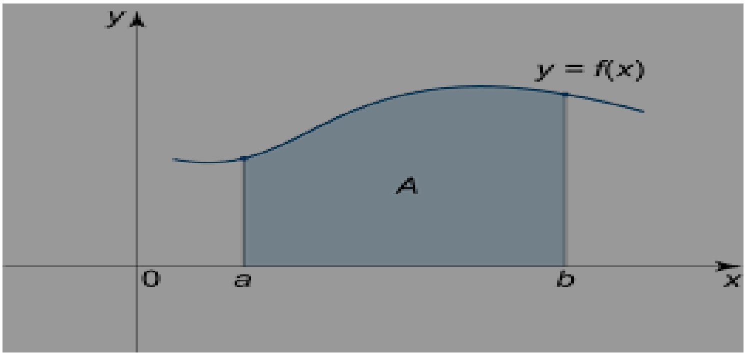
13.Integration application/area under the curve :-

1. The area under the curve

Define: Let F be continues function over the closed value [a, b], then the area under the curve define:-



$$A=\int_a^b f(x)\,dx \quad with\ x-axis$$

$$Or \quad A=\int_a^b f(y)\,dy \quad with\ y-axis$$

Example 1: Find the area under the curve bounded by the curve $y=\sqrt{x} \quad and\ 0 \le x \le 1\ \ with\ the\ x-axis$

Solution //

$$A=\int_a^b f(x)\,dx \quad \rightarrow A=\int_0^1 \sqrt{x}\,dx = \left.\frac{x^{\frac{3}{2}}}{3/2}\right|_0^1 = \frac{2}{3} unit^2$$

Example 1: Find the area bounded by the curve $\mathbf{y = x - x^2}$. with x − axis .

Solution /

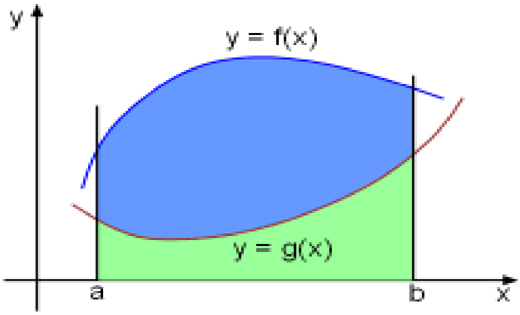
$$\mathbf{y = x - x^2. \text{with } x - \text{axis} \rightarrow 0 = x - x^2 \rightarrow x(1 - x^2) = 0}$$

x=0 & x=1

$$A = \int_0^1 (x - x^2)\, dx = \frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1 = \frac{1}{6} unit^2$$

2. The area between two curve

Define: Let F1&F2 are two functions over the closed value [a,b],then between two curves define as follows:-



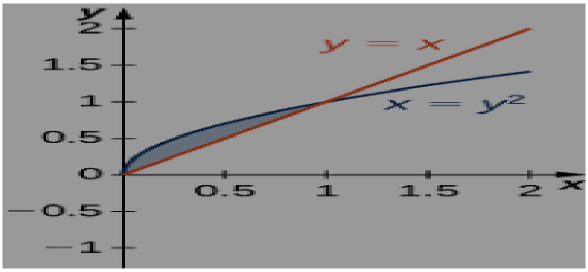
$$A=\int_a^b |f1(x) - f2(x)|\, dx \quad \textit{with } x - axis$$

Or $$A=\int_a^b |f1(y) - f2(y)| dx \quad \textit{with } y - axis$$

Example 1: Find the area of region bonded by the curves

$$y=\sqrt{x} \quad \& \quad y = x$$

Solution //



$$\sqrt{x} = x \rightarrow x - x^2 = 0 \rightarrow x(1 - x) = 0$$

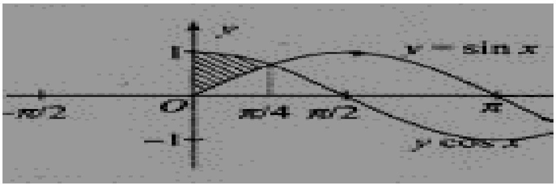
$$x=0 \ \& \ x=1$$

$$A = \int_0^1 (\sqrt{x} - x)\, dx = \frac{x^{\frac{3}{2}}}{3/2} - \frac{x^2}{2}\Bigg|_0^1 = \frac{1}{6} unit^2$$

Example 2: Find the area of region bonded by the curves

$$y=\sin x \ \& \ y = \cos x \ bounded\ by\ the\ lines\ x = 0 \ \& \ x = \frac{\pi}{2}$$

solution //



$$\sin x = \cos x \rightarrow \frac{\sin x}{\cos x} = 1 \rightarrow \tan x = 1$$

$$\mathbf{x}=\frac{\pi}{4} \in (\mathbf{0}, \frac{\pi}{2})$$

$$A = \int_{0}^{\frac{\pi}{4}} (\cos x - \sin x)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (\sin x - \cos x)dx$$

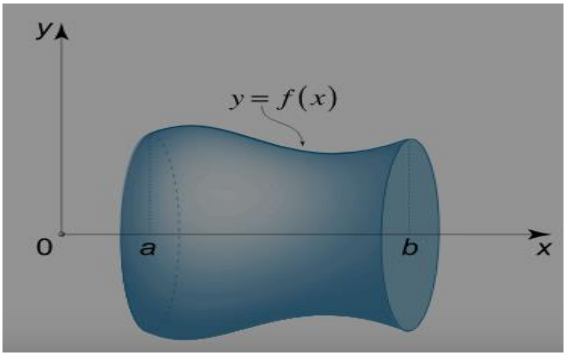
$$= \sin x + \cos x\Big|_{0}^{\frac{\pi}{4}} + (-\cos x - \sin x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{2}} =$$

$$(\frac{2}{\sqrt{2}} - 1) - (1 - \frac{2}{\sqrt{2}}) = \frac{4 - 2\sqrt{2}}{\sqrt{2}} unit^2$$

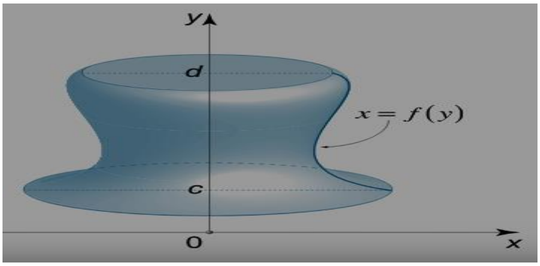
<u>14. Integration application/volume.</u>

A) by disk

1. Rotation with x-axis   [V=$\pi \int_a^b [f(x)]^2 \, dx$]



2. Rotation with y-axis [V=$\pi \int_{a}^{b}[f(y)]^2\,dy$]



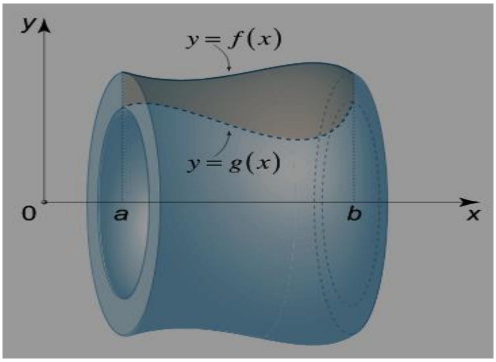
Example: Find the volume of the solid generated by revolving curve $y = \sqrt{x}$ $with\ x - axis\ from\ x = 0\ to\ x = 1$

Solution/

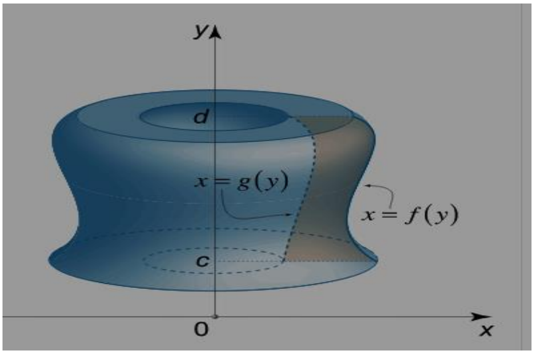
V=$\pi \int_{0}^{1}[\sqrt{x}]^2\,dx = \frac{\pi}{2} unit^3$

B) by washer

1. Rotation with x-axis [V=$\pi \int_{a}^{b}[f1(x)^2 - f2(x)^2]dx$



2. Rotation with y-axis $[V=\pi \int_a^b [f1(y)^2 - f2(y)^2]dy]$



Example: Find the volume of the solid bounded by revolving curve $y = \sqrt{x}$ &$y = x$. *is revolving with*

a). $x - axis$

b). $y - axis$

Solution//

a). $x - axis$ $\sqrt{x} = x \rightarrow x - x^2 = 0 \rightarrow x(1 - x) = 0$

x=0 & x=1

$$V = \pi \int_0^1 ((\sqrt{x})^2 - x^2)dx = \pi \left(\frac{x^2}{2} - \frac{x^3}{3}\Big|_0^1\right) = \frac{\pi}{6} unit^3$$

b). $y - axis$ $x1 = x2 \rightarrow y = y^2 \rightarrow y - y^2 = 0$

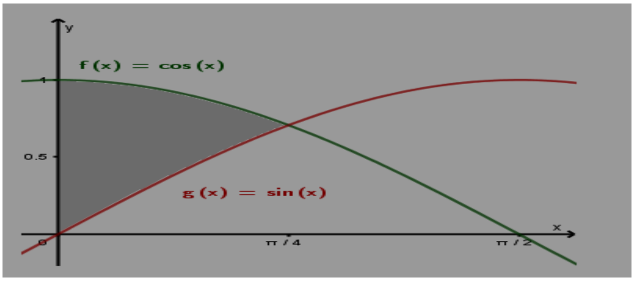
y=0 & y=1

$$V = \pi \int_0^1 (y)^2 - y^4)\, dx = \pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15} unit^3$$

Example: Find the volume of the solid bonded by the curves

$y=\sin x$ & $y=\cos x \,.\, 0 \le x \le \frac{\pi}{2}$ *is revolved about the x − axis*

Solution//



$$\sin x = \cos x \rightarrow \frac{\sin x}{\cos x} = 1 \rightarrow \tan x = 1$$

$$x=\frac{\pi}{4} \in (0.\frac{\pi}{2})$$

$$V = \pi[\int_{0}^{\frac{\pi}{4}} ((\cos x)^2 - \sin x)^2)dx + \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} ((\sin x)^2 - \cos x)^2)dx]$$

V=? (H.W.) unit^3