7. Integration methods/ powers of trigonometric function $\sin\theta$ & $\cos\theta$.

Some formulas of integration

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + c$

2. $\int u(x)^n dx = \frac{u(x)^{n+1}}{n+1} + c$

3. $\int \frac{du}{u} dx = ln\,|u| + c$

4. $\int e^u du = e^u + c$

5. $\int a^u du = \frac{1}{ln\,a} a^u + c$

6. $\int \frac{du}{\sqrt{1-u^2}} = \{ {\sin^{-1}(u)+c \atop -\cos^{-1}(u)+c} \}$

7. $\int \frac{du}{1+u^2} = \{ {\tan^{-1}(u)+c \atop -\cot^{-1}(u)+c} \}$

8. $\int \frac{du}{u\sqrt{u^2-1}} = \{ {\sec^{-1}(u)+c \atop -\text{ccsc}^{-1}(u)+c} \}$

powers of trigonometric function $\sin\theta$ & $\cos\theta$.

First ///If odd powers of ($\sin\theta$ & $\cos\theta$)

→ **using relation**($\sin^2 x = 1 - cos\; x^2$)or ($cos^2\, x = 1 -\; sin^2 x$)

Example 1: Find $\int cos^5 5x\, dx$.

Solution/ /

$$= \int (cos^2 5x)^2\, cos5xdx = \int (1 - sin^2 5x)^2\, cos5x\, dx$$

$$= \int (1 - 2sin^2 5x + sin^4 5x)\, cos5x\, dx$$

$$= \int (cos\ 5x\ dx - 2 \int sin^2 5x . cos\ 5x\ dx + \int sin^4 5x\ cos5x\ dx)$$

$$= \frac{sin\ 5x}{5} - \frac{2}{5*3} sin^3 5x + \frac{sin^5 5x}{25} + c$$

Example 2: ***Find*** $\int sin^5 x . cos^2 x\, dx$.

Solution //

$$= \int (sin^2 x)^2\ \ sin\ x\ \ cos^2 x\ dx$$

$$= \int (1 - cos^2 x)^2\ sin\ x\ \ cos^2 x\ dx$$

$$= \int (1 - 2cos^2 x + cos^4 x)\ sin\ x\ \ cos^2 x\ dx$$

$$= \int (sin\ x\ cos^2 x\ dx - 2 \int cos^4 x\ sin\ x\ dx + \int cos^6 x\ \ sinx\ dx)$$

$$= \frac{-cos^3 x}{3} + \frac{2}{5} cos^5 x - \frac{cos^7 x}{7} + c$$

H.W. // **1.** $\int sin^3 x\ \ dx$ **2.** $\int sin^3 x\ \ cos^2 x\, dx$

<u>Second</u> /// If even powers of ($\sin\theta$ & $\cos\theta$)

$\rightarrow$ **using relation**$(\sin^2 x = \frac{1}{2}(1 - \cos 2x) \quad or \; (\cos^2 x = \frac{1}{2}(1 + \cos 2x)$

Example: Find $\int \cos^2 2x \, dx$ **.**

Solution//

$$= \int \frac{1}{2}(1 + \cos 4x)dx = \frac{x}{2} + \frac{\sin 4x}{8} + c$$

Example : Find $\int \sin^4 3x \; dx$

Solution //

$$\int (\sin^2 3x)^2 \; dx = \int [\frac{1}{2}(1 - \cos 6x)]^2 \; dx$$

$$= \frac{1}{4}\int (1 - 2\cos 6x + \cos^2 6x) \, dx$$

$$= \frac{1}{4}\left(x - \frac{1}{3}\sin 6x + \int \frac{1}{2}(1 + \cos 12x)dx\right)$$

$$\frac{1}{4}x - \frac{1}{12}\sin 6x + \frac{1}{8}x + \frac{1}{4}\frac{1}{2}\frac{1}{12}\sin 12x + c$$

$$= \frac{1}{4}x - \frac{1}{12}\sin 6x + \frac{1}{8}x + \frac{1}{96}\sin 12x + c$$

Example : Find $\int \sin^2 x \cos^2 x \; dx$

Solution //

$$= \int \frac{1}{2}(1 - \cos 2x) \; \frac{1}{2}(1 + \cos 2x)dx$$

$$= \int \frac{1}{4}[(1 + \cos 2x - \cos 2x - \cos^2 2x]dx$$

$$=\int \frac{1}{4}[(1-\boldsymbol{cos^2 2x}]\boldsymbol{dx} = \int \frac{1}{4}[(1-\frac{\mathbf{1}}{\mathbf{2}}(\mathbf{1}+\boldsymbol{cos\ 4x}]\boldsymbol{dx}$$

$$=\frac{1}{4}[(x-\frac{\boldsymbol{x}}{\mathbf{2}}-\frac{\mathbf{1}}{\mathbf{8}}\boldsymbol{sin\ 4x}]+\boldsymbol{c}=\frac{\boldsymbol{x}}{\mathbf{8}}-\frac{\mathbf{1}}{\mathbf{32}}\boldsymbol{sin\ 4x}+\boldsymbol{c}$$

H.W.// $\int \boldsymbol{sin^4 x\ \ dx}$

<u>Third</u> //If multiplication of $\sin\theta$ & $\cos\theta$ with different angles

→ using relations

1-$\sin mx\ \ \sin nx = \frac{1}{2}[\cos(m-n)x - \cos(m+n)x]$

2-$\sin mx\ \ \cos nx = \frac{1}{2}[\sin(m-n)x + \sin(m+n)x]$

3-$\cos mx\ \ \cos nx = \frac{1}{2}[\cos(m-n)x + \cos(m+n)x]$

Example： Find $\int \boldsymbol{sin\ 3x\ \ cos\ 5x\ \ dx}$

Solution /

$$=\frac{1}{2}\int[sin(3-5)x + sin(3+5)x]\text{dx}$$

$$=\frac{1}{2}\int[sin(-2)x + sin(8)x]dx = \frac{1}{4}\cos 2x - \frac{1}{16}\cos 8x + c$$

H.W. // $\int_{\mathbf{0}}^{\boldsymbol{\pi}} \boldsymbol{cos\ 3x.cos\ 4x\ \ dx}$