Al-Mustaqbal University College

Chemical Engineering and Petroleum Industries department

DIFFERENTIAL EQUATIONS

2.2.1 First Order Differential Equations

Matlab can solve linear ordinary differential equations with or without initial/boundary conditions. Do not expect Matlab can solve nonlinear ordinary differential equations which typically have no analytical solutions. Higher derivatives can be handled as well. The command for finding the symbolic solution of differential equations is dsolve. For that command, the derivative of the function y is represented by Dy. For example, suppose that we want to find the solution of the equation $x\,y' - y = 1$. We will have:

>> dsolve('x*Dy-y=1', 'x')

ans =

-1+x*C1

This means that the solution is any function of the form $y = -1 + cx$, where c is any constant. The letter "D" has a special meaning and cannot be used otherwise inside **dsolve**. It means "first derivative of". The C1 is a constant of integration.

If we have the initial condition $y(1) = 5$, we can get the particular solution on the following way:

>> dsolve('x*Dy-y=1', 'y(1)=5', 'x')

ans =

-1+6*x

>>dsolve('Dy+y=cos(t)')

ans =

1/2*cos(t)+1/2*sin(t)+exp(-t)*C1

>> dsolve('Dy+2*y=12*t^2')

ans =

6*t^2-6*t+3+exp(-2*t)*C1

2.2.2 Second Order Equations

The second order linear equations can be solved similarly as the first order differential equations by using dsolve. For the command dsolve, the second derivative of y is represented with D2y. The letters "D2" mean second derivative.

For example, the command for solving $y''-3y'+2y = \sin x$.

>> dsolve('D2y-3*Dy+2*y=sin(x)', 'x')

ans =

3/10*cos(x)+1/10*sin(x)+C1*exp(x)+C2*exp(2*x)

If we have the initial conditions $y(0) = 1$, $y'(0)=-1$, we would have:

>> dsolve('D2y-3*Dy+2*y=sin(x)', 'y(0)=1', 'Dy(0)=-1', 'x')

ans =

3/10*cos(x)+1/10*sin(x)+5/2*exp(x)-9/5*exp(2*x)

Example: $d^2y/dx^2 -2dy/dx -3y=x^2$

>> dsolve('D2y - 2*Dy - 3*y=x^2', 'x')

ans =

-14/27+4/9*x-1/3*x^2+C1*exp(3*x)+C2*exp(-x)

Example: $d^2y/dx^2 -2dy/dx -3y=x^2$, **with y(0)=0,** and **dy/dx =1 at x=1**

>> dsolve('D2y - 2*Dy - 3*y=x^2','y(0)=0, Dy(1)=1','x')

ans =

-1/3*x^2+4/9*x-14/27+1/9*(-11+14*exp(3))/(3*exp(3)+exp(-1))*exp(-x)

+1/27*(33+1 4*exp(-1))/(3*exp(3)+exp(-1))*exp(3*x)

2.2.3 Higher Order Differential Equations

Similarity you can use the same way to solve the higher order differential equations.

3. Representing Functions

There is a way to define functions in MATLAB that behave in the usual manner. To represent a function in Matlab, we use "inline" command. For example to declare $f(x)=x2+3x+1$ you write:

>> f=inline('x^2+3*x+1')

f=

Inline function:

f(x) = x^2+3*x+1

Therefore to find f(2), to get the answer you write:

>> f(2)

ans =

11

The function $g(x,y)=x^2-3xy+2$ is defined as follows.

>> g=inline('x^2-3*x*y+2')

g =

Inline function:

g(x,y) = x^2-3*x*y+2

Now we can evaluate $g(2,3)$ in the usual way.

>>g(2,3)

ans =

 -12

In some cases, if we need to define function f as a vector. Then we use:

>> f = inline(vectorize('x^2+3*x-2'))

f =

Inline function:

f(x) = x.^2+3.*x-2

In this case, we can evaluate a function at more than one point at the same time. For example, to evaluate the above function at 1, 3 and 5 we have:

>> f([1 3 5])

ans =

2 16 38

4. Differentiation

The Matlab function that performs differentiation is **diff**. These operations show how it works:

>> syms x

>>diff(x^2)

ans =

2*x

>>diff(x^n)

ans =

x^n*n/x

>>diff(sin(x)^2)

ans =

2*sin(x)*cos(x)

For example, let's find the derivative of $f(x)=\sin(e^x)$.

>> syms x

>> diff(sin(exp(x)))

and get the answer as:

ans =

cos(exp(x))*exp(x)

Note: Instead of using syms to declare of variables you can use two Quotes '   ' to declare that the variable x is the interested variable in equation; you can use the same example in otherwise

>>diff('sin(exp(x))')

ans =

cos(exp(x))*exp(x)

The nth derivative of f is in the written in the form *diff(f,n).* then to find the second derivative we write;

>> diff(sin(exp(x)),2)

ans =

-sin(exp(x))*exp(x)^2+cos(exp(x))*exp(x)

For example to find the first derivative of $x3+3x2+8x$ you simply write:

>> syms x

>> diff(x^3+3*x^2+8*x)

ans =

3*x^2+6*x+8

Moreover to get the 3rd derivative, write:

>> diff(x^3+3*x^2+8*x ,3)

ans =

6

Note: To get higher derivatives, you can write the degree in place of 3.

To compute the partial derivative of an expression with respect to some variable we specify that variable as an additional argument in diff. For example to find the derivative for x in equation $f(x,y)=x^3y^4+y\sin x$.

>>syms x y

>> diff(x^3*y^4+y*sin(x),x)

ans =

3*x^2*y^4+y*cos(x)

Next we compute diff for y

>> diff(x^3*y^4+y*sin(x),y)

ans =

4*x^3*y^3+sin(x)

Finally we compute $d^3 fx^3$.

>> diff(x^3*y^4+y*sin(x),x,3)

ans =

6*y^4-y*cos(x)

5. Integration

By using the Symbolic Toolbox, you can find both definitive and in-definitive integrals of functions. We can use MATLAB for computing both definite and indefinite integrals using the command int. If **f** is a symbolic expression in **x**, then:

int(f) $\rightarrow \int f(x)dx$

For the indefinite integrals, consider the following example:

>> int('x^2')

ans =

1/3*x^3

Similarly as for diff command, we do not need the quotes if we declare x to be a symbolic variable. Therefore the above command can be re-written in otherwise such as:

>> syms x

>> int(x^2)

ans =

1/3*x^3

For example to find the in-definitive integral of $x^3+sin(x)$, you write:

>> syms x

>> int(x^3+sin(x))

ans =

1/4*x^4-cos(x)

Try these examples:

int(x)

int(a^x)

int(x*sin(x))

A definite integral can be taken by giving three arguments. The second and third arguments in that case are the first and second limits of integration.

int(f, a, b) $\rightarrow \int_{x=a}^{x=b} f(x)dx$

For the definitive integrals:

>> int(x^2, 0, 1)

ans =

1/3

Try these examples,

int(x,1,2)

int(x*sin(x),-2,7)

Moreover to get definitive integral to ln(x)+1/(x+1) from x=1 to x=2 write, you simply write:

>> int('ln(x) + 1/(x+1)', 1, 2)

ans =

-1+log(2)+log(3)

6. Limits

You can use limit to compute limits. For example, to evaluate the limit when x goes to 2 of the function $(x^2-4)/(x-2)$,we have:

>> syms x

>> limit((x^2-4)/(x-2), x, 2)

ans =

4

Limits at infinity:

>> limit(exp(-x^2-5)+3, x, Inf)

ans =

3