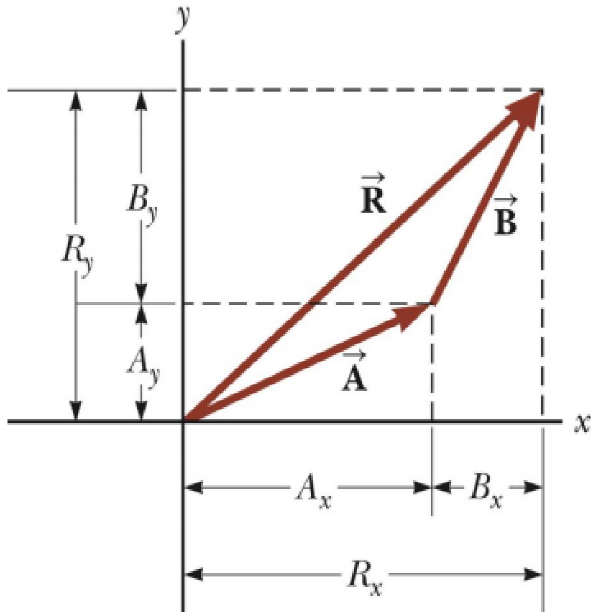
The direction of the vector to the x-axis $\tan\theta = \frac{A_y}{A_x} \rightarrow \theta = \tan^{-1}\frac{A_y}{A_x}$

Adding Vectors Using Unit Vectors

$$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$$
$$\vec{\mathbf{R}} = \left(A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}}\right) + \left(B_x\hat{\mathbf{i}} + B_y\hat{\mathbf{j}}\right)$$
$$\vec{\mathbf{R}} = \left(A_x + B_x\right)\hat{\mathbf{i}} + \left(A_y + B_y\right)\hat{\mathbf{j}}$$
$$\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}}$$

$R_x = A_x + B_x$ and $R_y = A_y + B_y$

$$R = \sqrt{R_x^2 + R_y^2} \qquad \theta = \tan^{-1}\frac{R_y}{R_x}$$



Example

Two vectors are given by $\vec{A} = 3i - 2j$ and $\vec{B} = -i - 4j$. Calculate (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$, (c) $|\vec{A} + \vec{B}|$, (d) $|\vec{A} - \vec{B}|$, and (e) the direction of $\vec{A} + \vec{B}$ and $|\vec{A} - \vec{B}|$.

(a) $\vec{A} + \vec{B} = (3i - 2j) + (-i - 4j) = 2i - 6j$

(b) $\vec{A} - \vec{B} = (3i - 2j) - (-i - 4j) = 4i + 2j$

(c) $|\vec{A} + \vec{B}| = \sqrt{2^2 + (-6)^2} = 6.32$

(d) $|\vec{A} - \vec{B}| = \sqrt{4^2 + 2^2} = 4.47$

(e) For $\vec{A} + \vec{B}$, $\theta = \tan^{-1}(-6/2) = -71.6^\circ$

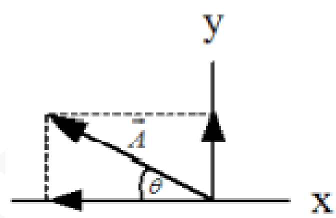
For $\vec{A} - \vec{B}$, $\theta = \tan^{-1}(2/4) = 26.6^\circ$

Example

vector A has a negative x component 3 units in length and positive y component 2 units in length.

a) Determine an expression for **A** in unit vector notation.
b) Determine the magnitude and direction of **A**.

Solution



A_x = -3 units & A_y = 2 units

(a) **A**= A_xi+ A_yj = -3i+2j units

(b) $|A| = \sqrt{A_x^2 + A_y^2} = \sqrt{(-3)^2 + (2)^2} = 3.61$

$\theta = \tan^{-1}(2/-3) = -33.7^{o}$ (relative to the –x axis)