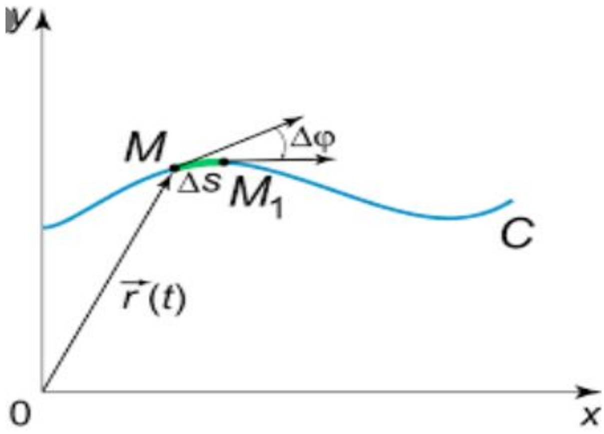
8. Curvature, Torsion & binormal vector: -

Curvature for curves in space



In space there is no natural way to find an angle like ∅ with which to measuring the change in **T** along a differential curve .but we still have **S** ,the directed distance along the curve and can define the curvature to be

$$\mathbf{K}=\left|\frac{dT}{dS}\right|$$

OR

$$\mathbf{K}=\frac{|vXa|}{|V^3|}$$

Example: Find the curvature of the curve, where a&b> *o*

$$\mathbf{r}=(\mathbf{a}\cos t\,\boldsymbol{i}+\mathbf{a}\sin t\,\boldsymbol{j}+\boldsymbol{btk})$$

Solution//

$$\mathbf{K}=\frac{|vXa|}{|V^3|}$$

$$V=\frac{dr}{dt}=-a\sin t\, i+a\cos t\, j+bk$$

$$a=\frac{dv}{dt}=\ (-a\cos t\, i-a\sin t\, j+0k)$$

$$|V|=\sqrt{(-a\sin t)^2+(a\cos t)^2)+b^{\,2}}$$
$$=\sqrt{(a^2\sin^2 t)+(a^2\cos^2 t)+b^{\,2}}$$
$$=\sqrt{a^2(\sin^2 t+\cos^2 t)+b^{\,2}}$$

$$|V|=\sqrt{a^2+b^2}$$

$$v\ X a=\begin{vmatrix} i & j & k \\ -a\sin t & a\cos t & b \\ -a\cos t & -a\sin t & 0 \end{vmatrix}$$

=+ab sin t i-ab cos t j+ka^2

$$|vXa|=\sqrt{(ab\sin t)^2+(-ab\cos t)^2+(a^2)^2}$$
$$=\sqrt{a^2b^2\sin^2 t+a^2b^2\cos^2 t+a^4}=\sqrt{a^2b^2+a^4}$$

$$|V|=\sqrt{a^2+b^2}$$

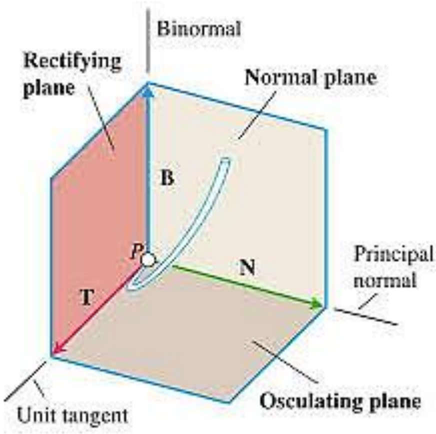
$$|V|^3=(a^2+b^2)^{\frac{3}{2}}$$

$$K=\frac{\sqrt{a^2b^2+a^4}}{(a^{2+}b^2)^{\frac{3}{2}}}$$

H.W.// Find the curvature of the curve,

r= $(c\cos t\, i+c\sin t\, j)$

<u>Torsion & binormal vector</u>



B=T x N

Binormal vector is perpendicular to both normal (N)& tangent(T) vector

The torsion $\tau = \left|\frac{dB}{dS}\right|$

It is measure of how mach the curve twists

$$\tau = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|v\ X\ a|^2} \qquad \text{if } \mathbf{v\ X\ a} \neq \mathbf{0}$$

Example: Find the torsion of the r= $(\cos t\, i + \sin t\, j + tk)$

Solution/

$$\tau = \frac{\begin{vmatrix} \dot{x} & \dot{y} & \dot{z} \\ \ddot{x} & \ddot{y} & \ddot{z} \\ \dddot{x} & \dddot{y} & \dddot{z} \end{vmatrix}}{|v\ X\ a|^2}$$

$$V=\frac{dr}{dt}=-\sin t\, i+\cos t\, j+k$$

a=-cos t i-sintj+0k

$$\dot{a}=\sin t\, i-\cos t\, j+0k$$

$$=\tau=\frac{\begin{vmatrix}-\sin t & \cos t & 1\\ -\cos t & -\sin t & 0\\ \sin t & -\cos t & 0\end{vmatrix}}{\begin{vmatrix} i & j & k\\ -\sin t & \cos t & 1\\ -\cos t & -\sin t & 0\end{vmatrix}^2}=\frac{\cos^2 t+\sin^2 t}{|\sin t\, i-\cos t\, j+k|^2}=\frac{1}{2}$$

H.W. Find the torsion for the curve r= $(3\sin t\, i+3\cos t\, j+4tk)$