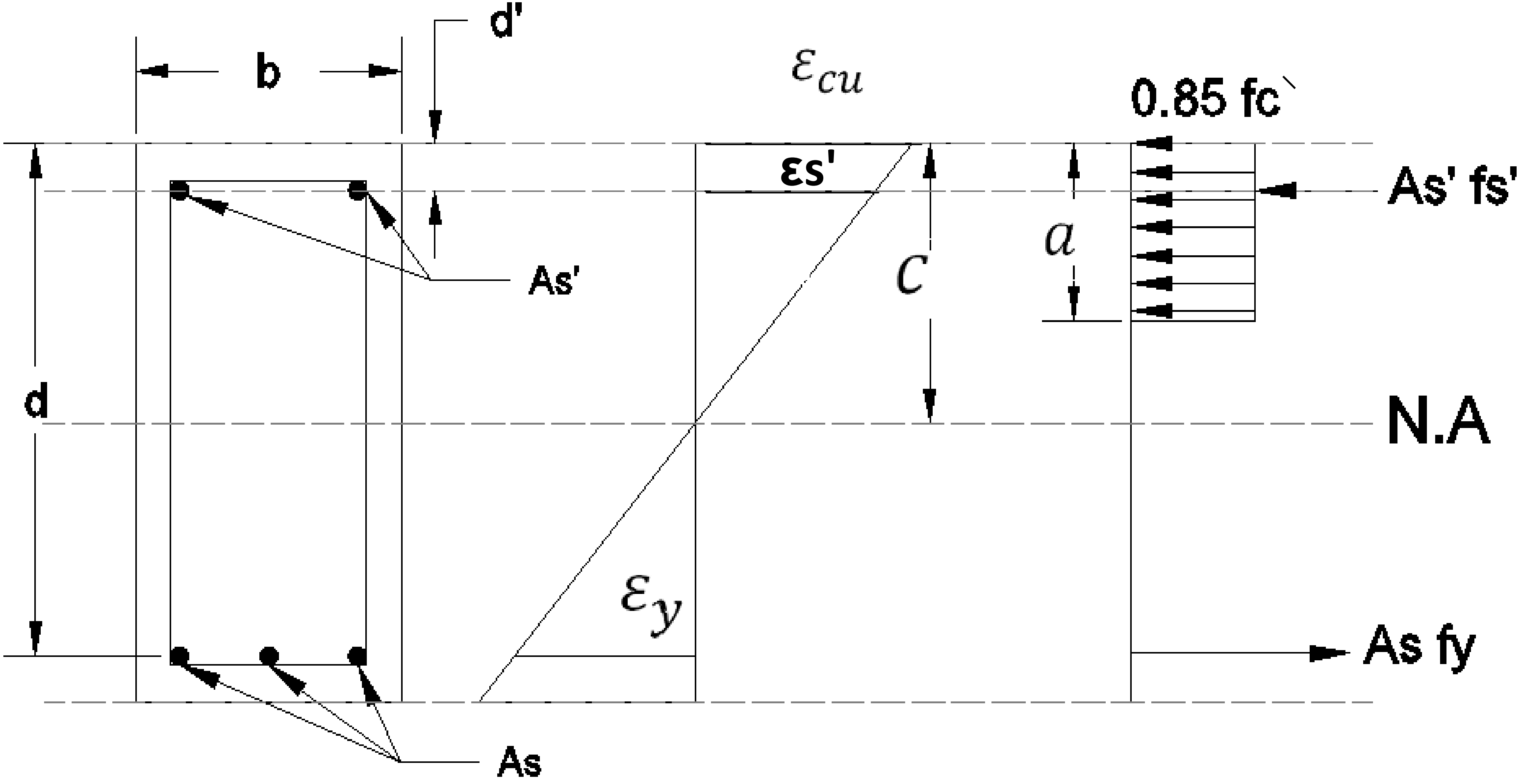
Doubly reinforce section

1. To increase moment capacity of sections having limited dimensions
2. To reduce the amount of long-term deflection
3. To support shear reinforcement (Stirrups)


d'
b
ε_{cu}
0.85 fc`
εs'
As' fs'
As'
a
c
d
N.A
ε_y
As fy
As

Balanced steel ratio:

$\varepsilon_s = \varepsilon_y$

$\varepsilon_c = \varepsilon_{cu}$ balance failure

Let

$$\bar{\rho} = \frac{As}{bd} \;, \qquad \rho' = \frac{As'}{bd}$$

$\bar{\rho}_b = \frac{As_b}{bd}$ balance steel ratio

$\sum Fx = 0$

$As_b\, fy = As'fs' + 85fc'ba \quad \div bdfy$

$$\frac{As_b}{bd} = \frac{As'}{bd} * \frac{fs'}{fy} + 0.85\frac{fc'}{fy} * \frac{a}{d}$$

$$\bar{\rho}_b = \rho' \frac{fs'}{fy} + 0.85\frac{fc'}{fy} * \frac{a}{d} \text{ ....(1)}$$

From strain diagram $\frac{\varepsilon_{cu}}{c_b} = \frac{\varepsilon_{cu}+\varepsilon_y}{d}$

$$c_b = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} * d$$

$$a = \beta_1 c = \beta_1 * \frac{\varepsilon_{cu}}{\varepsilon_{cu}+\varepsilon_y} d = \beta_1 * \frac{0.003}{0.003 + \frac{fy}{200000}} * d = \beta_1 * \frac{600}{600+fy} * d \text{....(2)}$$

Sub eq. 2 into eq. 1

$$\bar{\rho}_b = \rho' \frac{fs'}{fy} + 0.85\beta_1 \frac{fc'}{fy} \frac{600}{600+fy}$$

$$\bar{\rho}_b = \rho' * \frac{fs'}{fy} + \rho_b \qquad \text{for } fs' < fy$$

$$\bar{\rho}_b = \rho' + \rho_b \qquad \text{for } fs' = fy$$

fs' at balance condition:

$$\frac{\varepsilon_{cu} - \varepsilon_s'}{d'} = \frac{\varepsilon_{cu} + \varepsilon_y}{d}$$

$$\varepsilon_s' = \varepsilon_{cu} - \frac{d'}{d}\left(\varepsilon_{cu} + \varepsilon_y\right)$$

$fs' = fy$ if $\varepsilon_s' \geq \varepsilon_y$

$fs' < fy$ if $\varepsilon_s' < \varepsilon_y$

$$\therefore fs' = E_s \varepsilon_s' = E_S[\varepsilon_{cu} - \frac{d'}{d}\left(\varepsilon_{cu} + \varepsilon_y\right)$$

$E_s = 200000$ MPa , $\varepsilon_{cu} = 0.003$

$$fs' = 600 - \frac{d'}{d}(600 + fy)$$

$$\frac{d'}{d} = \frac{600 - fs'}{600 + fy}$$

$$\text{If} \begin{bmatrix} \frac{d'}{d} > 0.2 & fs' < fy \\ \frac{d'}{d} \leq 0.2 & fs' = fy \end{bmatrix} \text{for } fy = 400 \text{ MPa}$$

$$\text{If} \begin{bmatrix} \frac{d'}{d} > 0.333 & fs' < fy \\ \frac{d'}{d} \leq 0.333 & fs' = fy \end{bmatrix} \text{for } fy = 300\text{MPa}$$

To find $\bar{\rho}_{max}$,flow the same steps above for $\bar{\rho}_b$, but $\varepsilon_s = 0.004$

$$fs' = E_s\left[\varepsilon_{cu} - \frac{d'}{d}(\varepsilon_{cu} + 0.004)\right] \rightarrow fs' = 600 - 1400\frac{d'}{d} \leq fy$$

$$\bar{\rho}_{max} = 0.85\beta_1 * \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004} + \rho'\frac{fs'}{fy}$$

$$\bar{\rho}_{max} = 0.364\beta_1\frac{fc'}{fy} + \rho'\frac{fs'}{fy}$$

Bending moment capacity of doubly reinforced rectangular section

$$Mu = \emptyset Mn$$

1. $fs' = fy$

$$Mu = \emptyset \left[0.85\ fc'\ ba\left(d - \frac{a}{2}\right) + As' fy\ (d - d')\right]$$

$$\sum Fx = 0$$

$$As\ fy = As' fy + 0.85\ fc' ba \qquad \div b\ a\ fy$$

$$a = (\bar{\rho} - \rho') \frac{fy}{0.85 fc'} * d$$

2. $fs' < fy$

$$Mu = \phi \left[0.85 fc' ba \left(d - \frac{a}{2}\right) + As' fs' (d - d')\right]$$

$$\sum Fx = 0$$

$$As\ fy = As' fs' + 0.85 fc' ba \qquad \div bdfy$$

$$a = \left(\bar{\rho} - \rho' * \frac{fs'}{fy}\right) * \frac{fy}{0.85\ fc'} * d \ldots\ldots\ldots (1)$$

From strain diagram

$$\frac{\varepsilon_{cu} - \varepsilon_s'}{d'} = \frac{\varepsilon_{cu}}{c}$$

$$c=\frac{\varepsilon_{cu}}{\varepsilon_{cu}-\varepsilon_s'}*d'=\frac{600}{600-fs'}*d'$$

$$\therefore a=\beta_1.c=\frac{600\beta_1}{600-fs'}*d'\ldots\ldots\ldots\ldots(2)$$

$$Eq_1=Eq_2$$

$$\left(\bar{\rho}-\rho'\frac{fs'}{fy}\right)*\frac{fy}{0.85fc'}*d=\frac{600\beta_1}{600-fs'}*d'$$

2^{nd} order eq. to be solved for fs' then fined (a) from eq1 or eq2

Condition for yielding of compression reinforcement

For large $\bar{\rho}$, $\varepsilon_s' \geq \varepsilon_y \rightarrow fs' = fy$

For small $\bar{\rho}$, $\varepsilon_s' < \varepsilon_y \rightarrow fs' < fy$

For min. $\bar{\rho}$, $\varepsilon_s' = \varepsilon_y \rightarrow fs' = fy$

$\bar{\rho}_{min}$: The min. steel ratio required to insure yielding of compression reinforcement

$$\sum Fx = 0$$

$$As\ fy = As' fy + 0.85 fc' ba \qquad \div b\ d\ fy$$

$$\bar{\rho}_{min} = \rho' + 0.85 \frac{fc'}{fy} * \frac{a}{d}$$

From strain diagram

$$\frac{\varepsilon_{cu} - \varepsilon_s^{'}}{d^{'}} = \frac{\varepsilon_{cu}}{c} \quad sub \quad \varepsilon_s^{'} = \varepsilon_y$$

$$\frac{\varepsilon_{cu} - \varepsilon_y}{d^{'}} = \frac{\varepsilon_{cu}}{c}$$

$$c = \frac{\varepsilon_{cu} . d^{'}}{\varepsilon_{cu} - \varepsilon_y} = \frac{600}{600 - fy} * d^{'}$$

$$\therefore a = \beta_1 c = \frac{600\beta_1}{600\text{-}fy} * d^{'}$$

$$\bar{\rho}_{min} = \rho^{'} + 0.85\beta_1 \frac{fc^{'}}{fy} * \frac{d^{'}}{d} * \frac{600}{600\text{-}fy}$$ **for analysis problems**

IF $\bar{\rho} \geq \bar{\rho}_{min} \rightarrow fs' = fy$ For moment equation

IF $\bar{\rho} < \bar{\rho}_{min} \rightarrow fs' < fy$ For moment equation

For design problems set $\bar{\rho}_{min} = \bar{\rho}_{max}$ (*to ensure that* $fs' = fy$ for moment equation)

$$\rho' + 0.85\beta_1 \frac{fc'}{fy} * \frac{d'}{d} * \frac{600}{600 - fy} = 0.85\beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004} + \rho' \frac{fs'}{fy} \rightarrow$$

$$\rightarrow \mathbf{\frac{d'}{d} = \frac{3}{4200}(600 - fy)}$$

For fy=400MPa , $if\ \frac{d'}{d} \leq 0.143 \rightarrow fs' = fy$

For fy=300MPa , $if\ \frac{d'}{d} \leq 0.210 \rightarrow fs' = fy$

Reduction factor Ø

To find $\bar{\rho}_t$,flow the same steps above, but $\varepsilon_s = 0.005$

$$\bar{\rho}_t = 0.85\beta_1 \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} + \rho' = \rho_t + \rho' \qquad \text{For } fs' = fy$$

If $\bar{\rho} \leq \bar{\rho}_t \rightarrow \varepsilon_s \geq 0.005 \rightarrow \emptyset = 0.9$

If $\bar{\rho} > \bar{\rho}_t \rightarrow \varepsilon_s < 0.005 \rightarrow \emptyset = 0.483 + 83.3\varepsilon_t$

$$\bar{\rho}_t = 0.85\,\beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005} + \bar{\rho}\frac{fs'}{fy} \qquad \text{for } fs' < fy$$

$$\bar{\rho}_t = \rho_t + \bar{\rho}\frac{fs'}{fy} \qquad \text{for } fs' < fy, \qquad fs' = E_s\left[\varepsilon_{cu} - \frac{d'}{d}(\varepsilon_{cu}+0.005)\right] \rightarrow$$

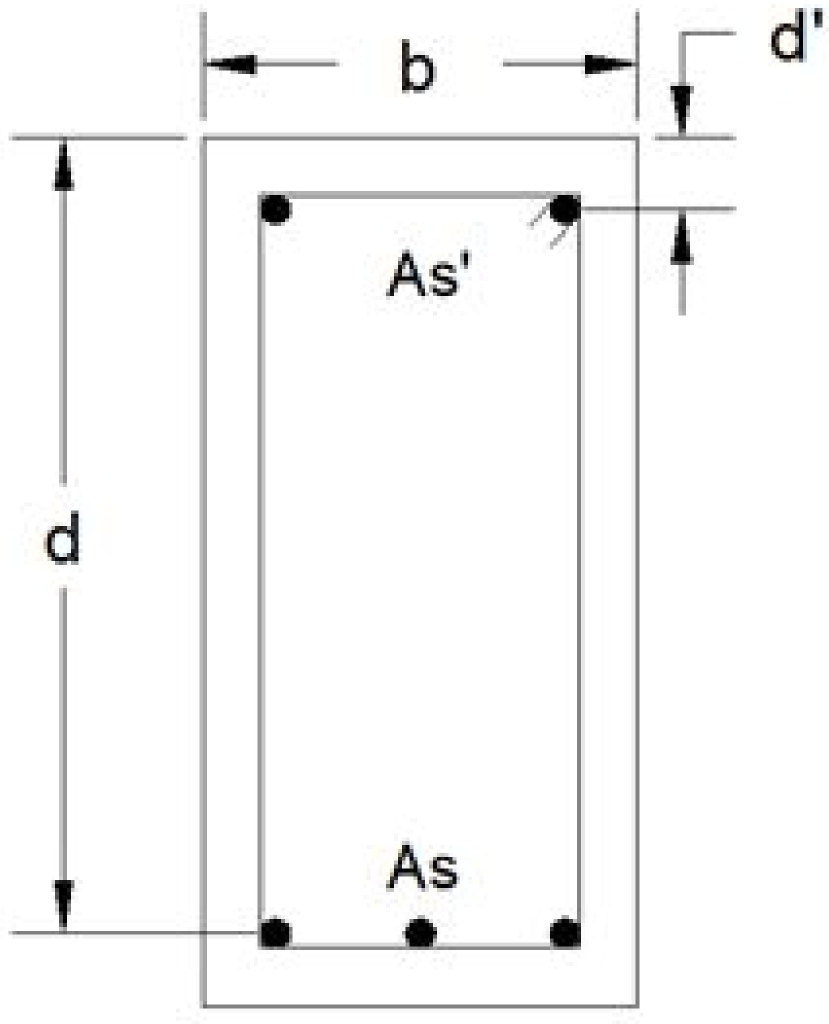
$$fs' = 600 - 1600\frac{d'}{d} \leq fy$$

$$\text{if } \bar{\rho} \leq \rho_t \rightarrow \emptyset = 0.9$$

$$\text{if } \bar{\rho} > \rho_t \rightarrow \emptyset = 0.483 + 83.3\,\varepsilon_t$$

$$\varepsilon_t = \varepsilon_{cu} * \frac{dt - c}{c}$$

Summary



$$\bar{\rho} = \frac{As}{bd} \quad , \qquad \rho' = \frac{As'}{bd}$$

1. Check

$$\rho_{max} \quad iF \begin{bmatrix} \bar{\rho} \leq \rho_{max\,(Single)} & \therefore \text{Singly reinforced section} \\ \bar{\rho} > \rho_{max(Single)} & \therefore \text{Doubly reinforced section} \end{bmatrix}$$

2. Check $\bar{\rho} \leq \bar{\rho}_{max} = 0.85\beta_1 \frac{fc'}{fy}\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.004} + \rho'\frac{fs'}{fy}$

$$fs' = 600 - 1400\frac{d'}{d} \leq fy \quad \text{for } \bar{\rho}_{max} \text{ equation}$$

3. Check $\bar{\rho}_{min} = 0.85\beta_1 \frac{fc'}{fy}\frac{d'}{d}\frac{600}{600-fy} + \rho'$

a. If $\bar{\rho} \geq \bar{\rho}_{min} \rightarrow fs' = fy$

$$Mu = \emptyset\left[0.85\ fc'ba\left(d - \frac{a}{2}\right) + As'fy(d - d')\right]$$

$$a = (\bar{\rho} - \rho')\frac{fy}{0.85\ fc'} * d \rightarrow a = (As - As')\frac{fy}{0.85fc'b}$$

$$\text{If } \bar{\rho} \leq \bar{\rho}_t = 0.85\beta_1 \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005} + \rho' \rightarrow \emptyset = 0.9$$

$$\text{If } \bar{\rho} > \bar{\rho}_t \rightarrow \emptyset = 0.483 + 83.3\varepsilon_t$$

b. If $\bar{\rho}<\bar{\rho}_{min} \rightarrow fs'<fy$

$$Mu=\emptyset\left[0.85fc'ba\left(d-\frac{a}{2}\right)+As\ fs'(d-d')\right]$$

$$\left[\begin{array}{l} a=\left(\bar{\rho}-\rho'\frac{fs'}{fy}\right)\frac{fy}{0.85\ fc'}d\ldots\ldots\ldots(1) \\ a=\frac{600\beta_1}{600-fs'}d'\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2) \end{array}\right] \text{solve for } fs' \text{and a}$$

$$\text{If } \bar{\rho} \leq \bar{\rho}_t = 0.85\beta_1\frac{fc'}{fy}*\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005}+\rho'\frac{fs'}{fy} \rightarrow \emptyset = 0.9$$

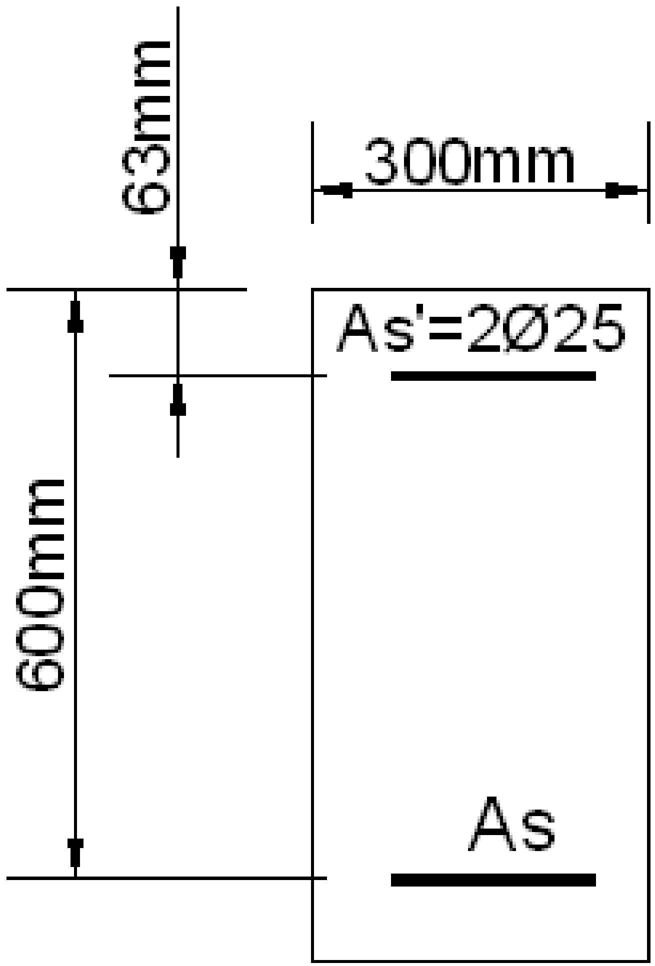
$$fs'=600-1600\frac{d'}{d} \leq fy$$

$$\text{If } \bar{\rho} > \bar{\rho}_t \rightarrow \emptyset = 0.483 + 83.3\varepsilon_t$$

$$\varepsilon_t = \varepsilon_{cu}\frac{dt - c}{c}$$

Ex1: $\frac{S}{C} = \frac{414}{35}\ MPa$, find Mu, if

1. 3Ø32, As=2413mm^2
2. 6Ø32, As=4826mm^2
3. 5Ø32 +1Ø25, As=4513mm^2



Solution:

As'=2Ø25=982mm^2 , $\rightarrow \rho' = \frac{As'}{bd} = \frac{982}{300*600} = 0.0055$

$1-\ 3Ø32$,

$$\mathbf{As} = 2413\text{mm}^2 \rightarrow \bar{\rho} = \frac{As}{bd} = \frac{2413}{300*600} = 0.0134$$

$$\rho_{max} = 0.85\beta 1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004}$$

$$for\ fc' = 35MPa > 28,$$

$$\beta 1 = 0.85 - 0.05 * \frac{fc' - 28}{7} = 0.85 - 0.05 * \frac{35 - 28}{7}$$

$$= 0.80$$

$$\rho_{max} = 0.85 * 0.8 * \frac{35}{400} \frac{0.003}{0.003 + 0.004} = 0.0246$$

$$\bar{\rho} = 0.0134 < \rho_{max} = 0.0246 \rightarrow singly\ R.S$$

$$\rho_{min} = \max\left(\frac{1.4}{fy} = \frac{1.4}{414} = 0.0034\,, \frac{\sqrt{fc'}}{4fy} = \frac{\sqrt{35}}{4*414} = 0.0036\right) = 0.0036 < \bar{\rho} = 0.0134\ O.K$$

$$Mu = \phi\rho bd^2 fy(1 - 0.59\rho\frac{fy}{fc'})$$

$$\rho_t = 0.85\beta 1\frac{fc'}{fy}\frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} = 0.85 * 0.80 * \frac{35}{414}\frac{0.003}{0.003 + 0.005} = 0.0215$$

$$\bar{\rho} = 0.0134 < \rho_t = 0.0215 \rightarrow \emptyset = 0.9$$

$$Mu = 0.9 * 0.0134 * 0.3 * 0.6^2 * 414 * \left(1 - 0.59 * 0.0134 * \frac{414}{35}\right) = 0.489 MN.m$$

$$\mathbf{2-\ 6Ø32, As =} 4826\text{mm}^2 \rightarrow \bar{\rho} = \frac{As}{bd} = \frac{4826}{300*600} = 0.0268$$

$$\bar{\rho} = 0.0268 > \rho_{max} = 0.0246 \rightarrow doubly\ R.S$$

$$\bar{\rho}_{\max} = 0.85\beta 1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004} + \rho' \frac{fs'}{fy}$$

$$fs' = 600 - 1400 \frac{d'}{d} = 600 - 1400 * \frac{63}{600} = 453MPa > fy \rightarrow$$

$$fs' = 414MPa$$

$$\bar{\rho}_{\max} = 0.85*0.8* \frac{35}{414} * \frac{0.003}{0.003+0.004} + 0.0055 * \frac{414}{414} = 0.0301$$

$$\bar{\rho} = 0.0268 < \bar{\rho}_{\max} = 0.0301\ \ O.K$$

$$\bar{\rho}_{\min}=0.85\beta 1\frac{fc'}{fy}\frac{d'}{d}\frac{600}{600-fy}+\rho'$$

$$\bar{\rho}_{\min}=0.85*0.8*\frac{35}{414}*\frac{63}{600}*\frac{600}{600-414}+0.0055=0.025$$

$\bar{\rho}=0.0268>\bar{\rho}_{\min}=0.025\rightarrow fs'=fy$ for moment equation

$$\bar{\rho}=0.0268<\bar{\rho}_t=0.85\beta_1\frac{fc'}{fy}*\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.005}+\rho'=\rho_t+\rho'=0.0215+0.0055=0.027\rightarrow\emptyset=0.9$$

$$\sum fx = 0$$

As fy=0.85fc' ba + As'fy →[As-As']fy=0.85fc' ba

$$a = \frac{(4826 - 982) * 10^{-6} * 414}{0.85 * 35 * 0.3} = 0.178m$$

$$c = \frac{a}{\beta 1} = \frac{178}{0.8} = 222mm$$

$$Mu = \emptyset[0.85 f c' b a \left(d - \frac{a}{2}\right) + As' f s' (d - d')]$$

$$Mu = 0.9\left[0.85 * 35 * 0.3 * 0.178\left(0.6 - \frac{0.178}{2}\right) + \right.$$

$$\left. 928 * 10^{-6} * 414(0.6 - 0.063)\right]$$

$$= 0.927 MN.m$$

$$for\ check: \frac{\varepsilon s'}{c - d'} = \frac{\varepsilon_{cu}}{c} \rightarrow \frac{\varepsilon s'}{222 - 63} = \frac{0.003}{222} \rightarrow \varepsilon s^{'}$$

$$= 0.00215 \rightarrow fs^{'} = Es * Es^{'} = 200000 * 0.00215$$

$$= 430 MPa > fy \rightarrow fs^{'} = 414 MPa\ for\ moment\ eq.$$

$\mathbf{3-\ 5Ø32\ +1Ø25,}\qquad \mathbf{As} = 4513\text{mm}^2 \rightarrow$

$$\bar{\rho} = \frac{As}{bd} = \frac{4513}{300*600} = 0.025$$

$$\bar{\rho} = 0.025 > \rho_{max} = 0.0246 \rightarrow doubly\ R.S$$

$$\bar{\rho} = 0.025 < \bar{\rho}\text{max} = 0.0301\ \ O.K$$

$$\bar{\rho} = 0.025 < \bar{\rho}\text{min} = 0.025\ \rightarrow fs' < fy$$

$$a = \left(\bar{\rho} - \rho' \frac{fs'}{fy}\right) \frac{fy}{0.85\ fc'} d \ldots\ldots\ldots \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$$a=\left(0.025-0.0055*\frac{fs'}{414}\right)\frac{414}{0.85\ *35}*0.6\ldots\ldots\ldots(1)$$

$$a=\frac{600\beta_1}{600-fs'}d'\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$$a=\frac{600*0.80}{600-fs'}*0.063\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$0.0055(fs')^2-13.65fs'+4690.62=0$ solve for fs'

$fs'=412MPa< fy$

$a=0.163m$

$$\bar{\rho}_t = 0.85\beta_1 \frac{fc'}{fy} * \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} + \rho' \frac{fs'}{fy}$$

$$fs' = 600 - 1600 \frac{d'}{d} = 600 - 1600 \frac{63}{600} = 432MPa > fy \rightarrow fs'$$

$$= 414MPa$$

$$\bar{\rho}_t = 0.85 * 0.8 * \frac{35}{414} * \frac{0.003}{0.003 + 0.005} + 0.0055 * \frac{414}{414}$$

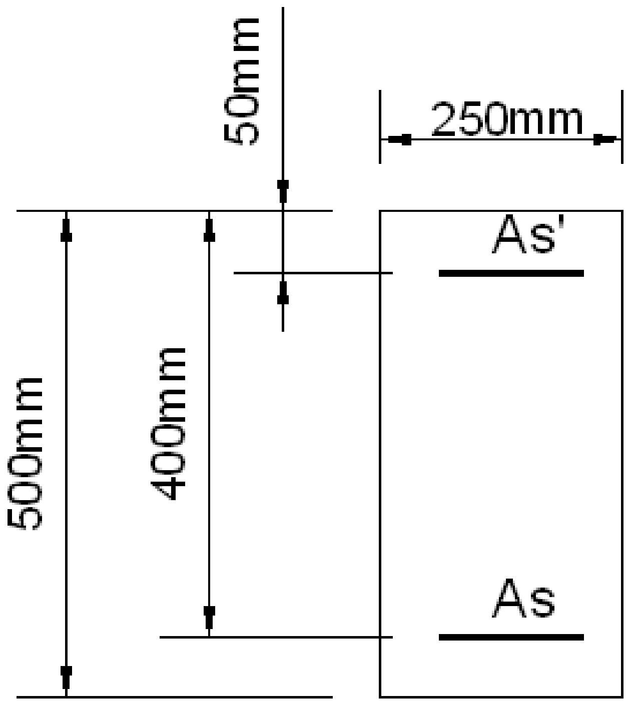
$$= 0.0271$$

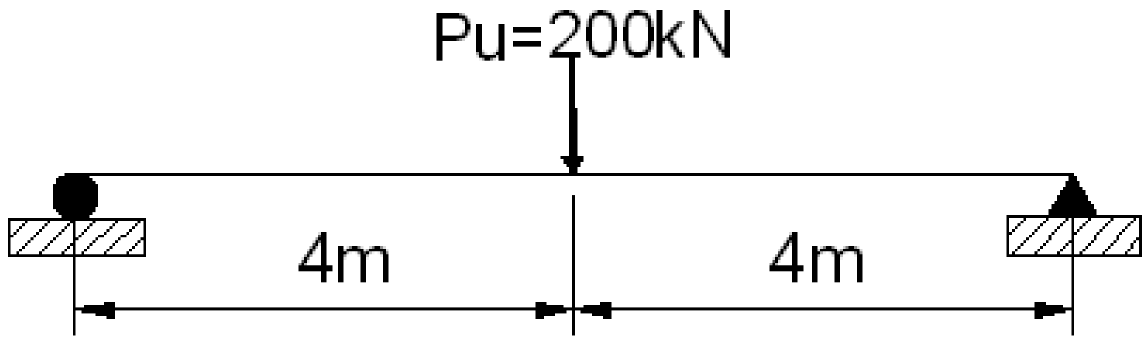
$$\bar{\rho} = 0.025 < \bar{\rho}t = 0.0271 \rightarrow \emptyset = 0.9$$

$$Mu = \emptyset\left[0.85fc'ba\left(d - \frac{a}{2}\right) + As'fs'(d - d')\right]$$

$$Mu = 0.9\left[0.85 * 35 * 0.3 * 0.163\left(0.6 - \frac{0.163}{2}\right) + 928 * 10^{-6} * 412 * (0.6 - 0.063)\right] = 0.874MN.m$$

Ex2: $\frac{S}{C} = \frac{400}{30}\ MPa$, factored load Pu=200 kN, ignore beam wt., Design beam for flexure.





Solution: $Mu_{ext} = \frac{Pu.l}{4} = \frac{200*8}{4} = 400\ kN.m$

Let Ø=0.9

$$Mu = \phi\rho b d^2 fy(1 - 0.59\rho\frac{fy}{fc'})$$

$$0.4 = 0.9 * \rho * 0.25 * 0.4^2 * 400(1 - 0.59\rho * \frac{400}{30})$$

$$\rho = \begin{pmatrix} 0.041 \\ 0.086 \end{pmatrix}\ choose\ min.postive\ value, \rho = 0.041$$

$$\rho_{max} = 0.85\beta 1\frac{fc'}{fy}\frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004}$$

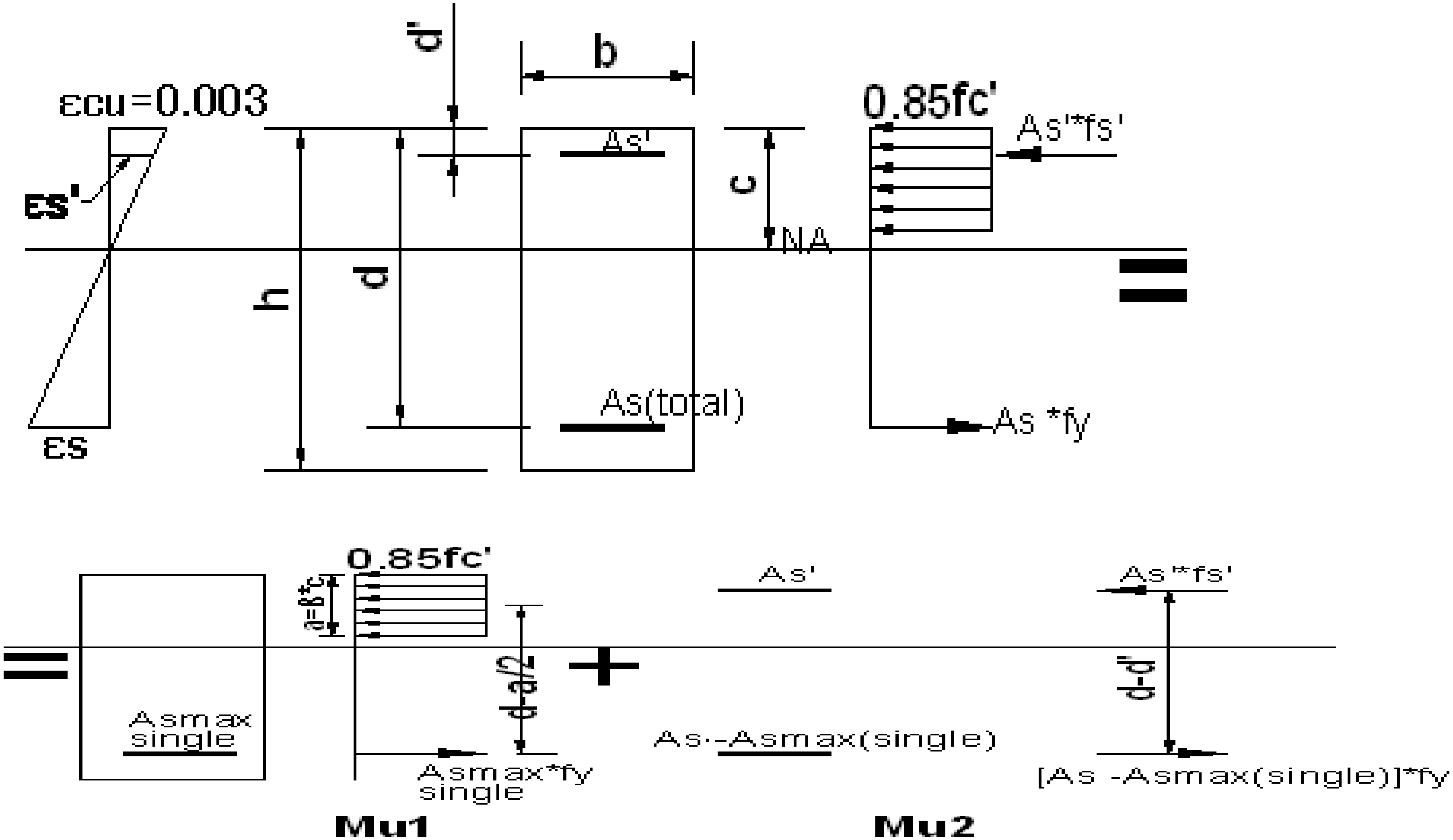
$$for\ fc' = 30MPa > 28,$$

$$\beta 1 = 0.85 - 0.05 * \frac{fc' - 28}{7} = 0.85 - 0.05 * \frac{30 - 28}{7}$$

$$= 0.83$$

$$\rho_{max} = 0.85 * 0.83 * \frac{30}{400}\ \frac{0.003}{0.003 + 0.004} = 0.0227$$

$$\rho = 0.041 > \rho_{max} = 0.0227\ \rightarrow doubly\ RCS$$


εcu=0.003
εs'
εs
d'
b
As'
h
d
As(total)
c
NA
0.85fc'
As'*fs'
As *fy
0.85fc'
a=ß*c
d-a/2
Asmax
single
Asmax*fy
single
As'
As-Asmax(single)
As'*fs'
d-d'
[As -Asmax(single)]*fy
Mu1
Mu2

$$for\ \rho = \rho_{max}, \rightarrow \varepsilon_t = 0.004, \rightarrow \emptyset = 0.483 + 83.3 * 0.004$$
$$= 0.816$$

$$Mu_1 = \phi \rho_{max} b d^2 fy(1 - 0.59\rho \frac{fy}{fc'})$$

$$Mu_1$$
$$= 0.816 * 0.0227 * 0.25 * 0.4^2$$
$$* 400\left(1 - 0.59 * 0.0227 * \frac{400}{30}\right) = 0.243 MN.m$$

$$Mu\ total = Mu1 + Mu2$$

$$400 = 243 + Mu2$$

$Mu2 = 157kN.m$

Let fs'=fy to be check later

OR $\frac{d'}{d} = \frac{50}{400} = 0.125 < 0.143 \rightarrow \therefore fs' = fy$

Mu2=Ø As' fs' (d-d')

0.157=0.816*As'*400(0.4-0.05)

As'=1.37*10^{-3} m^2=1370$mm^2$$\rightarrow \rho' = \frac{As'}{bd} = \frac{1370}{250*400} = 0.0137$

Total tension reinforcement(As)= $\rho_{max}bd + As'\frac{fs'}{fs}$

$$As = 0.0227 * 250 * 400 + 1370 * \frac{400}{400} = 3640mm^2$$

$$\bar{\rho} = \frac{As}{bd} = \frac{3640}{250 * 400} = 0.0364$$

$$\bar{\rho}min = 0.85\beta 1 \frac{fc'}{fy}\frac{d'}{d}\frac{600}{600 - fy} + \rho'$$

$$\bar{\rho}min = 0.85 * 0.83 * \frac{30}{400} * \frac{50}{400} * \frac{600}{600 - 400} + 0.0133$$

$$= 0.03354$$

$\bar{\rho}=0.0364$

$> \bar{\rho}min=0.03354$ → The previous assumption is valid, fs'=fy O.K

$$\bar{\rho}max=0.85\beta1\frac{fc'}{fy}\frac{\varepsilon_{cu}}{\varepsilon_{cu}+0.004}+\rho'\frac{fs'}{fy}$$

$$fs^{'}=600-1400\frac{d'}{d}=600-1400*\frac{50}{400}=425MPa>fy$$

$$\rightarrow fs^{'}=400MPa$$

$$\bar{\rho}max=0.85*0.83*\frac{30}{400}*\frac{0.003}{0.003+0.004}+0.0137*\frac{400}{400}$$

$$=0.03637$$

$\rho = 0.0364 < \bar{\rho}max=0.03637\ O.K$

Use 2Ø36+3Ø28=3883mm^2 tension Reinf.

Use 3Ø28=1847mm^2 comp. Reinf.

Check reduction factor Ø=0.816

As_{single} provided=Ast-As'*(fs'/fy)

=3883-1847*(400/400)=2036 mm^2

$$\sum fx = 0$$

As_{single} fy=0.85fc' ba

$$a = \frac{2036 * 10^{-6} * 400}{0.85 * 30 * 0.25} = 0.128m$$

$$c = \frac{a}{\beta 1} = \frac{128}{0.83} = 154mm$$

$$\frac{\varepsilon t}{d - c} = \frac{\varepsilon_{cu}}{c} \rightarrow \frac{\varepsilon t}{400 - 154} = \frac{0.003}{154} \rightarrow \varepsilon t = 0.0048 \rightarrow \therefore \emptyset$$

$$= 0.483 + 83.3 * 0.0048 = 0.88 > 0.816\ O.K$$

Mu=Ø(Mu1+Mu2)

$$Mu = \emptyset[0.85 fc' ba \left(d - \frac{a}{2}\right) + As' fs' (d - d')]$$

$$\frac{\varepsilon s'}{c-d'} = \frac{\varepsilon_{cu}}{c} \rightarrow \frac{\varepsilon s'}{154-50} = \frac{0.003}{154} \rightarrow \varepsilon s' = 0.002 \rightarrow fs'$$

$$= Es * Es' = 200000 * 0.002 = 400MPa$$

Mu

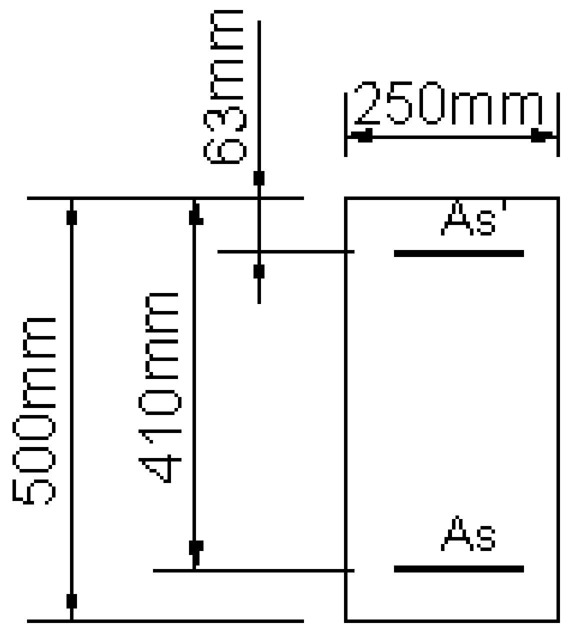
$$= 0.88\left[0.85 * 30 * 0.25 * 0.128\left(0.4 - \frac{0.128}{2}\right)\right.$$

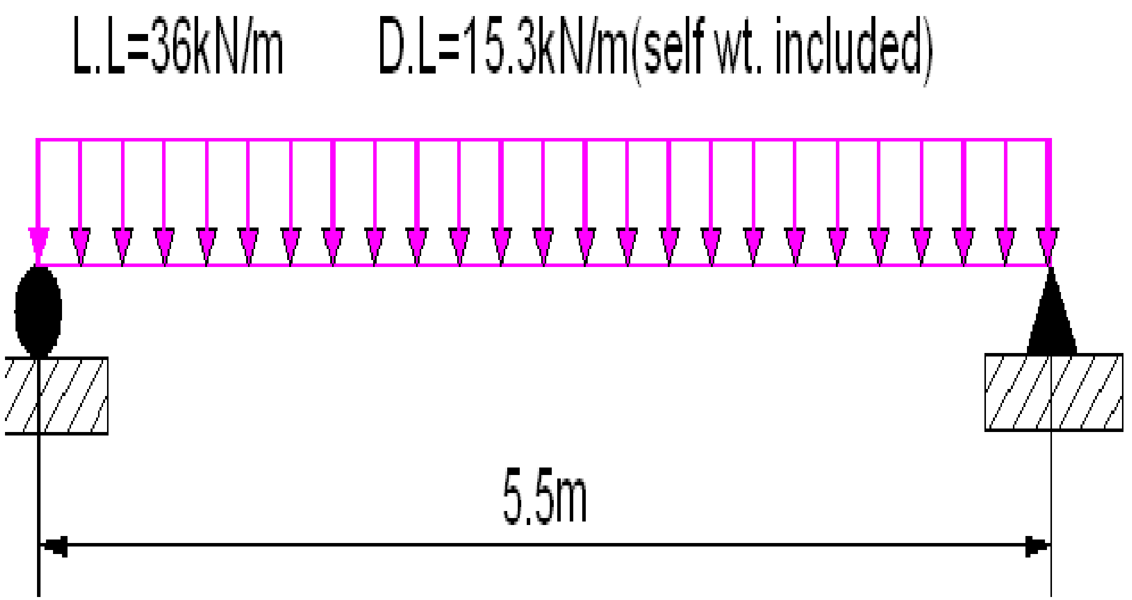
$$\left. + 1847 * 10^{-6} * 400(0.4 - 0.05)\right] = 0.469MN.m$$

$$> Muext = 0.400MN.m$$

$\therefore$ *no further reinforcement is necessary*

Ex3: $\frac{S}{C} = \frac{414}{28}\ MPa$, service load (L.L=36 kN/m, D.L=15.3kN/m(self wt included)). Design beam for flexure.





Solution:

Wu=1.2*15.3+1.6*36=76kN/m

$$Mu_{ext} = \frac{Wu\ l^2}{8} = \frac{76\ * 5.5^2}{8} = 287kN.m$$

Mu=ØMn

$$Mu = \phi\rho bd^2 fy(1 - 0.59\rho\frac{fy}{fc'})$$

$$0.287 = 0.9 * \rho * 0.25 * 0.41^2 * 414\left(1 - 0.59 * \rho * \frac{414}{28}\right)$$

$$130\rho^2 - 14.9\rho + 0.287 = 0 \rightarrow \rho = 0.0245$$

$$\rho_{max} = 0.85\beta 1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004}$$

$$for\ fc^{'} = 28MPa \leq 28MPa, \qquad \beta 1 = 0.85$$

$$\rho_{max} = 0.85 * 0.85 * \frac{28}{414} \frac{0.003}{0.003 + 0.004} = 0.0209$$

$$\rho = 0.0245 > \rho_{max} = 0.0209 \rightarrow doubly\ RCS$$

$$\rho_{\mathrm{t}} = 0.85 * 0.85 * \frac{28}{414} * \frac{0.003}{0.003+0.005} = 0.0183$$

Let $\rho = \rho_{\mathrm{t}} = 0.0183(slightly\ from \rho_{max}) \rightarrow \emptyset = 0.9$

$$Mu_1 = 0.9 * 0.0183 * 0.25 * 0.41^2 * 414\left(1 - 0.59 * 0.0183 * \frac{414}{28}\right) = 0.241 MN.m$$

$$Mu\ total = Mu1 + Mu2$$

$$287 = 241 + Mu2 \rightarrow Mu2 = 46\ kN.m$$

$$\frac{d'}{d} = \frac{63}{410} = 0.153 > 0.143 \rightarrow fs' < fy$$

As_{single} fy=0.85fc' ba

$$\rho_t bd\ fy = 0.85fc'ba$$

$$a = \frac{0.0183 * 0.41 * 414}{0.85 * 28} = 0.13m$$

$$c = \frac{a}{\beta 1} = \frac{130}{0.85} = 153mm$$

$$\frac{\varepsilon s'}{c - d'} = \frac{\varepsilon_{cu}}{c} \rightarrow \frac{\varepsilon s'}{153 - 63} = \frac{0.003}{153} \rightarrow \varepsilon s' = 0.00176 \rightarrow fs'$$

$$= Es * Es' = 200000 * 0.00176 = 352MPa$$

Mu2=ØAs'fs'(d-d')→0.046=0.9*As'*352(0.41-0.063)→

As'=4.18*10^{-6} m^2 =418mm^2

$$As = \rho_t bd + As' * \left(\frac{fs'}{fy}\right)$$

$$= 0.0183 * 250 * 410 + 418 * \left(\frac{352}{414}\right) = 2231mm^2$$

Use 4Ø28 (2463 mm^2) , tension reinforcement

Use 2Ø20 (628 mm^2) , compression reinforcement

$$\sum Fx = 0 \rightarrow (As - As' * \frac{fs'}{fy})fy = 0.85fc'ba \rightarrow a$$

$$= \frac{\left(2463 - 628 * \frac{352}{414}\right) * 10^{-6} * 414}{0.85 * 28 * 0.25} = 0.134m$$

$$c = \frac{a}{\beta 1} = \frac{134}{0.85} = 158mm$$

$$\frac{\varepsilon_t}{d - c} = \frac{\varepsilon_{cu}}{c} \rightarrow \frac{\varepsilon_t}{410 - 158} = \frac{0.003}{158} \rightarrow \varepsilon t = 0.00478 < 0.005$$

$$\rightarrow \emptyset = 0.483 + 83.3 * 0.00478 = 0.881 < 0.9$$

$$Mu = \emptyset[0.85 f c' b a \left(d - \frac{a}{2}\right) + As' fs' (d - d')]$$

$$\frac{\varepsilon s'}{c - d'} = \frac{\varepsilon_{cu}}{c} \rightarrow \frac{\varepsilon s'}{158 - 63} = \frac{0.003}{158} \rightarrow \varepsilon s' = 0.0018 \rightarrow$$

$$fs' = Es * Es' = 200000 * 0.0018 = 361 MPa$$

Mu

$$= 0.881\left[0.85 * 28 * 0.25 * 0.134\left(0.41 - \frac{0.134}{2}\right)\right.$$

$$\left. + 628 * 10^{-6} * 361(0.41 - 0.063)\right] = 0.310 MN.m$$

$$> Muext = 0.287 MN.m$$

$$\therefore no\ further\ reinforcement\ is\ necessary$$