Derivative of a^x function *and logarithm* $(log\frac{u}{a}$

3. a^x function: (a=constant)& a> 0

Y= a^x

Domain function=$-\infty < x < +\infty$

Range function=$y > 0$

Properties of a^x

1.$a^x = e^{\ln a^x} = e^{x \ln a}$

2.$x^x = e^{\ln x^x} = e^{x \ln x}$

Derivative of a^x

$$y=a^u \rightarrow \frac{dy}{dx} = a^u \, du \; ln\, a$$

Integration of a^x

$$\int \boldsymbol{a^u du} = \frac{\boldsymbol{a^u}}{\boldsymbol{ln\, a}} + \boldsymbol{c}$$

Example: $y=3^{\tan^{-1} x^2}$ $Find\ \bar{y}$

Solution //

$$\bar{y} = \boldsymbol{3^{tan^{-1} x^2}} \frac{2x}{1+(x^2)^2} * \ln 3$$

Example: $y=\ln x\, 3^{\sin x}$ $\quad \boldsymbol{Find}\ \bar{y}$

Solution //

$$\bar{y} = \ln x * (\mathbf{3}^{\boldsymbol{\sin x}} * \cos x * \ln 3) + \mathbf{3}^{\boldsymbol{\sin x}} * \frac{1}{x}$$

Example: $y=\boldsymbol{\pi}^{\boldsymbol{x}} + \boldsymbol{x}^{\boldsymbol{\pi}}$ $\quad \boldsymbol{Find}\ \bar{y}$

Solution //

$$\bar{\boldsymbol{y}} = \boldsymbol{\pi}^{\boldsymbol{x}} * (\mathbf{1}) * \ln \boldsymbol{\pi} + \boldsymbol{\pi}\, \boldsymbol{x}^{\boldsymbol{\pi-1}}$$

Example: Find $\bar{\boldsymbol{y}}$. $\boldsymbol{\pi}^{\boldsymbol{y}} = (\boldsymbol{sin\ x})^{\boldsymbol{x}} + \boldsymbol{ln\ x} + \boldsymbol{\pi}^{\boldsymbol{cos\ x}} + \boldsymbol{x}^{\boldsymbol{2\pi}}$

Solution //

$$(\boldsymbol{sin\ x})^{\boldsymbol{x}} = \boldsymbol{e}^{\boldsymbol{ln\ (sin\ x)^{x}}} = \boldsymbol{e}^{\boldsymbol{x\ ln\ sin\ x}}$$

$$\boldsymbol{\pi}^{\boldsymbol{y}} = \boldsymbol{e}^{\boldsymbol{x\ ln\ sin\ x}} + \boldsymbol{ln\ x} + \boldsymbol{\pi}^{\boldsymbol{cos\ x}} + \boldsymbol{x}^{\boldsymbol{2\pi}}$$

$$\boldsymbol{\pi}^{\boldsymbol{y}} * \bar{\boldsymbol{y}} * \boldsymbol{ln\ \pi} = \boldsymbol{e}^{\boldsymbol{x\ ln\ sin\ x}}\left(\boldsymbol{x}.\frac{\boldsymbol{cos\ x}}{\boldsymbol{sin\ x}} + \boldsymbol{ln\ sin\ x} * \mathbf{1}\right) + \frac{\mathbf{1}}{\boldsymbol{x}} + \boldsymbol{\pi}^{\boldsymbol{cos\ x}} * (-\boldsymbol{sin\ x}) * \boldsymbol{ln\ \pi} + \mathbf{2}\boldsymbol{\pi}\boldsymbol{x}^{\boldsymbol{2\pi-1}}$$

$$\bar{y} = \frac{e^{x\ ln\ sin\ x}(x.\frac{cos\ x}{sin\ x} + ln\ sin\ x * 1) + \frac{1}{x} + \pi^{cos\ x} * (-sin\ x) * ln\ \pi + 2\pi x^{2\pi-1}}{\boldsymbol{\pi}^{\boldsymbol{y}} * \boldsymbol{ln\ \pi}}$$

Example: Find $\int 2^{cos\,x} sin\,x dx$

Solution //

$$= \frac{2^{cos\,x} * (-1)}{\ln 2} + c$$

Example: Find $\int \frac{2^{sin^{-1}x}}{\sqrt{1+x^2}} dx$

Solution//

$$= \frac{2^{sin^{-1}x}}{\ln 2} + c$$

Example: Find $\int \frac{2^x}{1+4^x} dx$

Solution //

$$= \int \frac{2^x}{1 + (2^x)^2} dx$$

$$= \frac{\tan^{-1} 2^x}{\ln 2} + c$$

4. Function $of\ logarithm\ (log_a^u$

$$Y = log_a^u = \frac{\ln u}{\ln a}$$

Relation between logarithm &natural logarithm

Properties of logarithm function

1. $log_a\left(\frac{x}{y}\right) = log_a^x - log_a^y$

2. $log_a(x.y) = log_a^x + log_a^y$

3. $log\ {}_a^1 = 0. log\ {}_a^a = 1.\ log\ {}_a^{a^x} = x. log\ {}_2^{10} = \frac{ln\ 10}{ln\ 2} = \frac{2.3}{0.6}$

Derivative

$$\boldsymbol{y = log\ {}_a^u} \rightarrow \frac{\mathrm{dy}}{\mathrm{dx}} = \frac{1}{\ln a}\frac{du}{u}$$

Example: $\boldsymbol{y = log\ {}_3^{sin\ x}. Find\ \bar{y}}$

Solution //

$$\bar{y} = \frac{1}{\ln 3}\frac{\cos x}{\sin x}$$