

10.Differentiation and Integration of inverse trigonometric functions:

$$\frac{d}{dx} \sin^{-1} u = \frac{\frac{du}{dx}}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \cos^{-1} u = \frac{-\frac{du}{dx}}{\sqrt{1-u^2}}$$

$$\frac{d}{dx} \tan^{-1} u = \frac{\frac{du}{dx}}{1+u^2}$$

$$\frac{d}{dx} \cot^{-1} u = \frac{-\frac{du}{dx}}{1+u^2}$$

$$\frac{d}{dx} \sec^{-1} u = \frac{\frac{du}{dx}}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx} \csc^{-1} u = \frac{-\frac{du}{dx}}{|u|\sqrt{u^2-1}}$$

Example 1: find \bar{y} . $y = \tan^{-1}(x \sin x)$

Solution//

$$\bar{y} = \frac{1}{1 + (x \sin x)^2} * [x * \cos x + \sin x * 1]$$

$$\bar{y} = \frac{x * \cos x + \sin x}{1 + (x \sin x)^2}$$

Example 2: find \bar{y} . $y = \sin^{-1} \sqrt{x} \sec^{-1} (\sin x)$

Solution //

$$\bar{y} = \sin^{-1} \sqrt{x} * \left(\frac{\cos x}{|\sin x| \sqrt{\sin^2 x - 1}} \right) + \sec^{-1} (\sin x) * \frac{\frac{1}{2\sqrt{x}}}{\sqrt{1 - (\sqrt{x})^2}}$$

Integration of inverse:

$$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + c$$

$$\int \frac{-du}{\sqrt{a^2 - u^2}} = \cos^{-1} \frac{u}{a} + c$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + c$$

$$\int \frac{-du}{a^2 + u^2} = \frac{1}{a} \cot^{-1} \frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + c$$

$$\int \frac{-du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \csc^{-1} \frac{u}{a} + c$$

Example 1: Find $\int \frac{dx}{\sqrt{9-x^2}}$

Solution //

$$\int \frac{dx}{\sqrt{9-x^2}} = \int \frac{dx}{\sqrt{3^2-x^2}} = \sin^{-1} \frac{x}{3} + c$$

Example 2: Find $\int \frac{-dx}{\sqrt{4-(x-2)^2}}$

Solution //

$$\int \frac{-dx}{\sqrt{4-(x-2)^2}} = \int \frac{-dx}{\sqrt{2^2-(x-2)^2}} = \cos^{-1} \frac{x-2}{2} + c$$

11. Differentiation and Integration of hyperbolic functions:

hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

Example: prove that $\sinh 2x = 2 \sinh x \cosh x$

Solution //

$$\begin{aligned} 2 \sinh x \cosh x &= 2 * \frac{e^x - e^{-x}}{2} * \frac{e^x + e^{-x}}{2} \\ &= \frac{1}{2} (e^x - e^{-x}) \cdot (e^x + e^{-x}) \\ &= \frac{1}{2} (e^{2x} + e^0 - e^0 - e^{-2x}) = \frac{e^{2x} - e^{-2x}}{2} = \sinh 2x \end{aligned}$$

H.W. Example: prove that $\cosh^2 x - \sinh^2 x = 1$

H.W. Example: prove that $\cosh x + \sinh x = e^x$

Differentiation of hyperbolic functions

$$\frac{d}{dx}(\sinh ax) = a \cosh ax$$

$$\frac{d}{dx}(\cosh ax) = a \sinh ax$$

$$\frac{d}{dx}(\tanh ax) = a \operatorname{sech}^2 ax$$

$$\frac{d}{dx}(\operatorname{csch} ax) = -a \operatorname{csch} ax \coth ax$$

$$\frac{d}{dx}(\operatorname{sech} ax) = -a \operatorname{sech} ax \tanh ax$$

$$\frac{d}{dx}(\coth ax) = -a \operatorname{csch}^2 ax$$

Integration of hyperbolic functions

$$\int \sinh x \, dx = \cosh x + c$$

$$\int \cosh x \, dx = \sinh x + c$$

$$\int \tanh x \, dx = \ln(\cosh x) + c$$

$$\int \coth x \, dx = \ln|\sinh x| + c$$

$$\int \operatorname{sech} x \, dx = \tan^{-1}|\sinh x| + c$$

$$\int \operatorname{csch} x \, dx = \ln \left| \tanh \frac{x}{2} \right| + c$$