

Mathematic I

**4.Matrix.**

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

**Example: -Find the matrix A+B and A-B**

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 2 \\ 2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 5 \\ 1 & 2 \\ 0 & 1 \end{bmatrix}$$

**Solution//**

$$C = A + B = \begin{bmatrix} 3 + 2 & 1 + 5 \\ 4 + 1 & 2 + 2 \\ 2 + 0 & -1 + 1 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 5 & 4 \\ 2 & 0 \end{bmatrix}$$

$$D = A - B = \begin{bmatrix} 3 - 2 & 1 - 5 \\ 4 - 1 & 2 - 2 \\ 2 - 0 & -1 - 1 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 3 & 0 \\ 2 & -2 \end{bmatrix}$$

**Example:-Find the matrix  $A \times B$** 

$$A = \begin{bmatrix} 1 & -1 \\ 2 & 0 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 4 & 0 & -1 \\ 1 & 1 & 2 \end{bmatrix}$$

**Solution//**

$$= \begin{bmatrix} (1 \times 4) + (-1 \times 1) & (1 \times 0) + (-1 \times 1) & (1 \times -1) + (-1 \times 2) \\ (2 \times 4) + (0 \times 1) & (2 \times 0) + (0 \times 1) & (2 \times -1) + (0 \times 2) \\ (-1 \times 4) + (3 \times 1) & (-1 \times 0) + (3 \times 1) & (-1 \times -1) + (3 \times 2) \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 3 & -1 & -3 \\ 8 & 0 & -2 \\ -1 & 3 & 7 \end{bmatrix}$$

**5.Determinates:-**

## 1. Minors and Cofactors:

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

$$[A] = a_{11} \times \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12} \times \begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13} \times \begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

2. Special method is used only when matrix  $3 \times 3$ 

$$[A] = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \end{bmatrix}$$

**Properties of Determinates:-**

1. If all elements of row or column are equal a zero →  
*determinate value = 0*

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 0 & 0 \\ 5 & 3 & -2 \end{bmatrix} = [D] = 0$$

2. If similar elements a row two or column two →  
*determinate value = 0*

$$\begin{bmatrix} -1 & 2 & 3 \\ 3 & 0 & -2 \\ -1 & 2 & 3 \end{bmatrix} = [D] = 0 \quad \text{or} \quad \begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 2 \\ 4 & 2 & 4 \end{bmatrix} = [D] = 0$$

**Example //Find the Determinate of [A]**

$$A = \begin{bmatrix} 3 & 1 & 2 \\ 4 & 2 & -2 \\ 2 & -1 & 3 \end{bmatrix}$$

**Solution/**

$$\begin{aligned} [A] &= 3 \begin{bmatrix} 2 & -2 \\ -1 & 3 \end{bmatrix} - 1 \begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix} + 2 \begin{bmatrix} 4 & 2 \\ 2 & -1 \end{bmatrix} \\ &= 3[6 - 2] - 1[12 - (-4)] + 2[-4 - 4] = 12 - 16 - 16 \\ &= -20 \end{aligned}$$

OR

$$\begin{vmatrix} 3 & 1 & 2 \\ 4 & 2 & -2 \\ 2 & -1 & 3 \end{vmatrix} \begin{matrix} 3 & 1 \\ 4 & 2 \\ 2 & -1 \end{matrix}$$

$$= 3 * 2 * 3 + 1 * -2 * 2 + 2 * 4 * (-1) - 2 * 2 * 2 - 3 * (-2) * (-1) - 1 * 4 * 3$$

$$=-20$$

H.W.  $\begin{vmatrix} 3 & 1 & 2 \\ 4 & 2 & -K \\ 2 & -1 & 3 \end{vmatrix} = 0$ . Find value  $K$

### 6.Grammer Rule:

$$x = \frac{Dx}{D}, \quad y = \frac{Dy}{D}, \quad z = \frac{Dz}{D}$$

Example//Find  $x, y$  and  $z$  from these equations by using Grammer Rule.

$$x+y+z=2 \quad \dots\dots 1$$

$$3x-y+2z=2 \quad \dots\dots 2$$

$$2x-3y+3z=1 \quad \dots\dots 3$$

Solution //

$$\begin{aligned}
 D &= \begin{bmatrix} 1 & 1 & 1 \\ 3 & -1 & 2 \\ 2 & -1 & 3 \end{bmatrix} \\
 &= 1 \begin{bmatrix} -1 & 2 \\ -3 & 3 \end{bmatrix} - 1 \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} + 1 \begin{bmatrix} 3 & -1 \\ 2 & -3 \end{bmatrix} \\
 &= 3 - 5 - 7 = -9
 \end{aligned}$$

$$\begin{aligned}
 Dx &= \begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & 2 \\ 1 & -1 & 3 \end{bmatrix} \\
 &= 2 \begin{bmatrix} -1 & 2 \\ -3 & 3 \end{bmatrix} - 1 \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} + 1 \begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix} \\
 &= 6 - 4 - 5 = -3
 \end{aligned}$$

$$x = \frac{Dx}{D} = \frac{-3}{-9} = \frac{1}{3}$$

$$\begin{aligned}
 Dy &= \begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 2 & 1 & 3 \end{bmatrix} = 1 \begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} - 2 \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} + 1 \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \\
 &= 4 - 10 - 1 = -7
 \end{aligned}$$

$$y = \frac{Dy}{D} = \frac{-7}{-9} = \frac{7}{9}$$

$$\begin{aligned}
 DZ &= \begin{bmatrix} 1 & 1 & 2 \\ 3 & -1 & 2 \\ 2 & -3 & 1 \end{bmatrix} \\
 &= 1 \begin{bmatrix} -1 & 2 \\ -3 & 1 \end{bmatrix} - 1 \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} + 2 \begin{bmatrix} 3 & -1 \\ 2 & -3 \end{bmatrix} \\
 &= 5 + 1 - 14 = -8 \\
 Z &= \frac{DZ}{D} = \frac{-8}{-9} = \frac{8}{9}
 \end{aligned}$$

**H.W.** Find x,y and z from these equations by using Grammer Rule.

$$x+y=1 \dots\dots 1$$

$$y+z=1 \dots\dots 2$$

$$x+y+z=1 \dots\dots 3$$