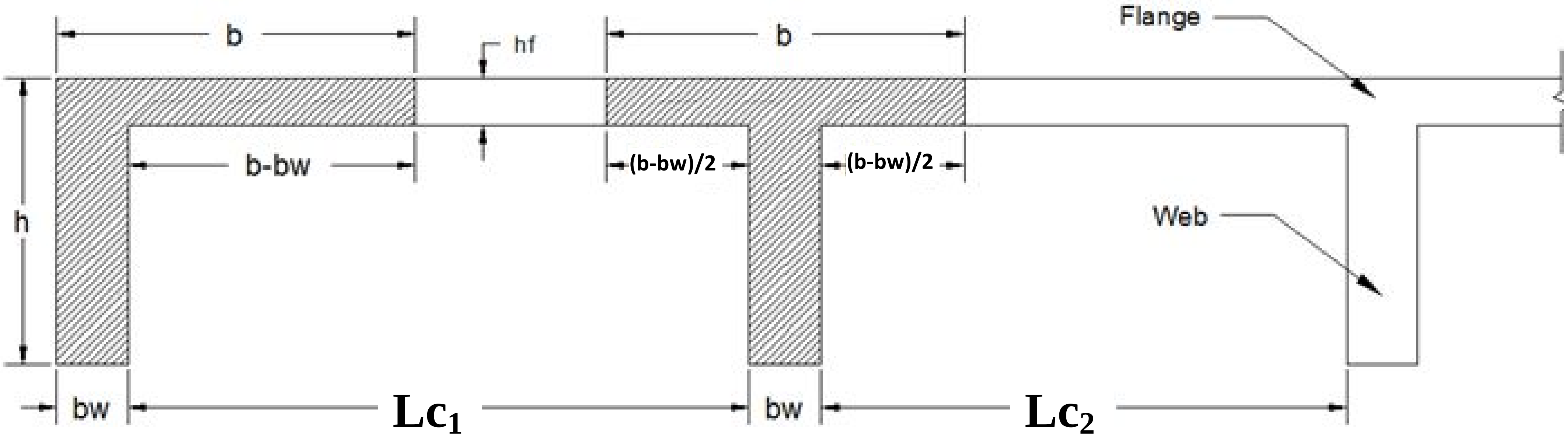
T-Beam section (ACI code 8.10)



b:Effective flange width

$(b - bw) or \frac{b-bw}{2}$: Over hange

L:span length

hf:slab thickness

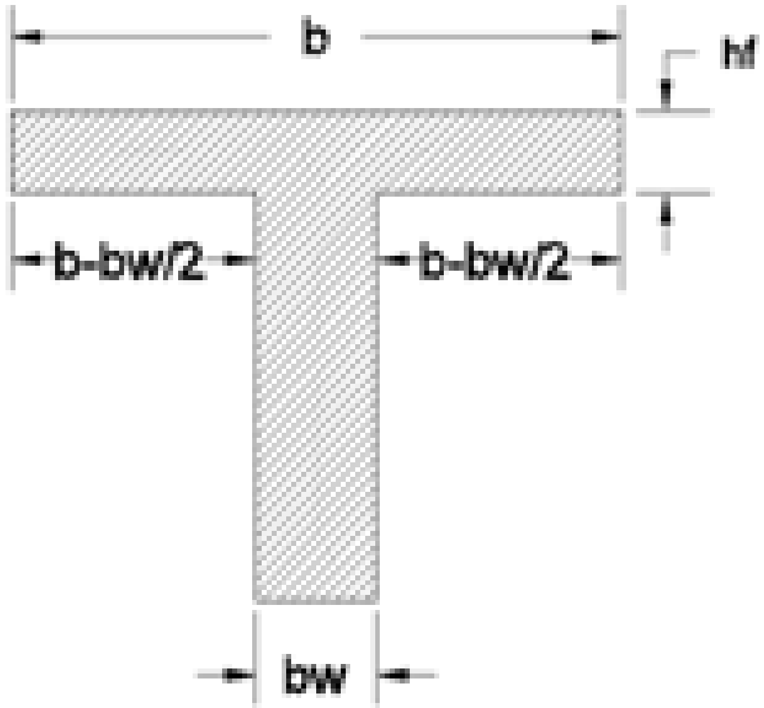
lc_1, lc_2:clear distance to next beam

bw:web width

1. Symmetrical T-Beam

$$\begin{bmatrix} b \le \frac{L}{4} \\ \frac{b-bw}{2} \le 8hf \\ \frac{b-bw}{2} \le \frac{lc_1+lc_2}{4} \end{bmatrix}$$

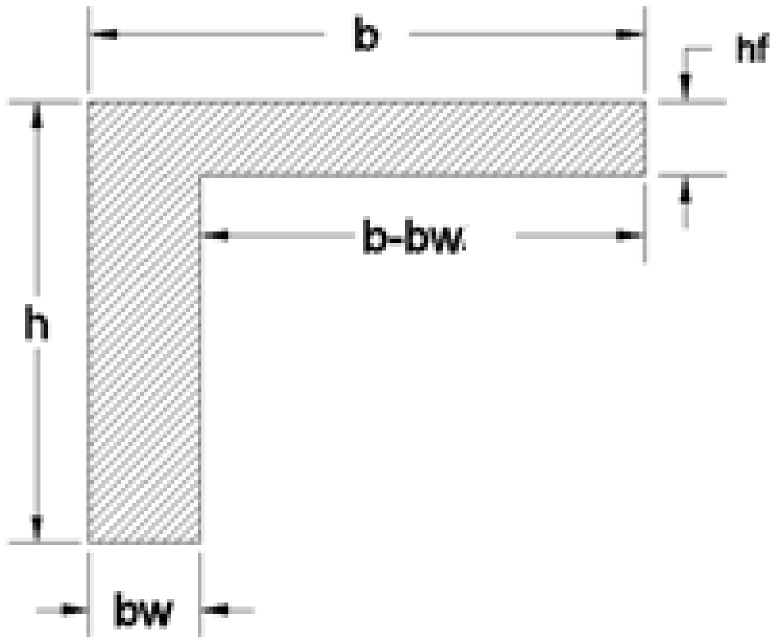
Choose minimum value of (b)



2. L-section (Spandrel beam)

$$\begin{bmatrix} b - bw \leq \frac{L}{12} \\ b - bw \leq 6hf \\ b - bw \leq \frac{lc_1}{2} \end{bmatrix}$$

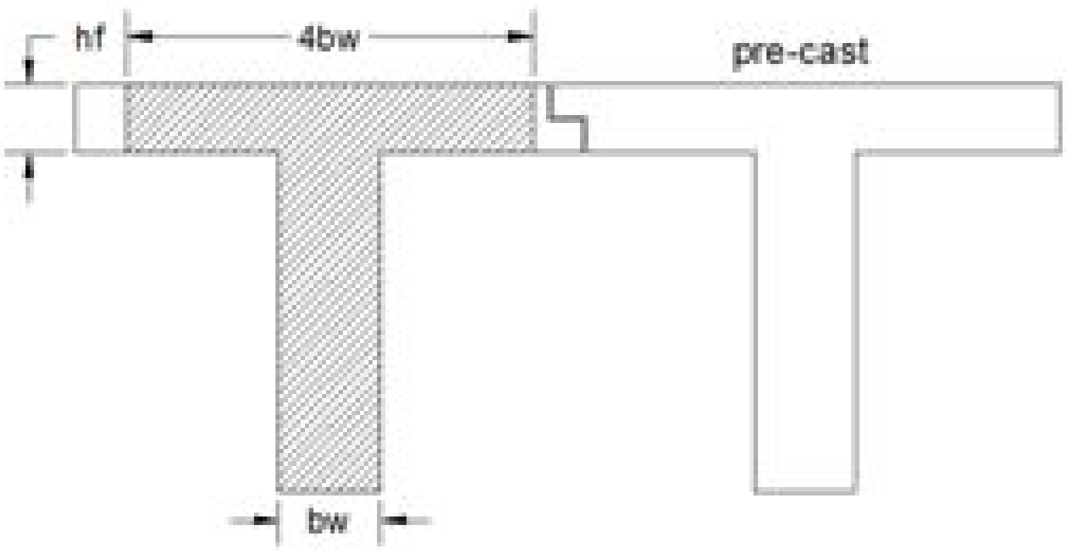
Choose minimum value of (b)



3. Isolated T-Beam

$$hf \geq \frac{bw}{2}$$

$$b \leq 4bw$$



Analysis of T-beam

1. **$a \le h_f$ → Rectangular section**

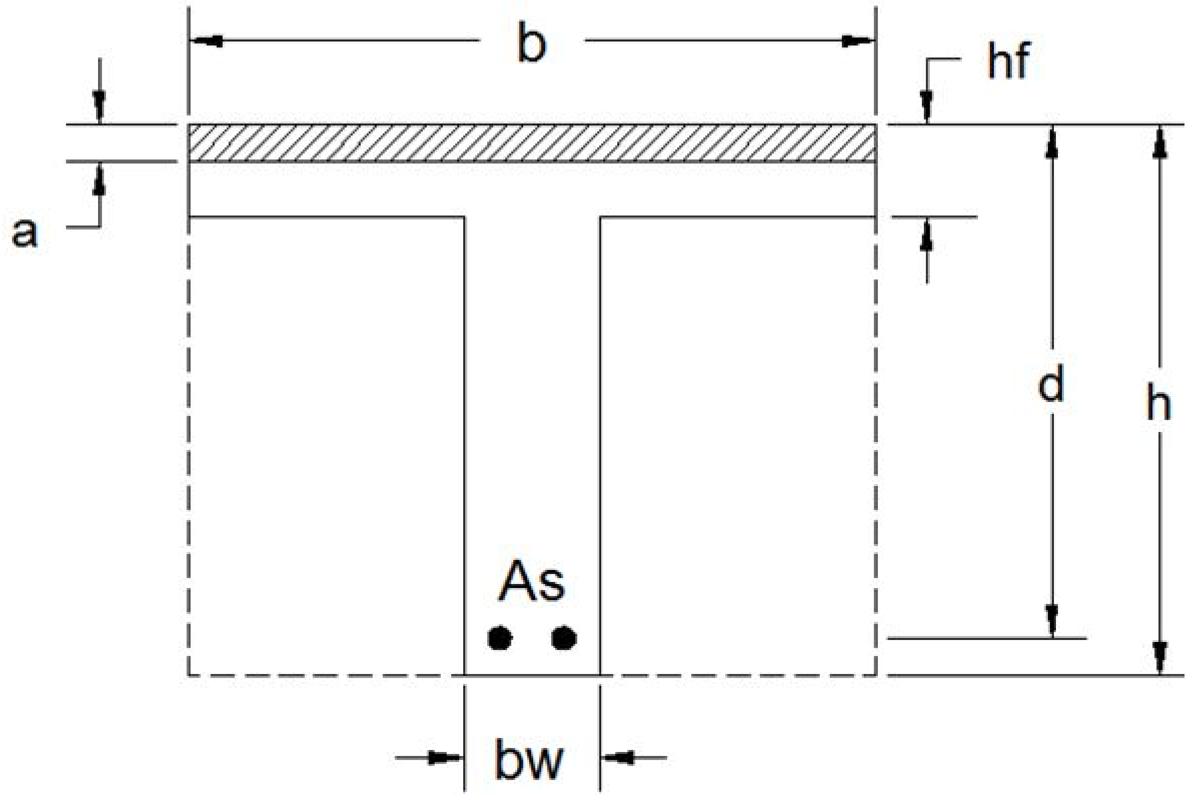
$$\sum Fx = 0$$

$$Asfy = 0.85fc'ba$$

$$a = \frac{Asfy}{0.85fc'b} \le h_f$$

$$Mu = \emptyset \rho bd^2 fy \left(1 - 0.59\rho \frac{fy}{fc'}\right)$$

$$= \emptyset Asfy \left(d - \frac{a}{2}\right)$$



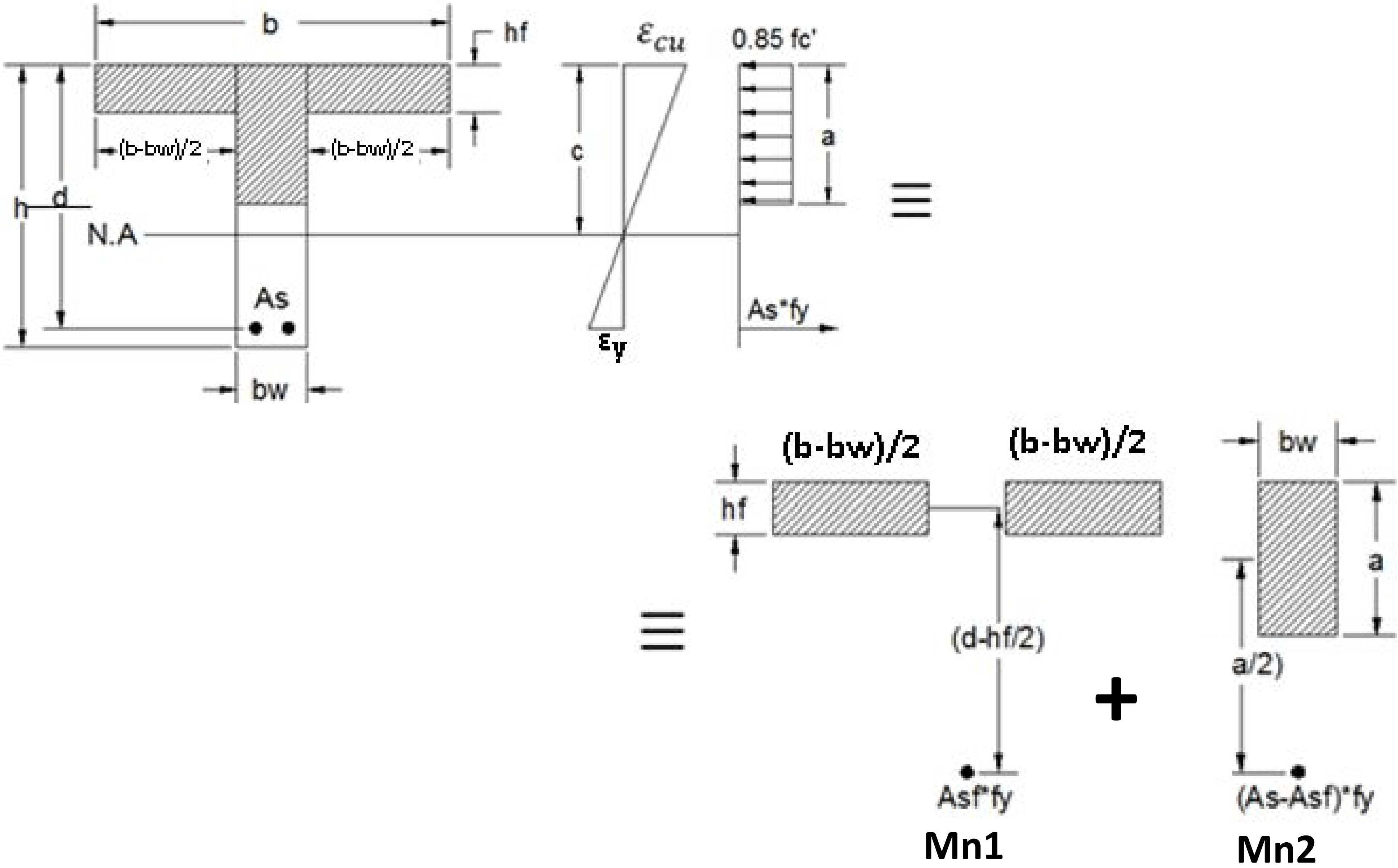
$$\rho = \frac{As}{bd}$$

$$\rho_{max} = 0.85\beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004}$$

$$\rho_{min} = \max\left(\frac{1.4}{fy}, \frac{\sqrt{fc'}}{4fy}\right) * \frac{bw}{b}$$

$$\rho_{min} \leq \rho \leq \rho_{max}$$

2. $a > h_f \rightarrow$ T-Beam section



$$Mn=Mn_1+Mn_2=(As-Asf)fy\left(d-\frac{a}{2}\right)+Asf*fy\left(d-\frac{h_f}{2}\right)$$

$$Mu = \emptyset Mn$$

$$\sum Fx = 0$$

$$Asf*fy=0.85fc^{'}(b-bw)h_f$$

$$Asf = 0.85\frac{fc^{'}}{Fy}(b-bw)h_f \ldots\ldots\ldots.(1)$$

$$\sum Fx = 0$$

$$(As-Asf)fy=0.85\ fc^{'}bw\ a$$

$$a = \frac{(As-Asf)}{bw}\frac{fy}{0.85\ fc^{'}} \ldots\ldots.(2)$$

$$\text{Let}\rho_w = \frac{As}{b_w d} \quad , \qquad \rho_f = \frac{Asf}{b_w d}$$

$$eq1 \rightarrow Asf = 0.85\frac{\text{fc}^{'}}{\text{fy}}(b - b_w)h_f \qquad \div b_w d$$

$$\rho_f = 0.85\frac{\text{fc}^{'}}{\text{fy}}\left(\frac{b}{b_w} - 1\right)\frac{h_f}{d}$$

$$eq2 \rightarrow a = \frac{As - Asf}{bw}\frac{\text{fy}}{0.85\ \text{fc}^{'}} * \frac{d}{d}$$

$$a = \left(\rho_w - \rho_f\right)\frac{\text{fy}}{0.85\ \text{fc}^{'}} * d$$

Balanced Steel Ratio for T-Beam

$$\sum Fx = 0$$

$$AsFy = 0.85fc^{'}b_{w}a + 0.85\ fc^{'}(b - b_{w})h_{f} \quad \div b_{w}d\ fy$$

$$\frac{As}{b_{w}d} = 0.85\frac{fc^{'}}{fy}\frac{a}{d} + 0.85\frac{fc^{'}}{fy}(b - b_{w})hf * \frac{1}{b_{w}d}$$

$$\rho_{w_{b}} = 0.85\frac{fc^{'}}{fy}\frac{a}{d} + \rho_{f} \ldots\ldots(1)$$

From strain digram

$$\frac{\varepsilon_{cu}}{c_{b}} = \frac{\varepsilon_{cu} + \varepsilon_{y}}{d} \rightarrow c_{b} = \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{y}}.d$$

$$a = \beta_1 c = \beta_1 \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_y} d = \beta_1 \frac{600}{600+\text{fy}} d \ldots .(2)$$

$sub\ eq_2 in\ eq_1$

$$\rho_{w_b} = 0.85\beta_1 \frac{\text{fc}'}{\text{fy}} \frac{600}{600+\text{fy}} + \rho_f$$

$$\rho_{w_b} = \rho_b + \rho_f$$

To find $\rho_{w_{max}}$, flow the steps above , but $\varepsilon_s = 0.004$

$$\rho_{w_{max}} = 0.85\ \beta_1 \frac{\text{fc}'}{\text{fy}} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004} + \rho_f$$

$$\rho_{min} = \max\left(\frac{1.4}{\mathrm{fy}}, \frac{\sqrt{\mathrm{fc}'}}{4\mathrm{fy}}\right)$$

$$\rho_{min} \leq \rho_w \leq \rho_{w_{max}}$$

Reduction factor $\emptyset$

To find ρ_{w_t}, flow the steps above , but $\varepsilon_s = 0.005$

$$\rho_{w_t} = 0.85\,\beta_1 \frac{\mathrm{fc}'}{\mathrm{fy}} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} + \rho_f = \rho_t + \rho_f$$

$$IF\ \rho_w \leq \rho_{w_t} \rightarrow \emptyset = 0.9$$

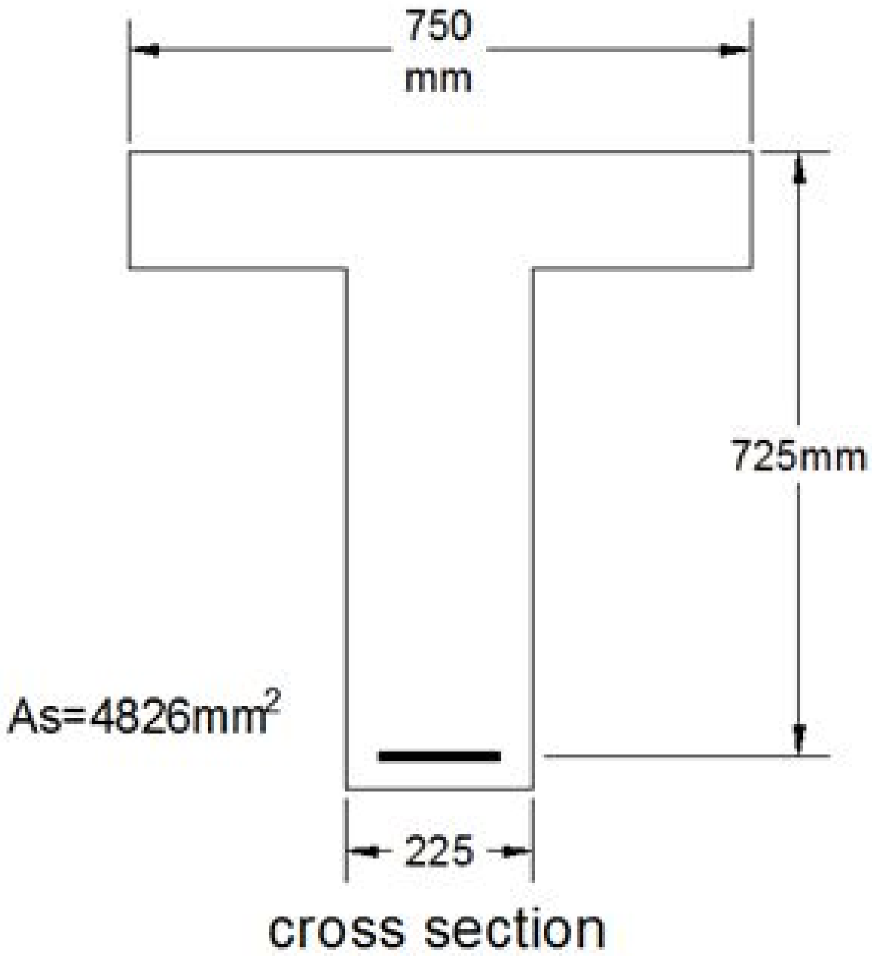
$$IF\ \rho_w > \rho_{w_t} \rightarrow \emptyset = 0.483 + 83.3\varepsilon_t$$

$$\frac{\varepsilon_{cu}}{c} = \frac{\varepsilon_t + \varepsilon_{cu}}{dt}$$

$$\varepsilon_t = \frac{\varepsilon_{cu} * dt}{c} - \varepsilon_{cu} \qquad (\varepsilon_{cu} = 0.003)$$

Ex1: Isolated T-Beam $\frac{S}{c} = \frac{400}{20}$MPa find Mu

$$if \begin{bmatrix} h_f = 140mm \\ h_f = 180mm \end{bmatrix}$$



Solution:

1)$\boldsymbol{h_f = 140mm}$

$$h_f = 140mm > \frac{b_w}{2} = 112.5\,mm(o.k)$$

$$b = 750mm < 4b_w = 900mm\ o.k$$

$$Let\ a < h_f\ \ R.c(b = 750mm, d = 725mm)$$

$$\sum Fx = 0$$

0.85fc'ba=Asfy

$$a = \frac{4826*10^{-6}*400}{0.85*20*0.75} = 0.151m > h_f\ \rightarrow \therefore T - section$$

$$\sum Fx = 0$$

$$As_f * \text{fy} = 0.85\text{fc}'(b - b_w)hf$$

$$Asf = \frac{0.85*20*(0.75-0.225)*0.14}{400} * 10^6 = 3124mm^2$$

$$\rho_f = \frac{Asf}{b_w d} = \frac{3124}{225 * 725} = 0.0192$$

$$\rho_w = \frac{As}{b_w d} = \frac{4826}{225 * 725} = 0.0296$$

$$\rho_{w_{max}} = 0.85\ \beta_1 \frac{\text{fc}'}{\text{fy}} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004} + \rho_f$$

$$\rho_{w_{max}} = \rho_{max} + \rho_f$$

$$= 0.85 * 0.85 * \frac{20}{400} * \frac{0.003}{0.003 + 0.004} + 0.0192 = 0.0347$$

$$\rho_{min} = \max\left(\frac{1.4}{fy} = 0.0035, \frac{\sqrt{fc'}}{4fy} = 0.0028\right) = 0.0035$$

$$\rho_{min} < \rho_w < \rho_{w_{max}} \quad \text{O.K}$$

$$\sum Fx = 0$$

$$(As - Asf)fy = 0.85fc'b_w a$$

$$a = \frac{(4826 - 3124) * 10^{-6} * 400}{0.85 * 20 * 0.225} = 0.178m > h_f \;\; o.k$$

$$\rho_{w_t} = 0.85\,\beta_1 \frac{\text{fc}'}{\text{fy}} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} + \rho_f = \rho_t + \rho_f$$

$$\rho_{w_t} = 0.85 * 0.85 * \frac{20}{400} * \frac{0.003}{0.003 + 0.005} + 0.0192 = 0.0327$$

$$\rho_w < \rho_{w_t} \rightarrow \emptyset = 0.9$$

$$Mu = \emptyset\left[(As - Asf)\text{fy}\left(d - \frac{a}{2}\right) + Asf * \text{fy}\left(d - \frac{h_f}{2}\right)\right]$$

$$Mu = 0.9\left[(4826 - 3124)*10^{-6}*400\left(0.725 - \frac{0.178}{2}\right) + 3124 * 10^{-6} * 400\left(0.725 - \frac{0.14}{2}\right)\right] = 1.126 MN.m$$

OR

$$Mu = \emptyset\left[0.85fc'b_w a\left(d - \frac{a}{2}\right) + 0.85fc'(b - b_w)h_f\left(d - \frac{h_f}{2}\right)\right]$$

$$Mu = 0.9\left[0.85 * 20 * 0.225 * 0.178\left(0.725 - \frac{0.178}{2}\right) + 0.85 * 20 * (0.75 - 0.225) * 0.14\left(0.725 - \frac{0.14}{2}\right)\right] = 1.126MN.m$$

2) hf = 180mm

$$hf = 180mm > \frac{b_w}{2} = 112.5mm\ o.k$$

$$a = 151mm < h_f = 180\text{mm} \quad \therefore Rectangular\ section$$

$$\rho = \frac{As}{b_w d} = \frac{4826}{750 * 725} = 0.0089$$

$$\rho_{max} = 0.85 * 0.85 * \frac{20}{400} * \frac{0.003}{0.003 + 0.004} = 0.0155$$

$$\rho_{min} = \max\left(\frac{1.4}{\text{fy}} * \frac{\text{b}_\text{w}}{\text{b}}, \frac{\sqrt{\text{fc}'}}{4\text{fy}} * \frac{b_w}{b}\right) = 0.00105$$

$$\rho_{min} < \rho < \rho_{max} \quad o.k$$

$$\rho_t = 0.85 * 0.85 * \frac{20}{400} * \frac{0.003}{0.003 + 0.005} = 0.0135$$

$$\rho < \rho_t \rightarrow \emptyset = 0.9$$

$$Mu = \emptyset \rho b \mathrm{d}^2 \mathrm{fy} \left(1\text{-}0.59\ \rho \frac{\mathrm{fy}}{\mathrm{fc}'}\right)$$

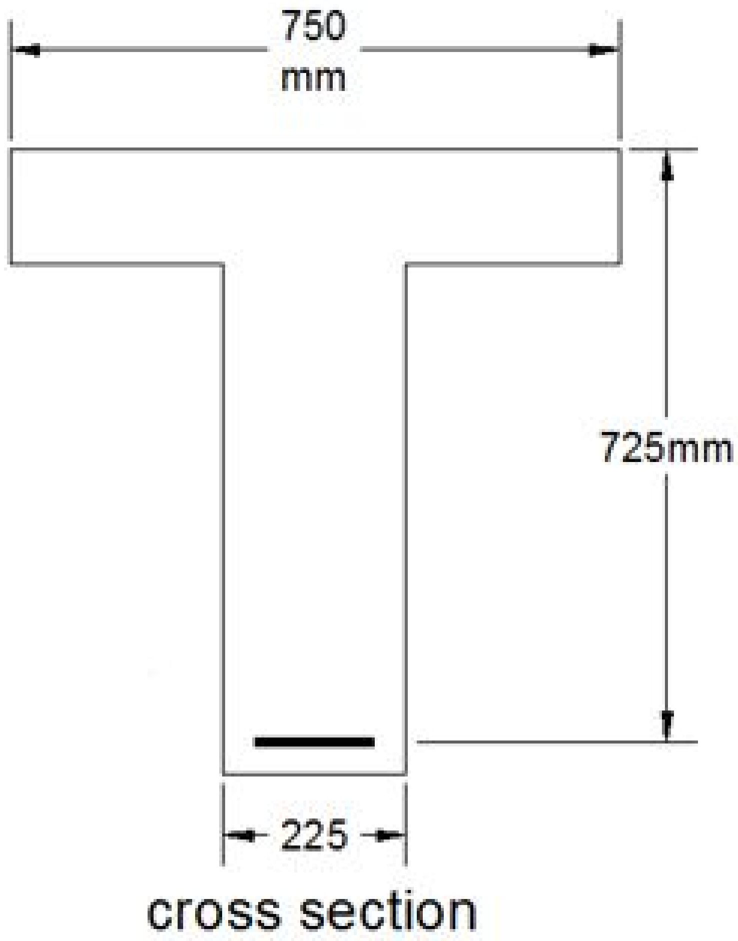
$$Mu$$

$$= 0.9 * 0.0089 * 0.75 * 0.725^2 * 400 \left(1\text{-}0.59 * 0.0089 * \frac{400}{20}\right)$$

$$= 1.128\ MN.m$$

Hence, hf, has no significant effect in increasing section moment capacity. To increase moment capacity, increase(d or As).

Ex2: Isolated T-Beam $\frac{s}{c} = \frac{400}{20}$MPa , $h_f = 140mm$,Muext=1.126MN.m,find As.



Solution:

$$h_f = 140mm > \frac{b_w}{2} = 112.5\,mm(o.k)$$

$$b = 750mm < 4b_w = 900mm\ o.k$$

$$Muf = \emptyset 0.85fc'bhf(d - \frac{hf}{2})$$

$$Muf = 0.90 * 0.85 * 20 * 0.75 * 0.14\left(0.725 - \frac{0.14}{2}\right)$$

$$Muf = 1.052 < Muext = 1.126MN.m \rightarrow \therefore a > hf(T - section)$$

$\sum Fx = 0$

$A_{sf}\, f_y = 0.85 f_c' h_f (b - b_w)$

Asf *400=0.85*20*0.14(0.75-0.225)→Asf=3124mm^2

OR

$$Mu1 = \emptyset 0.85 f c' h_f (b - b_w)\left(d - \frac{h_f}{2}\right)$$

$$Mu1 = 0.9 * 0.85 * 20 * 0.14(0.75 - 0.225)\left(0.725 - \frac{0.14}{2}\right) = 0.736 \text{MN.m}$$

$$Mu1 = \emptyset Asf * \mathrm{fy}\left(d - \frac{h_f}{2}\right)$$

$$0.736 = 0.9 * Asf * 400\left(0.725 - \frac{0.14}{2}\right) \rightarrow \mathrm{Asf}=3124mm^2$$

$$Muext = Mu1 + Mu2$$

$$1126 = 736 + Mu2 \rightarrow Mu2 = 390kN.m$$

$$Mu2 = \emptyset 0.85fc'b_w a\left(d - \frac{a}{2}\right)$$

$$0.39 = 0.9 * 0.85 * 20 * 0.225 * a\left(0.725 - \frac{a}{2}\right)$$

$$1.721a^2 - 2.496a + 0.39 = 0 \rightarrow a = \binom{0.178m}{1.268m} \rightarrow a$$

$$= 178mm$$

$$Mu2 = \emptyset(As - Asf)\text{fy}\left(d - \frac{a}{2}\right)$$

$$0.39 = 0.9(As - 3124 *^{-6})*400*\left(0.725 - \frac{0.178}{2}\right) \rightarrow As$$

$$= 4826mm^2$$

$$\rho_f = \frac{Asf}{b_w d} = \frac{3124}{225 * 725} = 0.0192$$

$$\rho_w = \frac{As}{b_w d} = \frac{4826}{225 * 725} = 0.0296$$

$$\rho_{w_{max}} = 0.85\,\beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.004} + \rho_f$$

$$\rho_{w_{max}} = \rho_{max} + \rho_f$$

$$= 0.85 * 0.85 * \frac{20}{400} * \frac{0.003}{0.003 + 0.004} + 0.0192 = 0.0347$$

$$\rho_{min} = \max\left(\frac{1.4}{fy} = 0.0035, \frac{\sqrt{fc'}}{4fy} = 0.0028\right) = 0.0035$$

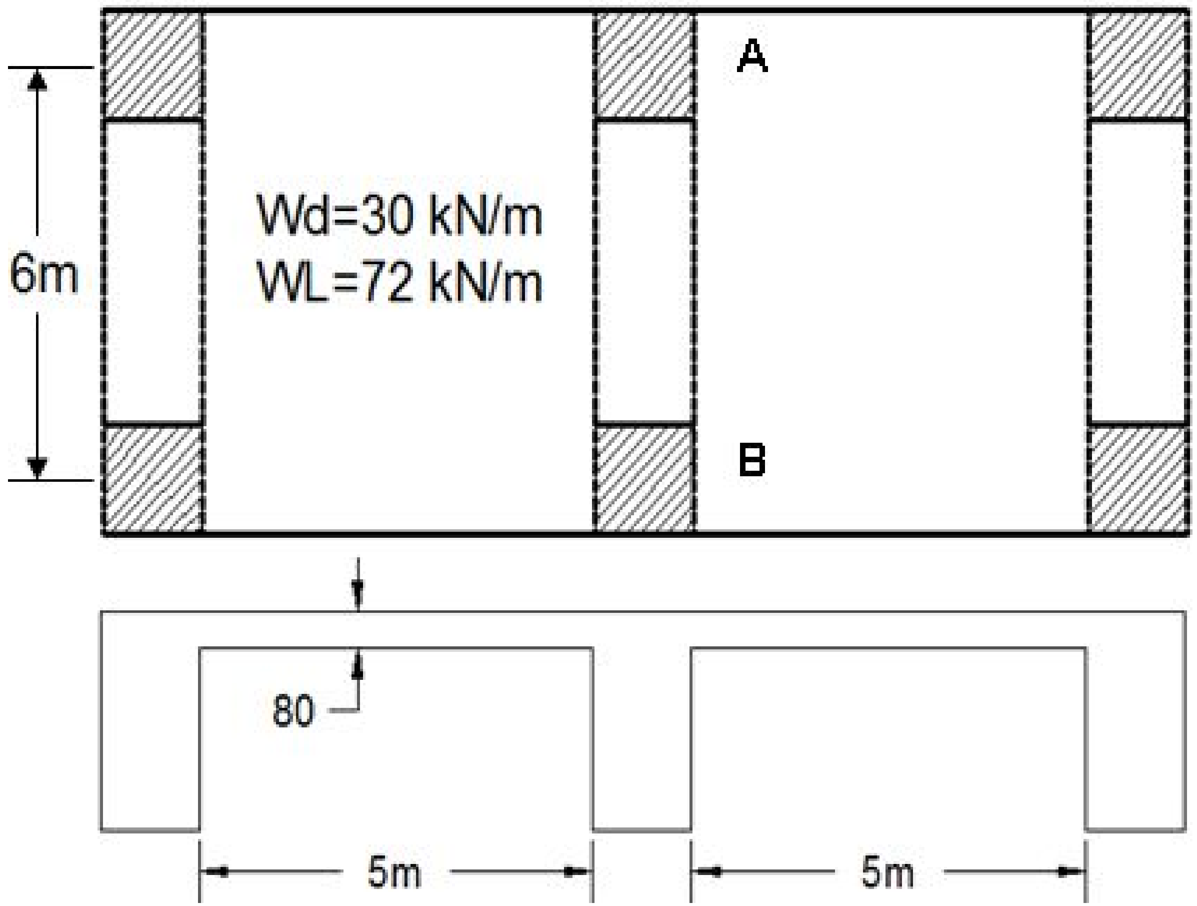
$$\rho_{min} < \rho_w < \rho_{w_{\max}} \quad \text{O.K}$$

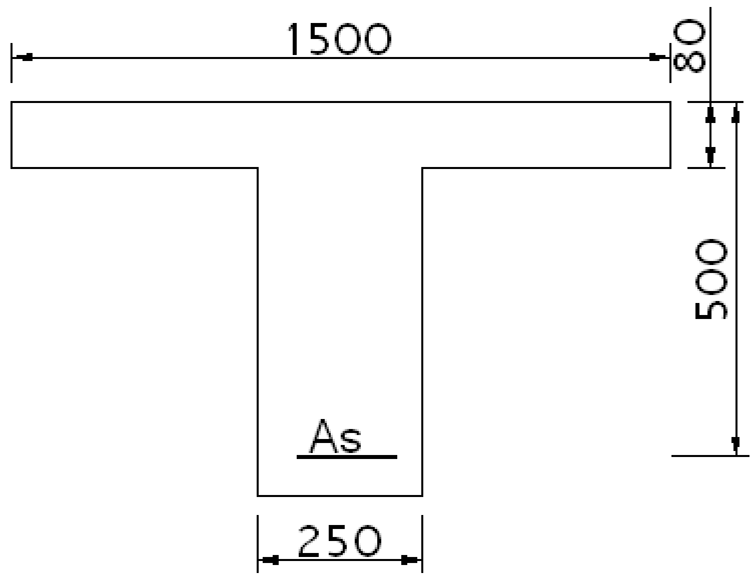
$$\rho_{w_t} = 0.85\,\beta_1 \frac{fc'}{fy} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + 0.005} + \rho_f = \rho_t + \rho_f$$

$$\rho_{w_t} = 0.85 * 0.85 * \frac{20}{400} * \frac{0.003}{0.003 + 0.005} + 0.0192 = 0.0327$$

$$\rho_w < \rho_{w_t} \rightarrow \emptyset = 0.9 \text{ O.K}$$

Ex3: $\frac{s}{c} = \frac{400}{30}$MPa, effective depth=500mm. Required area of steel for member (AB), $W_D = 30\frac{kN}{m}, W_L = 72\ kN/m$(not: member AB is simply supported)





Solution:

- $b \leq \frac{L}{4} = \frac{6000}{4} = 1500mm$
- $\frac{b-b_w}{2} \leq 8hf \rightarrow b = 1530mm$
- $\frac{b-b_w}{2} \leq \frac{1}{4}(lc_1 + lc_2) \rightarrow b = 5250mm$

choose min. value of b=1500mm

$$W_u = 1.2 * 30 + 1.6 * 72 = 151.2 \frac{kN}{m}$$

$$Mu_{ext} = \frac{W_u L^2}{8} = \frac{151.2 * 6^2}{8} = 681\ kN.m$$

$Let\ \emptyset = 0.9$ to be check later

$$Mu_f = \emptyset 0.85 fc'bh_f\left(\text{d-}\frac{h_f}{2}\right)$$

$$Mu_f = 0.9 * 0.85 * 20 * 1.5 * 0.08\left(0.5\text{-}\frac{0.08}{2}\right) = 0.845MN.m$$

$$> Mu_{ext} = 0.681MN.m \rightarrow a < h_f(RS)$$

$$Mu = \emptyset \rho b d^2 \text{fy}\left(1\text{-}0.59\ \rho\frac{\text{fy}}{\text{fc}'}\right)$$

$$0.681 = 0.9\rho * 1.5 * 0.5^2 * 400\left(1 - 0.59\rho\frac{400}{20}\right)$$

$$1593\rho^2 - 132\rho + 0.681 = 0 \rightarrow \rho = \begin{bmatrix} 0.00538 \\ 0.08 \end{bmatrix} = 0.00538$$

$$\rho_t = 0.85\beta_1 \frac{fc'}{fy} \frac{0.003}{0.003 + 0.005} = 0.0135 > \rho \rightarrow \emptyset = 0.9 \text{ O.K}$$

$$\rho_{max} = 0.85\beta_1 \frac{fc'}{fy} \frac{0.003}{0.003 + 0.004} = 0.0155 > \rho \ \therefore o.k$$

$$\rho_{min} = \max\left(\frac{1.4}{fy} = 0.0035, \frac{\sqrt{fc'}}{4fy} = 0.00279\right) \frac{b_w}{b}$$

$$= 0.00058 < \rho \therefore o.k$$

$$\rho_{min} < \rho < \rho_{max} \ \therefore o.k$$

$$As = \rho bd = 0.00538 * 1500 * 500$$

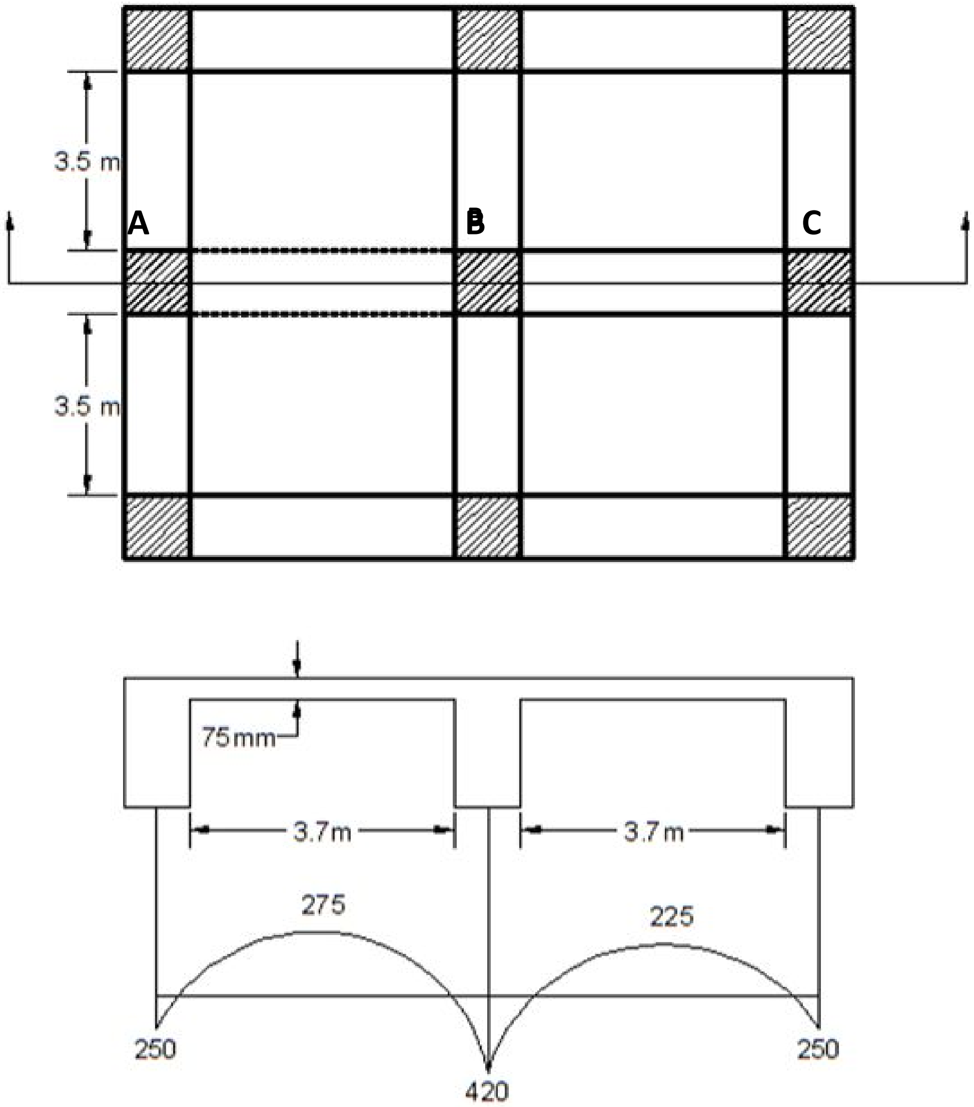
$$As = 4035mm^2$$

$$As_{min} = 0.0035 * 250 * 500 = 435mm^2$$

or

$$As_{min} = 0.00058 * 1500 * 500 = 435mm^2$$

Ex4: $\frac{S}{c} = \frac{400}{20} MPa, \quad hf = 75mm$, Design member(ABC)



Solution

1. **$-\text{ve Mu} = 420\ \text{kN.m}$ [R.S]**

$$\rho_{max} = 0.85 * 0.85 * \frac{20}{400} * \frac{3}{7} = 0.0155$$

Let $\rho = 0.6\rho_{max}, (R.S)$Economical steel ratio, $\rho = 0.6 * 0.0155 = 0.0093$

Let $d = 2.5 * b_{w,}$ $\quad (d = (2-3)b_w)$

$$\rho_t = 0.85 * 0.85 * \frac{20}{400} * \frac{3}{8} = 0.0135 > \rho = 0.0093 \rightarrow \therefore \emptyset = 0.9$$

$$Mu = \emptyset \rho b d^2 fy \left(1 - 0.59 * \rho \frac{fy}{fc'}\right)$$

0.42

$$= 0.9 * (0.0093) b d^2 * 400 \left(1 - 0.59 * 0.0093 * \frac{400}{20}\right)$$

$$b_w^3 = 0.0225\, m^3$$

$$b_w = 282\, mm \rightarrow use\ \ b_w = 280mm$$

$$\therefore d = 2.5 * 282 = 705 \cong 710mm$$

$$As = 0.0093 * 280 * 710 = 1849\, mm^2$$

$$use\ 4\emptyset 25 \rightarrow As = 1963\, mm^2\ \ (Top\ reinforcement)$$

2. $\underline{\mathbf{-Mu_2 = 250\ kN.m \rightarrow let\ \emptyset = 0.9}}$

$$0.25 = 0.9 * \rho * 0.28 * 0.71^2 * 400\left(1 - 0.59\, \rho \frac{400}{20}\right)$$

$$599.6\, \rho^2 - 50.81\rho + 0.25 = 0$$

$$\rho = 0.00524$$

$$\rho_{min} = \max\left(\frac{1.4}{fy}, \frac{\sqrt{fc'}}{4fy}\right) = 0.0035 < \rho \text{ o.k}$$

$$As = 0.00524 * 280 * 710 = 1042\, mm^2$$

$$\rho < \rho_t = 0.0135 \rightarrow \emptyset = 0.9\ O.K$$

3. **+ve M = 275 kN. m**

$$b < \frac{L}{4} = \frac{3.7}{4} = 0.925\ m$$

$$\frac{b - b_w}{2} \le 8hf \rightarrow b = 1.48m$$

$$\frac{b - b_w}{2} \le \frac{1}{4}(lc_1 + lc_2) \rightarrow b = 3.78m$$

choose min. value of b=925mm

$$let\ a = hf, \emptyset = 0.9$$

$$Muf = \emptyset 0.85 \mathrm{fc}' * b * hf\left(d - \frac{hf}{2}\right) =$$

$$Muf = 0.9*0.85*20*0.925*0.075\left(0.71 - \frac{0.075}{2}\right)$$

$$= 0.714\ MN.m = 714\ kN.m > Mu_{ext} = 275kN.m$$

$$\rightarrow\ (R.S)$$

$$Mu = \emptyset\ \rho\ b\ d^2 \mathrm{fy}\left(1 - 0.59 * \rho\frac{\mathrm{fy}}{\mathrm{fc}'}\right)$$

$$0.275 = 0.9\rho * 0.925 * 0.71^2 * 400\left(1 - 0.59\ \rho\frac{400}{20}\right)$$

$$1980.8\rho^2 - 167.85\rho + 0.275 = 0$$

$$\rho = 0.00167 < \rho_{max} = 0.0155$$

$$\rho_{min} = \max\left(\frac{1.4}{fy}, \frac{\sqrt{fc'}}{4fy}\right)\frac{b_w}{b} = 0.00106 < \rho \therefore o.k$$

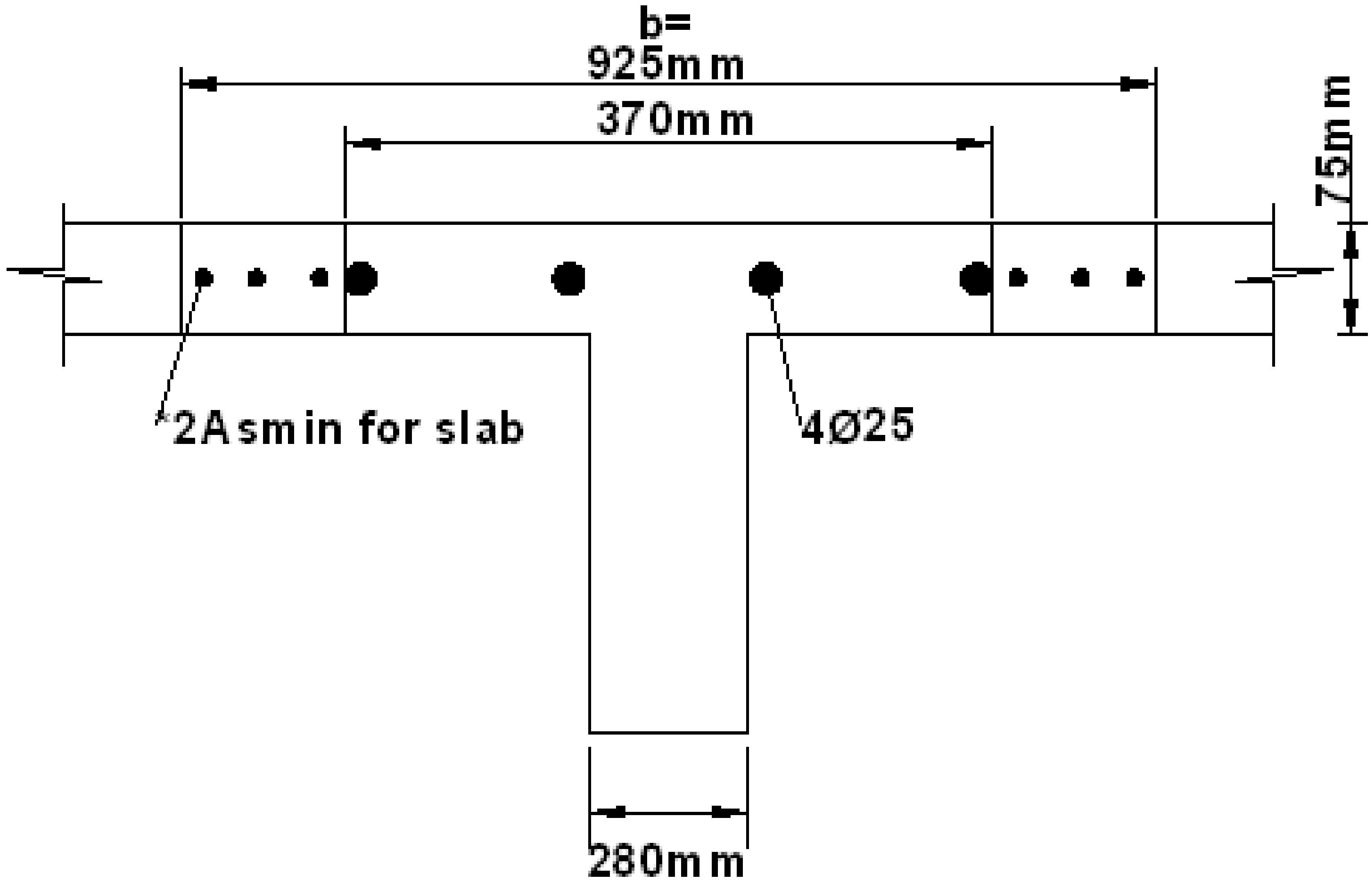
$check\ \emptyset$

$$\rho = 0.00167 < \rho_t = 0.0135 \rightarrow \emptyset = 0.9\ o.k$$

$$As^+ = 0.00167 * 925 * 710 = 1097mm^2$$

ACI 10.6.6 : negative reinforcement should be distributed over the width of smaller of(b, span/10), to control on crack

$$min = \left[\begin{matrix} b_{eff} = 925mm \\ or \\ \frac{span}{10} = \frac{3700}{10} = 370mm\ (o.k) \end{matrix} \right]$$


b=
925mm
370mm
75mm
*2Asmin for slab
4Ø25
280mm