Al-Mustaqbal University College

Chemical Engineering and Petroleum Industries department

Chapter (4)

Measures of Dispersion

Dispersion is the degree of data spread about an average. Several measures are used including:

Range, mean absolute deviation , standard deviation , variance and coefficient of variation.

Mean Absolute Deviation :

Is the arithmetic mean of the absolute deviations.

$$M.A.D = \frac{\sum |x_i - \bar{x}|}{N} \qquad \text{for raw data}$$

$$= \frac{\sum f_i \, |x_i - \bar{x}|}{N} \qquad \text{for grouped data}$$

Mean other than $\bar{x}$ may be used to obtain M.A.D from the respective mean.

Standard Deviation :

Is the root mean square of the deviation.

$$S = \sqrt{\frac{\sum(x_i - \bar{x})^2}{N}}$$ for raw material

$$S = \sqrt{\frac{\sum f_i(x_i - \bar{x})^2}{N}}$$ for grouped data

$$\bar{x} = \frac{\sum x_i}{N}$$ (for raw data)

$$\bar{x} = \frac{\sum f_i x_i}{\sum f_i}$$ (for grouped data, $\sum fi = N$)

Standard deviation of a sample (S) is related to the standard deviation of the population (σ) by :

a) Standard deviation of a sample (S) and mean ($\bar{x}$)
b) Standard deviation of a popuation (σ)and mean (μ)

$$\sigma = S\sqrt{\frac{N}{N-1}} = \sqrt{\frac{\sum f_i(x_i - \bar{x})^2}{N-1}}$$

$$\sigma = \sqrt{\frac{\sum fi\ (xi - \bar{x})^2}{N-1}}$$ (for sample)

$$\sigma = \sqrt{\frac{\sum fi\ (xi - \mu)^2}{N-1}}$$ (for population)

Variance :

Is the square of the standard deviation i.e. S^2 for sample, σ^2 for population.

Coefficient of Variation :

Is a relative dispersion measure (dimension less).

$$Relative\ Dispersion = \frac{absolute\ dispersion}{average}$$

$$Coefficient\ of\ Variation = \frac{S}{\bar{x}}$$

Where S = Standard deviation

$\bar{x}$ = The arithmetic mean

Properties of Standard Deviation :

* Of all standard deviations, the min. is that from the arithmetic mean.

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$$S = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}}$$

$$S = \sqrt{\frac{\Sigma f_i (x_i - \bar{x})^2}{N}}$$

$$S^2 = \frac{\Sigma f_i (x_i - \bar{x})^2}{N}$$

$$S^2 = \frac{\Sigma f_i (x_i^2 - 2x_i\bar{x} + \bar{x}^2)}{N}$$

$$S^2 = \frac{\Sigma f_i x_i^2 - 2\bar{x}\Sigma f_i x_i + \bar{x}^2 \Sigma f_i}{N}$$

$$\frac{\partial}{\partial \bar{x}} = \frac{-2\Sigma f_i x_i + 2\bar{x}\Sigma f_i}{N} = 0$$

$$-2\Sigma f_i x_i + 2\bar{x}\Sigma f_i = 0$$

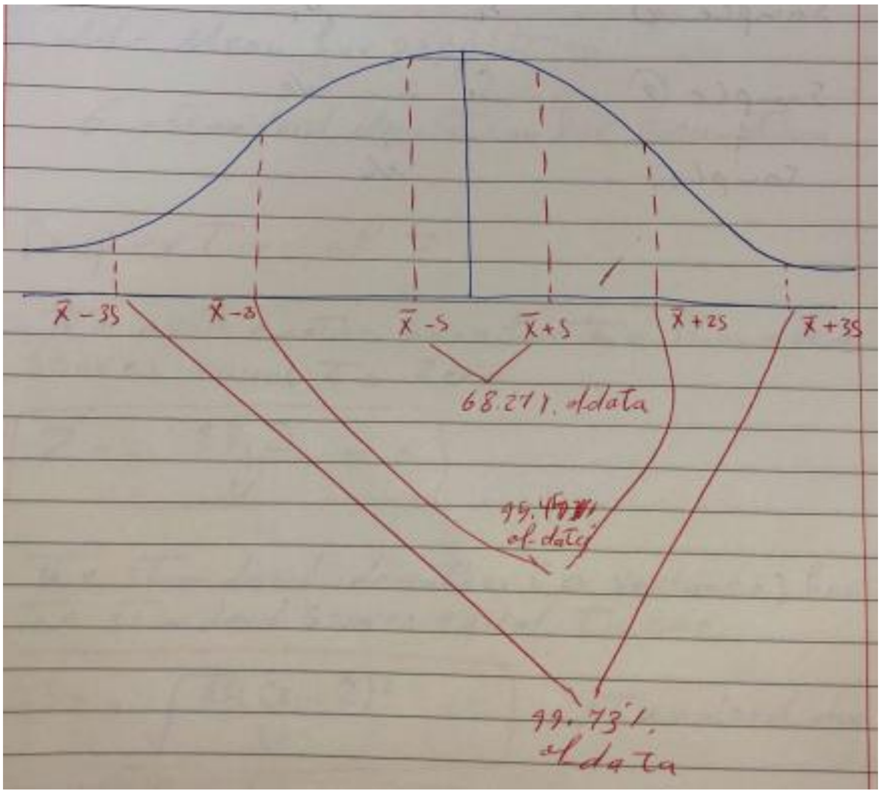
$$2\Sigma f_i x_i = 2\bar{x}\Sigma f_i$$

$$\bar{x} = \frac{\cancel{2}\Sigma f_i x_i}{\cancel{2}\Sigma f_i}$$

$$\bar{x} = \frac{\Sigma f_i x_i}{\Sigma f_i}$$

* For ideal normal distributions:

With in $\bar{x} \mp S$	$68.27\%\ of\ data$
$\bar{x} \mp 2\,S$	$95.45\%\ of\ data$
$\bar{x} \mp 3\,S$	$99.73\%\ of\ data$



* For several samples, the combined S is given by :

$$S^2 = \frac{N_1\,S_1^2 + N_2\,S_2^2 + \ \dots}{N_1 + N_2 + \cdots}$$

Standard Variable :

The dimensional measurements x_i may be expressed as dimension less standardized variables Z_i

$$Z_i = \frac{x_i - \bar{x}}{S} = \frac{(\bar{x} + S) - \bar{x}}{S} = 1$$

i.e. when Z=1 , the measurement is removed by one standard deviation from the mean.

Properties of Z :

1. The arithematic mean for the standard scores equal to zero.

$$\acute{Z} = \frac{\sum f_i Z_i}{N} = 0$$

2. The standard deviation (or variance) for the standard scores equal to **one**.

$$S_Z = \sqrt{\frac{\sum f_i (Z_i - \bar{Z})^2}{N}} = 1$$

$$S_Z^2 = \frac{\sum f_i (Z_i - \bar{Z})^2}{N} = 1$$

Q 2) For the following data :

Class limits	f
60 – 62	5
63 – 65	18
66 – 68	42
69 – 71	27
72 - 74	8

Obtain :

A) S , S^2 B) Z C) $\acute{Z}$, S_z

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Q2 For The following data:

class LimiTs	f	x_i	$f_i x_i$	$f_i(x_i-\bar{x})^2$	Z_i	$f_i z_i$
60 _ 62	5	61	305	208.0125	-2.209	-11.045
63 _ 65	18	64	1152	214.245	-1.182	-21.276
66 _ 68	42	67	2814	8.505	-0.154	-6.468
69 _ 71	27	70	1890	175.5675	-0.875	23.571
72 _ 74	8	73	584	246.42	1.9	15.2
			6745	852.75	-0.7713	-0.018

obTain:

A) S, S^2 B) Z C) $\bar{Z}, S_z$

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A. $\bar{X} = \dfrac{\Sigma f_i x_i}{\Sigma f_i} = \dfrac{6745}{100} = \boxed{67.45}$

$S = \sqrt{\dfrac{\Sigma f_i (x_i - \bar{x})^2}{\Sigma f_i}}$

$S = \sqrt{\dfrac{852.75}{100}} = \boxed{2.92}$

$S^2 = (2.92)^2 \Longrightarrow S^2 = \boxed{8.527}$

B. $Z_i = \dfrac{x_i - \bar{x}}{S}$

$Z_1 = \dfrac{61 - 67.45}{2.92} = -2.209$

$Z_2 = \dfrac{64 - 67.45}{2.92} = -1.182$

eTc

C. $\bar{Z} = \dfrac{\Sigma f_i z_i}{\Sigma f_i} = \dfrac{-0.018}{100} = -0.00018 \simeq 0$

$f_i(z_i - \bar{z})^2$

24.39

25.14

0.994

20.51

29.885

99.9

$$S_z = \sqrt{\frac{\sum f_i (z_i - \bar{z})^2}{\sum f}} = \sqrt{\frac{99.9}{100}} \simeq 1$$