

Al-Mustaqbal University College

Chemical Engineering and Petroleum Industries department



Chapter (5)

Probability Distribution

* Probability : When an event may happen in (x) ways out of a total of (n) possible equally likely ways, the probability of occurrence (success) is given by :

$$p = Pr(E) = \frac{x}{n}$$

Hence the prob. Of non-occurrence (failure) is :

$$q = Pr(\tilde{E}) = \frac{n - x}{n} = 1 - \frac{x}{n} = 1 - p$$

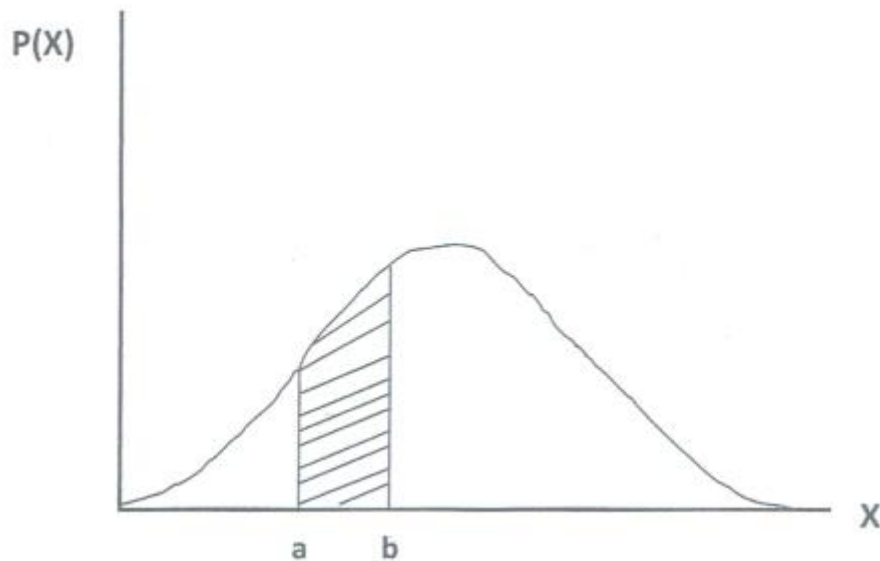
Thus $p + q = 1$

Continuous Probability Distribution :

* If a variable X may assume a continuous set of values, the prob. dist. Is a frequency curve where $p(x)=fr$.

Total area under the curve = $\sum fr = \sum p = 1$.

* Prob. that X may lie between a and b ; $Pr [a < X < b] =$ area under curve from a to b .



The Normal Distribution

* The normal distribution is the most important of all probability distribution. It is applied directly to many practical problems, and several very useful distributions are based on it .

It is some times called the Gaussin dist. .

Characteristics :

Many empirical freq. dist. Have the following characteristics :

1. They are approximately symmetrical, and the mode is close to the centre of the dist.
2. The mean, median, and mode are close together.
3. The shape of the dist. Can be approximated by a bell.

* The total area bounded by the curve and the X axis is one. Hence, the area under the curve between $X=a$ and $X=b$, Where $a < b$ represent the prob. that X lies between a and b .

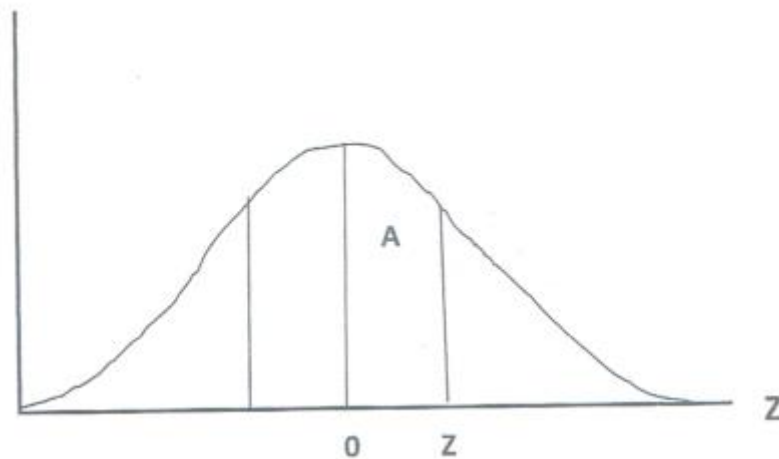
* Areas under the normal dist. Curve between 0 and Z are given in a :

Table (*). (Given below)

The prob. that Z lies between 0 and Z :

$$\Pr [0 < Z < z] = A$$

From table (*) the area between any two ordinates can be found by using the symmetry of the curve about $Z=0$.



* Some properties of the normal dist. :

$$\text{Mean} = \mu$$

$$\text{Variance} = \sigma^2$$

$$\text{Standard dev.} = \sigma$$

$$\text{Mean dev.} = \sigma \sqrt{\frac{2}{\pi}} = 0.7979 \sigma$$

* Areas under the normal curve :

When $Pr[0 < Z < Z_1] = A$

$Pr[-Z_1 < Z < 0] = A$ (symmetrical curve).

$Pr[Z_1 < Z] = 0.5 - A$ } Total area=1 so that area from
 $Pr[-Z_1 < Z] = 0.5 + A$ } $0 \rightarrow \infty$ is 0.5

When Z_1 and Z_2 are of same signs :

$$Pr[Z_1 < Z < Z_2] = A_{Z_2} - A_{Z_1}$$

When Z_1 and Z_2 are of different signs :

$$Pr[Z_1 < Z < Z_2] = A_{Z_2} + A_{Z_1}$$

* For bound of measurements, the bound in actual value is ∓ 0.5 units in L.S.D.

Example :

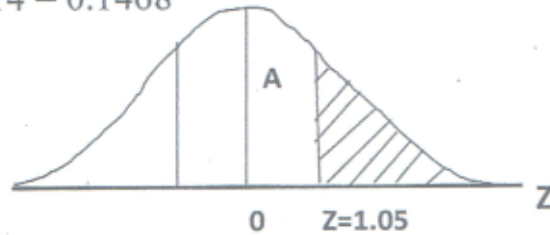
For measurements of $\mu = 160$, $\sigma = 10$, obtain the following :

1. $\Pr[X \text{ greater than } 170] = \Pr[X > 170]$

$$Z = \frac{X - \mu}{\sigma} = \frac{170.5 - 160}{10} = 1.05$$

$\therefore \Pr[Z > 1.05] = 0.5 - 0.35314 = 0.1468$

Where $A_{Z=1.05} = 0.35314$

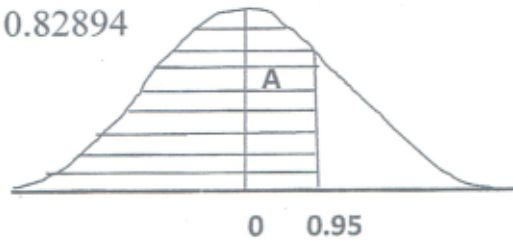


2. $\Pr[X \text{ less than } 170] = \Pr[X < 170]$

$x = 169.5 \rightarrow Z = 0.95$

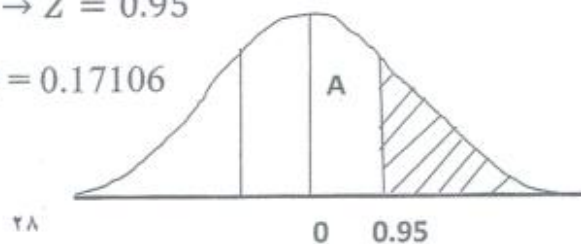
$\Pr [Z < 0.95] = 0.5 + 0.32894 = 0.82894$

Where $A_{Z=0.95} = 0.32894$



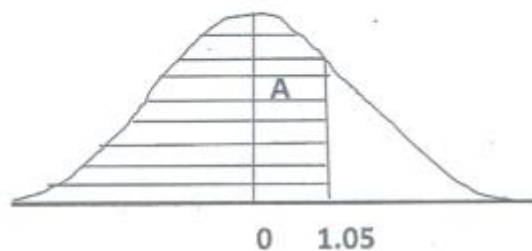
3. $\Pr[X \geq 170] \rightarrow x = 169.5 \rightarrow Z = 0.95$

$\Pr [Z > 0.95] = 0.5 - 0.32894 = 0.17106$



4. $\Pr[X \leq 170] \rightarrow x = 170.5 \rightarrow Z = 1.05$

$\Pr [Z > 1.05] = 0.5 + 0.35314 = 0.85314$



Linear inter polation :

When Z lies between successive Z_1 and Z_2 with respective A_1 and A_2 , A is obtained by linear interpolation "

$$\frac{Z - Z_1}{Z_2 - Z_1} = \frac{A - A_1}{A_2 - A_1} \quad , Z_1 < Z < Z_2$$

e.g. for $Z_1 = 2.32$ $A_1 = 0.48983$

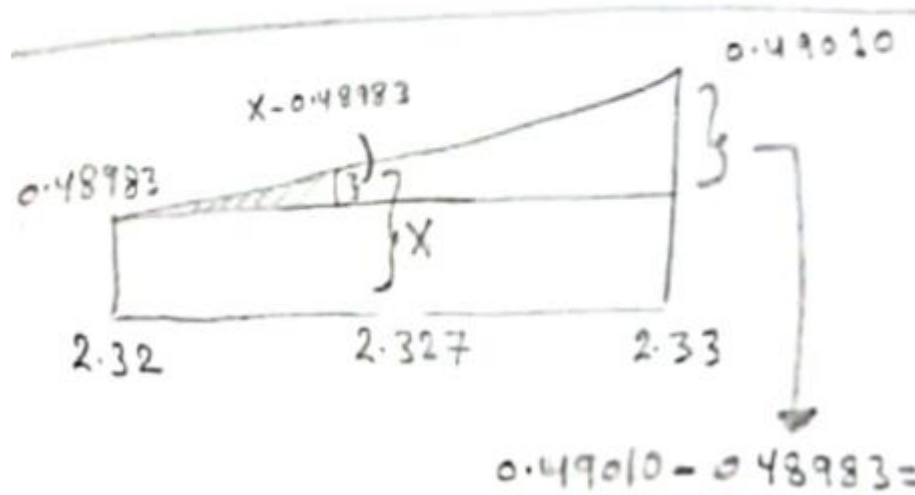
$Z_2 = 2.33$ $A_2 = 0.49010$

Then when $Z = 2.327 \rightarrow A = ?$

$$A = \frac{Z - Z_1}{Z_2 - Z_1} (A_2 - A_1) + A_1 = 0.49002$$

$$\frac{0.49010 - 0.48983}{2.33 - 2.32} = \frac{X - 0.48983}{2.327 - 2.32}$$

$$X = 0.49002$$



Example 1)

For a measurement of size $(N)=500$

$\mu = 151$, $\sigma = 15$, assuming normal dist. Find how many measurements :

a) between 120 and 155 = $\Pr[120 \leq X \leq 155]$

$$x_1 = 119.5 \rightarrow z_1 = -2.1 \rightarrow A_1 = 0.4821$$

$$x_2 = 155.5 \rightarrow z_2 = 0.3 \rightarrow A_2 = 0.1179$$

$$\Pr[-2.1 < Z < 0.3] = 0.4821 + 0.1179 = \left. \begin{array}{l} \text{No. of meas.} = \\ 500[0.4821+0.1179] \end{array} \right\} = 300$$

b) more than 185 = $\Pr[Z > 185]$

$$x = 185.5 \rightarrow Z = 2.3 \rightarrow A = 0.4893$$

$$\Pr[Z > 2.3] = 0.5 - 0.4893 = \left. \begin{array}{l} \text{No. of meas.} = \\ 500[0.51-0.4893] \end{array} \right\} = 5.$$

c) Less than 128 = $\Pr[X < 128]$

$$x = 127.5 \rightarrow Z = -1.57 \rightarrow A = 0.4418$$

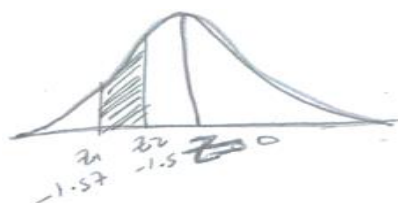
$$\text{No. of meas.} = 500[0.5-0.4418]=29$$

d) equal to 128 = $\Pr[X=128]$

$$x_1 = 127.5 \rightarrow z_1 = -1.57 \rightarrow A_1 = 0.4418$$

$$x_2 = 128.5 \rightarrow z_2 = -1.5 \rightarrow A_2 = 0.4332$$

$$\Pr[-1.57 < Z < -1.5] = 0.4418 - 0.4332 = \text{No. of meas.} = 500(0.4418 - 0.4332) = 4$$



e) Less than or equal to 128 = $\Pr [X \leq 128]$

$$x = 128.5 \rightarrow Z = -1.5 \rightarrow A = 0.4332$$

$$\text{No.} = 500[0.5 - 0.4332] = 33$$

f) Less than or equal to 185 = $\Pr [X \leq 185]$

$$x = 185.5 \rightarrow Z = 2.3 \rightarrow A = 0.4893$$

$$\text{No.} = 500[0.5 + 0.4893] = 495$$

Example 2)

For a sample of washers produced by a machine the mean inside dia. (μ) is 5.02 mm and the standard deviation is 0.05 mm. The max. useful tolerance in the dia. Is 4.96 to 5.08 mm, otherwise the washers are considered defective. Determine % of defective washers.

Solu.)

$$\Pr \text{ of max. tolerance} = \Pr (4.96 \leq X \leq 5.08)$$

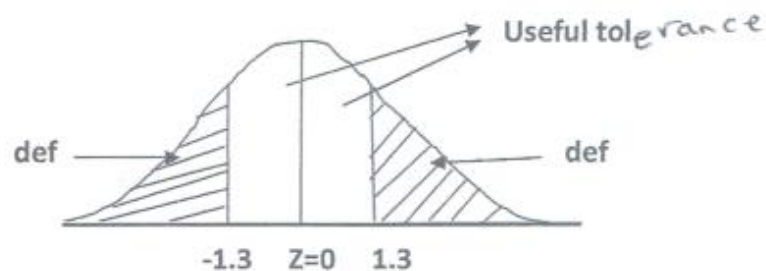
$$\text{One unit in L.S.D.} = 0.01$$

$$x_1 = 4.955 \rightarrow z_1 = -1.3 \rightarrow A_1 = 0.4032$$

$$x_2 = 5.085 \rightarrow z_2 = +1.3 \rightarrow A_2 = 0.4032$$

$$\Pr [-1.3 < Z < 1.3] = 2 * 0.4032 = 0.8064$$

$$\therefore \% \text{ of defective washers} = (1 - 0.8064) * 100 = 19.4 \%$$



Example 3)

Out of a large No. of examination applicant a sample of size 50 gave a mean mark of 64 and a standard dev. of 14 . What is the expected % of applicants achieving a min. pass mark of 50 ?

Solu. :

$\mu = 64$

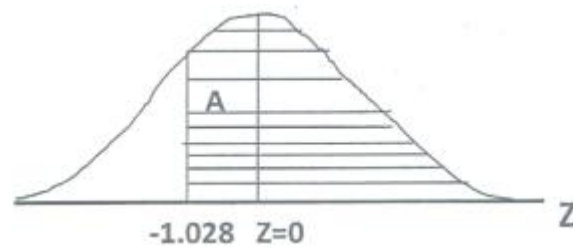
one unit = 1

$$\sigma = S \sqrt{\frac{N}{N-1}} = 14 \sqrt{\frac{50}{50-1}} = 14.1$$

Pr [app. Have a min. pass mark of 50] = Pr [50 ≤ X]

$x = 49.5 \rightarrow Z = -1.028 \rightarrow A = 0.3480$

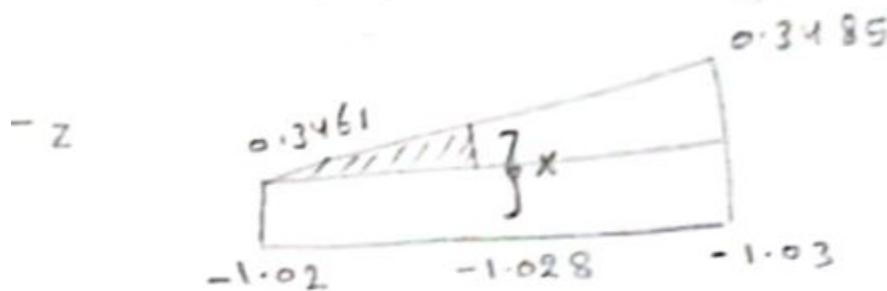
Pr [-1.028 < Z] = 0.348 + 0.5 = 0.848 = 84.8 %



interpolation

$$\% \frac{0.3485 - 0.3461}{-1.03 + 1.02} = \frac{x - 0.3461}{-1.028 + 1.02}$$

$x = 0.3480 = A$



Example 4)

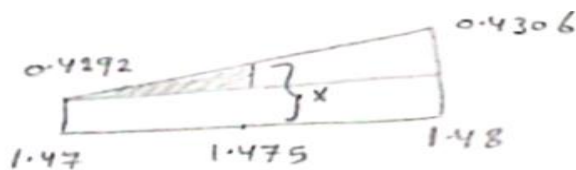
The strength of individual bars made by a certain manufacturing process are approximate normally distributed with mean 28.4 and standard dev. 2.95 . To ensure safety, a customer requires at least 95% of the bars to be stronger than 24.0 . (one unit = 0.1)

- a) Do the bars meet the specification ?
- b) By improved manufacturing techniques, the manufacturer make the bars more uniform (that is, decrease the standard dev.) what value of standard dev. will just meet the specification if the mean stays the same ?

Solu. :

$$\Pr [X > 24.0] \rightarrow Z_1 = \frac{X - \mu}{\sigma} = \frac{24.05 - 28.4}{2.95}$$

$$Z_1 = -1.475 \rightarrow A_1 = 0.4299$$

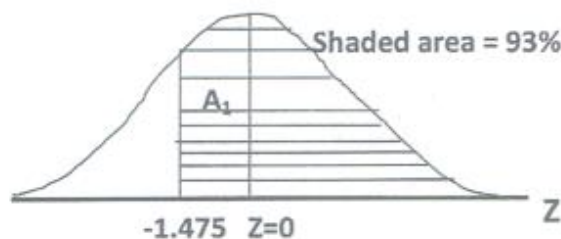


$$\frac{0.4306 - 0.4292}{1.48 - 1.47} = \frac{x - 0.4292}{1.475 - 1.47}$$

$$x = 0.4299 = A_1$$

$$\Pr [Z > -1.475] = 0.5 + 0.4299 = 0.9299 \approx 93\%$$

Since (93%) less than 95% , the bars do not meet the specification.



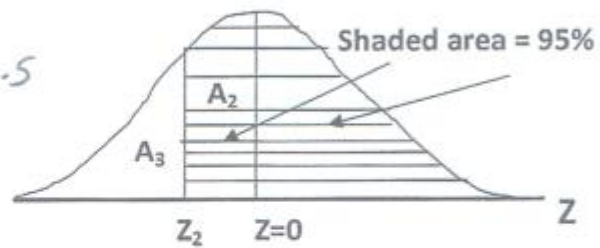
b) The specification is at least 95% of bars > 24.0

$$A_3 = 1 - 0.95 = 0.05$$

$$\therefore A_2 = 0.5 - 0.05 = 0.45$$

$$\text{or: } A_2 = 0.95 - 0.5$$

$$A_2 = 0.45$$



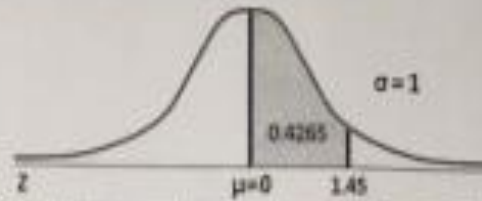
At $A_2 = 0.45 \rightarrow Z_2 = -1.645$ (from table)

$$\underline{Z_2} = \frac{X - \mu}{\sigma} \rightarrow -1.645 = \frac{24.05 - 28.4}{\sigma}$$

$\sigma = 2.644$ (if the σ can be reduced to 2.644 while keeping the mean constant, the specification will just be met)

Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
 For example, when Z score = 1.45 the area = 0.4265.



| Z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |
| 3.1 | 0.4990 | 0.4991 | 0.4991 | 0.4991 | 0.4992 | 0.4992 | 0.4992 | 0.4992 | 0.4993 | 0.4993 |
| 3.2 | 0.4993 | 0.4993 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4994 | 0.4995 | 0.4995 | 0.4995 |
| 3.3 | 0.4995 | 0.4995 | 0.4995 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4996 | 0.4997 |
| 3.4 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4997 | 0.4998 |
| 3.5 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 | 0.4998 |
| 3.6 | 0.4998 | 0.4998 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.7 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.8 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 | 0.4999 |
| 3.9 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5000 |

