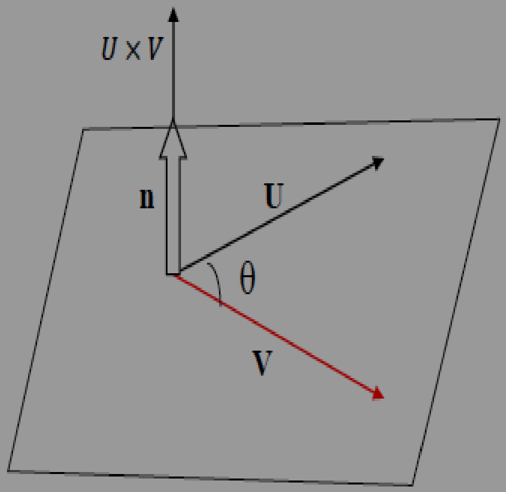
2. Vector/cross product.

If U an V are not parallel, they determine a plane. A unit vector (n) is perpendicular to the plane by (right hand rule).

This mean that we choose (n) to be unit vector. The cross product ($U \times V$) means that (U cross V).



Properties of cross product:

1. $\mathrm{U}\times \boldsymbol{V} = -\boldsymbol{V} \times \boldsymbol{U}$

2. $\mathrm{U}\times (\boldsymbol{V} + \boldsymbol{W}) = \boldsymbol{U} \times \boldsymbol{V} + \boldsymbol{U} \times \boldsymbol{W}$

3. $\mathbf{0}\times \boldsymbol{U} = \mathbf{0}$

Calculate cross products, by using Determinates.

$$If\ U = u_1 i + u_2 j + u_3 k$$

$$V = v_1 i + v_2 j + v_3 k$$

$$U \times V = \begin{bmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_2 \end{bmatrix}$$

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

Area of plane=$|U \times V|$

Example 1: Find U$\times \boldsymbol{V}. \boldsymbol{IF}\ \boldsymbol{U} = \boldsymbol{i} + \boldsymbol{j} + \boldsymbol{k}\ .\boldsymbol{V} = -2\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and find area of plane.

Solution //

$$\mathbf{U} \times \boldsymbol{V} = \begin{bmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{2} & \mathbf{1} \end{bmatrix}$$

$$= \boldsymbol{i} \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} \end{bmatrix} - \mathbf{j} \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{1} \end{bmatrix} + \boldsymbol{k} \begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{2} \end{bmatrix} = -\boldsymbol{i} - \mathbf{3}\boldsymbol{j} + \mathbf{4}\boldsymbol{k}$$

Area of plane=$|\boldsymbol{U} \times \boldsymbol{V}|$

$$=\sqrt{(-\mathbf{1})^2 + (-\mathbf{3})^2 + (\mathbf{4})^2}=\sqrt{\mathbf{26}}$$

Example 2: Find unit vector perpendicular to the plane .$P(1.-1.2).Q(2.0.-1).R(0.2.1)$ $and\ find\ area\ of\ plane$.

Solution //

PQ= (2-1)i+(0-(-1)j+(-1-2)k=i+j-3k

PR= (0-1)i+(2-(-1)j+(1-2)k=-i+3j-k

$$PQ\times PR = \begin{bmatrix} i & j & k \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{bmatrix}$$

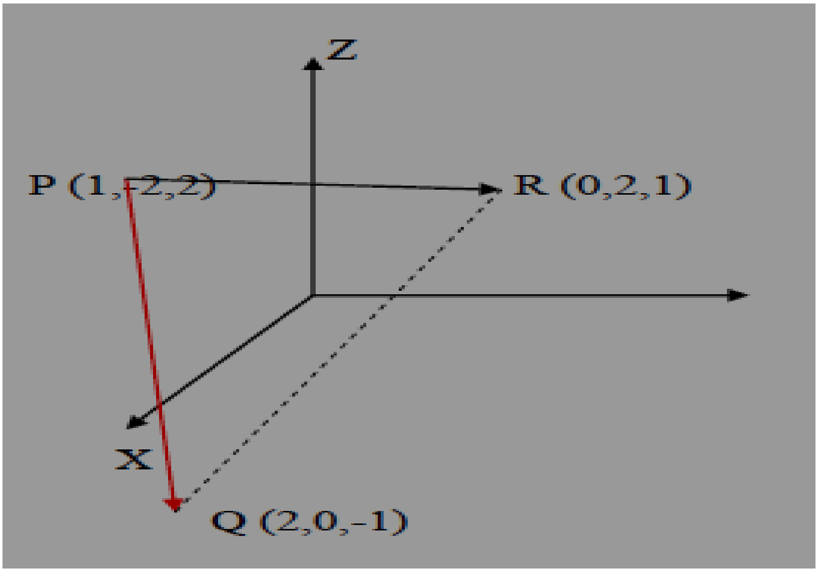
$$PQ \times PR = i\begin{bmatrix} 1 & -3 \\ 3 & -1 \end{bmatrix} - j\begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix} + k\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$$

$$= 8i + 4j + 4k$$

$$|PQ \times PR| = \sqrt{(8)^2 + (4)^2 + 4)^2} = \sqrt{96}$$

Unit vector=$\frac{PQ\times PR}{|PQ\times PR|} = \frac{8i+4j+4k}{\sqrt{96}}$

Area of plane=$|PQ \times PR| = \sqrt{96}$



H.W. //

Example: Find unit vector perpendicular to the plane $.P(1.1.1).Q(2.1.3).R(0.2.1) and\ find\ area\ of\ plane$