Mathematics II

Revision:

1. Double integration (Area, moment, centroid, moment of inertia & volume)

2. Ordinary Differential Equation (ODE).

3. Methods and techniques for solving differential equations and systems of differential equations, for first order

-variable separable.

-Homogenous.

-Exat

-linear.

4. Linear second order homogenous equation with constant coefficient

5.Linear second order Non homogenous equation with constant coefficient

6. Higher order linear equation with constant coefficent

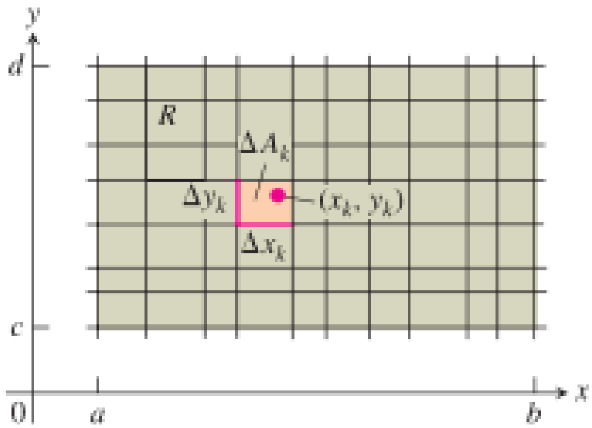
7. Infinite series.

8. Taylor polynomials

1. Double Integration (Area, moment, centroid, moment of inertia & volume)

-<u>Area</u>

$$A=\iint dx\,dy \quad or \quad A=\iint dy\,dx$$



Example //Find area enclosed by x=3, x=1, y=0,y=2.using double integration.

Solution/ /

$$A=\iint dx\,dy$$

$$A=\int_0^2\int_1^3 dx\,dy=\int_0^2 (x)\Big|_1^3 dy=\int_0^2 2\,dy=(2y)\Big|_0^2=4$$

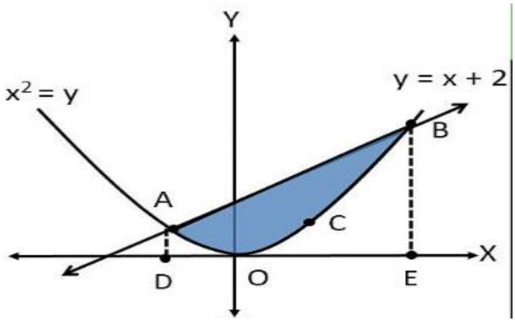
Or

$$A=\iint dy\,dx$$

$$A=\int_1^3\int_0^2 dy\,dx=\int_1^3 (y)\Big|_0^2 dx=\int_1^3 2\,dx=(2x)\Big|_1^3=4$$

Example //Find the area enclosed by y=x+2 & .
y=x^2

Solution //



$$y_1=y_2$$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

(X+1) (x-2)=0

X=-1 , x=2

A=$\iint dy\, dx$

A=$\int_{-1}^{2}\int_{x^2}^{x+2} dy\, dx = \int_{-1}^{2}(y)\Big|_{x^2}^{x+2} dx = \int_{-1}^{2}(x+2) - x^2)\, dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big|_{-1}^{2} = 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3} = 4.5$

Example//Evaluate $\int_0^{\pi}\int_x^{\pi} \frac{\sin y}{y} dy\, dx$

Solution // x_1=0 , x_2= $\pi. y1 = \pi. y2 = x$

$$\int_0^{\pi}\int_0^{y}\frac{\sin y}{y}\,dx\,dy=\int_0^{\pi}\left(\frac{\sin y}{y}x\right)\Big|_0^{y}dy$$

$$=\int_0^{\pi}\sin y\;dy=(-\cos x)\Big|_0^{\pi}=2$$

<u>Application of double integration:-</u>

1-volume $=\iint zdA=\iint z\,dx\,dy=\iint z\,dy\,dx$

2-moment

Mx$=\iint ydA=\iint y\,dx\,dy=\iint y\,dy\,dx$

My$=\iint xdA=\iint x\,dx\,dy=\iint x\,dy\,dx$

3-Center of area

$$\overline{X}=\frac{My}{A}.\quad \overline{Y}=\frac{Mx}{A}$$

$$\left[where\ A=\iint dA=\iint dx\,dy=\iint dy\,dx\right]$$

4. Moment of Inertia

Ix$=\iint y^2dA\quad .Iy=\iint x^2dA.Ir=\iint(x^2+y^2)dA$

Example// Find the centroid enclosed function

Y=6x-x^2 . $y = x$

Solution /

$$y_1=y_2$$

$$6x - x^2 = x$$

$$x^2 - 6x + x = 0$$

$$x^2 - 5x = 0$$

$$X\ (x-5\ ) = 0$$

$$X=0 \text{ or } x=5$$

$$\bar{X} = \frac{My}{A}. \quad \bar{Y} = \frac{Mx}{A}$$

A=$\iint dy\,dx = \int_0^5 \int_x^{6x-x^2} dy\,dx = \int_0^5 (y)|_x^{6x-x^2} dx = \int_0^5 (6x - x^2 - x)\,dx = 125/6$

Mx=$\iint y dA = \iint y\,dy\,dx$

Mx=$\int_0^5 \int_x^{6x-x^2} ydy\,dx = \int_0^5 \left(\frac{y^2}{2}\right)\Big|_x^{6x-x^2} dx = \int_0^5 (\frac{(6x-x^2)^2}{2} - \frac{(x)^2}{2})\,dx = 104.16$

Also My=625/12

$$\bar{X} = \frac{My}{A} = \frac{\frac{625}{12}}{\frac{125}{6}} = 2.5$$

$$\bar{Y} = \frac{Mx}{A} = \frac{104.16}{125/6} = 5$$

Example//Calculate second moment of inertia about y-axis for area enclosed by y=x^2 , y=x+2.

Solution //

$$y_1 = y_2$$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

X=2 or x=-1

$$Iy = \iint x^2 dA$$

$$= \iint x^2 dy\, dx = \int_{-1}^{2}\int_{x^2}^{x+2} x^2 dy\, dx =$$

$$= \int_{-1}^{2} (x^2 y)\Big|_{x^2}^{x+2} dx$$

$$= \int_{-1}^{2} x^2(x + 2 - x^2)dx = \frac{x^4}{4} + \frac{2x^3}{3} - \frac{x^5}{5}\Big|_{-1}^{2}$$

$$= 365\ unit\ of\ length$$

Example// Find the volume enclosed by two surfaces

$$z_1=2+x^2 \quad +y^2 \qquad ,z_2=4-x^2-y^2$$

Solution //

$$z_1=z_2$$

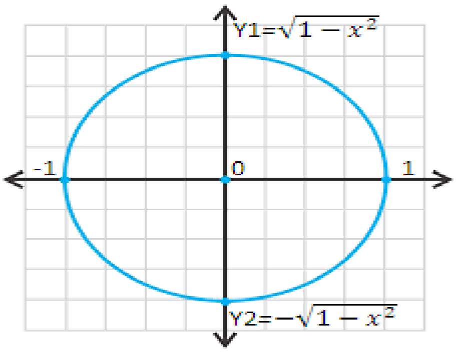
$$2+x^2+y^2=4-x^2-y^2$$

$$2+x^2+y^2-4+x^2+y^2=0$$

$$2x^2+2y^2-2=0$$

$$x^2+y^2=1 \leftrightarrow x^2+y^2=r^2$$

$$y=\sqrt{1-x^2}$$



$$\text{Volume} =\iint z dA=\iint z\, dy\, dx=\iint (Z2-Z1)\, dy\, dx= \iint (4-x^2-y^2-2-x^2-y^2)\, dy\, dx$$

$$\text{Volume}=2\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}(2-2x^2-2y^2)dy\, dx$$

Or Volume=$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}}(2-2x^2-2y^2)dy\,dx$

Or Volume=4 $\int_{0}^{1}\int_{0}^{\sqrt{1-x^2}}(2-2x^2-2y^2)dy\,dx$

v=4 $\int_{0}^{1}\left(2y-2x^2y-\frac{2y^3}{3}\right)\Big|_{0}^{\sqrt{1-x^2}}dx$

$$=4\int_{0}^{1}\left(2\sqrt{1-x^2}-2x^2(1-x^2)-\frac{2(\sqrt{1-x^2})^3}{3}\right)dx$$