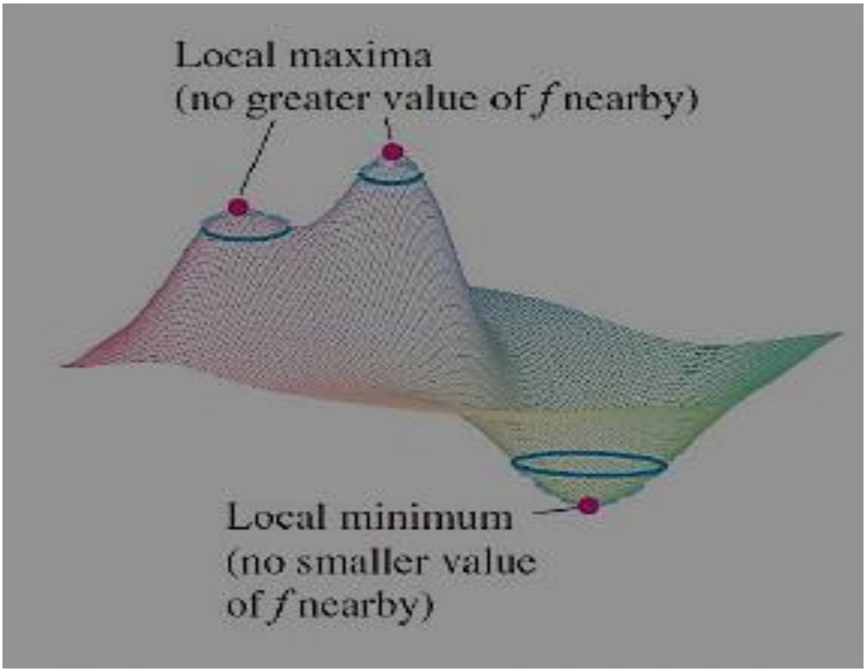
14.The extremes(max,min &saddle points):-

To find the local extreme values of a function of a single, we look for points where the graph has a horizontal tangent line. At such points, we then look for **local maxima**, **local minima**, and points of inflection. For a function $f(x,y)$ of two variables, we look for points where the surface $z=f(x,y)$ has a horizontal tangent plane. At such points, we then look for **local maxima**, **local minima**, and **saddle points** (more about saddle points

in a moment)



Then:

1. if $f_{xx} < 0$ and $f_{xx}f_{yy} - f_{xy}{}^2 > 0$ at $(a,b) \Longrightarrow$ then f has **a local maximum** at (a,b)

2. if $f_{xx} > 0$ and $f_{xx}f_{yy} - f_{xy}{}^2 > 0$ at $(a,b) \Longrightarrow$ then f has **a local minimum** at (a,b)

3. if $f_{xx}f_{yy} - f_{xy}{}^2 < 0$ at $(a,b) \Longrightarrow$ then f has **a saddle point** at (a,b)

4. if $f_{xx}f_{yy} - f_{xy}{}^2 = 0$ at $(a,b) \Longrightarrow$ then **the test inconclusive** at (a,b).
In this case we must find some other way to determine the behavior of f at (a,b)

<u>Note</u>:-

$f_x = 0$ and $f_y = 0 \Longrightarrow$ solve these equation to find the value of $(x,y) = (a,b) \Longrightarrow$ $(critical\ point)$

Example: find the extreme values of the function

$f(x,y)=xy-x^2-y^2-2x-2y+4$

Solution:

$$f_x=\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}(xy-x^2-y^2-2x-2y+4)=\boxed{y-2x-2}$$

$$f_y=\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}(xy-x^2-y^2-2x-2y+4)=\boxed{x-2y-2}$$

$$\left.\begin{aligned} f_x=0 &\Longrightarrow y-2x-2=0 \\ f_y=0 &\Longrightarrow x-2y-2=0 \end{aligned}\right\} \Longrightarrow \text{Solve these equation to find } (x,y) \Longrightarrow (a,b)$$

$$\left.\begin{aligned} x=-2 &\Longrightarrow a=-2 \\ y=-2 &\Longrightarrow b=-2 \end{aligned}\right\} \Longrightarrow \text{Critical point } (-2,-2)$$

$$f_{xx}=\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial}{\partial x}(y-2x-2)=-2$$

$$\therefore\ f_{xx}(-2,-2)=\boxed{-2}$$

$$f_{yy}=\frac{\partial^2 f}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial}{\partial y}(x-2y-2)=-2$$

$$\therefore\ f_{yy}(-2,-2)=\boxed{-2}$$

$$f_{xy}=\frac{\partial^2 f}{\partial x\partial y}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right)=\frac{\partial}{\partial x}(x-2y-2)=1$$

$$f_{xx}f_{yy}-f_{xy}^{\;2}=(-2)(-2)-(1)^2=4-1=\boxed{3}$$

$f_{xx}<0$ and $f_{xx}f_{yy}-f_{xy}^{\;2}>0 \Longrightarrow \therefore f$ has a local maximum at $(-2,-2)$

The value of f at this point is:

$$f(-2,-2)=(-2)(-2)-(-2)^2-(-2)^2-(2)(-2)-(2)(-2)+4$$
$$=4-4-4+4+4+4=\boxed{8}$$

Example: find the local maxima, local minima, and saddle point of the function

$$f(x,y)=x^2+3xy+3y^2-6x+3y-6$$

Solution:

$$f_x=\frac{\partial f}{\partial x}=\frac{\partial}{\partial x}(x^2+3xy+3y^2-6x+3y-6)=\boxed{2x+3y-6}$$

$$f_y=\frac{\partial f}{\partial y}=\frac{\partial}{\partial y}(x^2+3xy+3y^2-6x+3y-6)=\boxed{3x+6y+3}$$

$$\left.\begin{array}{l} f_x=0 \Longrightarrow 2x+3y-6=0 \\ f_y=0 \Longrightarrow 3x+6y+3=0 \end{array}\right\} \Longrightarrow$$

Solve these equation to find $(x,y) \Longrightarrow (a,b)$

$$\left.\begin{array}{l} x=15 \Longrightarrow a=15 \\ y=-8 \Longrightarrow b=-8 \end{array}\right\} \Longrightarrow \text{Critical point } (15,-8)$$

$$f_{xx}=\frac{\partial^2 f}{\partial x^2}=\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right)=\frac{\partial}{\partial x}(2x+3y-6)=2$$

$$\therefore\ f_{xx}(15,-8) = \boxed{2}$$

$$f_{yy} = \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial y}(3x + 6y + 3) = 6$$

$$\therefore\ f_{yy}(15,-8) = \boxed{6}$$

$$f_{xy} = \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial}{\partial x}(3x + 6y + 3) = 3$$

$$f_{xx} f_{yy} - f_{xy}{}^2 = (2)(6) - (3)^2 = 12 - 9 = \boxed{3}$$

$f_{xx} > 0$ and $f_{xx} f_{yy} - f_{xy}{}^2 > 0 \Longrightarrow \therefore f$ has a local minimum at $(15,-8)$

The value of f at this point is:

$$f(15,-8) = (15)^2 + (3(15)(-8) + (3)(-8)^2 - (6)(15) + (3)(-8) - 6$$

$$= 225 - 360 + 192 - 90 - 24 - 6 = \boxed{-63}$$