

4) Exact (first order first degree)**The general form**

$$M(x, y)dx + N(x, y)dy = 0$$

Condition (test) if $\frac{\partial M(x, y)}{\partial y} = \frac{\partial N(x, y)}{\partial x} \rightarrow$ *mean is Exact*

Assume $\frac{\partial F}{\partial x} = M(x, y)$ & $\frac{\partial F}{\partial y} = N(x, y)$

$$\left[F = \int M * dx + g(y) \right]$$

OR

$$\left[F = \int N dy + g(x) \right]$$

Example// Solve the equation

$$\sin y \, dx + (x \cos y - 2y)dy = 0$$

Solution/

$$\sin y \, dx + (x \cos y - 2y)dy = 0 \leftrightarrow M(x, y)dx + N(x, y)dy = 0$$

$$M(x, y) = \sin y \rightarrow \frac{\partial M}{\partial y} = \cos y$$

$$N(x, y) = (x \cos y - 2y) \rightarrow \frac{\partial N}{\partial x} = \cos y$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ this is Exact}$$

$$F = \int M * dx + g(y)$$

$$F = \int \sin y dx + g(y) \rightarrow F = \cos y + g(y)$$

$$\frac{\partial F}{\partial y} = \frac{\partial}{\partial y} (x \sin y + g(y))$$

$$\frac{\partial F}{\partial y} = N(x, y) = (x \cos y - 2y)$$

$$(x \cos y - 2y) = \frac{\partial}{\partial y} ((x \sin y) + g(y)) \rightarrow$$

$(x \cos y - 2y = x \cos y + \bar{g}(y))$ (Integration of the two parties)

$$x \sin y - y^2 = x \sin y + g(y) \rightarrow g(y) = -y^2$$

$$\therefore F = \cos y - y^2 = c$$

Example// Solve $(x - xy^2)dx + (8y - yx^2)dy = 0$

Solution /

$$(x - xy^2)dx + (8y - xy^2)dy = 0$$

$$\Leftrightarrow M(x, y)dx + N(x, y)dy = 0$$

$$M(x, y) = (x - xy^2) \rightarrow \frac{\partial M}{\partial y} = -2xy$$

$$N(x, y) = (8y - yx^2) \rightarrow \frac{\partial N}{\partial x} = -2xy$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \text{ this is Exact}$$

$$F = \int M * dx + g(y)$$

$$F = \int (x - xy^2) dx + g(y) \rightarrow F = \left(\frac{x^2}{2} - \frac{y^2x^2}{2}\right) + g(y)$$

$$\frac{\partial F}{\partial y} = \frac{\partial}{\partial y} \left(\left(\frac{x^2}{2} - \frac{y^2x^2}{2}\right) + g(y)\right)$$

$$\frac{\partial F}{\partial y} = N(x, y) = (8y - yx^2)$$

$$(8y - yx^2) = \frac{\partial}{\partial y} \left(\left(\frac{x^2}{2} - \frac{y^2x^2}{2}\right) + g(y)\right) \rightarrow$$

$$8y - yx^2 = -x^2y + \bar{g}(y)$$

(Integration of the two parties)

$$4y^2 - \frac{x^2y^2}{2} = -\frac{x^2y^2}{2} + g(y) \rightarrow g(y) = 4y^2$$

$$\therefore F = \left(\frac{x^2}{2} - \frac{y^2x^2}{2}\right) + 4y^2 = c$$