

10. Integration by long division: -

Using when partial fraction only formula $\frac{f(x)}{g(x)}$.

When the degree of the numerator is \geq than the degree of the denominator.

المتبقى
المنقسم عليه \pm الناتج

Example 1: Find $\int \frac{x^3+4x^2}{x^2+4x+3} dx$.

Solution /

$$\begin{array}{r}
 \overline{) x^3 + 4x^2} \\
 \underline{+x^3 + 4x^2 + 3x} \\
 0 -3x
 \end{array}$$

$$\int x dx - \int \frac{3x}{x^2 + 4x + 3} dx$$

$$= \int x dx - \int \frac{3x}{(x + 3)(x + 1)} dx$$

$$\int \frac{3x}{(x + 3)(x + 1)} dx = \int \frac{A}{(x + 3)} dx + \int \frac{B}{(x + 1)} dx$$

$$\frac{3x}{(x + 3)(x + 1)} = \frac{A(x + 1) + B(x + 3)}{(x + 3)(x + 1)}$$

$$3x = Ax + A + Bx + 3B$$

$$A+B=3.....(1)$$

$$A+3B=0.....(2)$$

$$\rightarrow A = \frac{9}{2} \rightarrow B = \frac{-3}{2}$$

$$\therefore \int x dx - \left(\int \frac{\frac{9}{2}}{(x+3)} dx + \int \frac{\frac{-3}{2}}{(x+1)} dx \right)$$

$$\frac{1}{2}x^2 - \frac{9}{2}\ln|x+3| + \frac{3}{2}\ln|x+1| + c$$

H.W. //find $\int \frac{x^3}{x^2+2x+1} dx$

11. Integration by Part :-

This method is used when there are two multiplying functions, one of which is different from the other.

$$\text{formula } \int g(x).f(x)dx$$

To solution using $(\int U dV = U.V - \int V dU)$

U=Easy derivation function, dV=Easy integration function

dU=Derivative of U, V=Integration of dV

Example 1: Find $\int x \cos x dx$.

Solution //

Let $U=x$, $dV=\cos x dx$

$dU=1dx$, $V=\int \cos x dx = \sin x$

$$\int U dV = (U.V - \int V dU)$$

$$\int x \cos x dx = x. \sin x - \int \sin x dx = x. \sin x + \cos x + c$$

Example 2: Find $\int \ln x dx$

Solution //

Let $U = \ln x$, $dV = dx$

$$dU = \frac{1}{x} dx \quad , V = \int dx = x$$

$$\int U dV = U.V - \int V dU$$

$$\int \ln x dx = x \ln x. - \int x. \frac{1}{x} dx = x. \ln x - x + c$$

H.W. // find $\int x e^x dx$

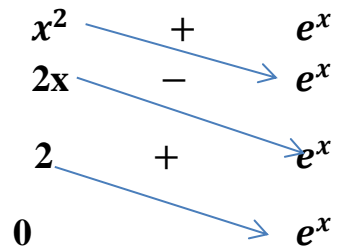
12. Tubular Integration :-

This method is used when there are two multiplying functions, one of which is integral and the other derivative to zero.

Example 1: Find $\int x^2 e^x dx$

Solution /

Derivative Integration

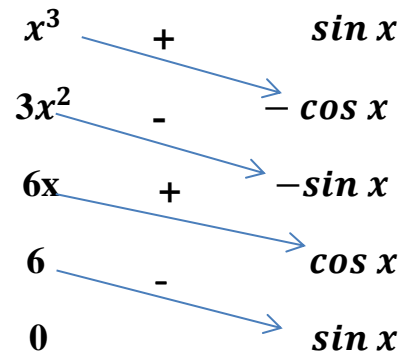


$$\rightarrow x^2 e^x - 2x e^x + 2e^x + c$$

Example 2: Find $\int x^3 \sin x dx$

Solution //

Derivative Integration



$$\rightarrow -x^3 \cos x + 3x^2 \sin x + 6x \cos x - 6 \sin x + c$$