



Class: 1st
Subject: Electrical Technology
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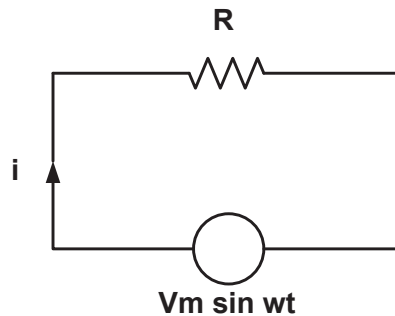
Lecture No. 20-25

“AC Circuits”



A . C Circuits

1. Pure resistance :



$$v = V_m \sin \omega t \quad \text{----- (1)}$$

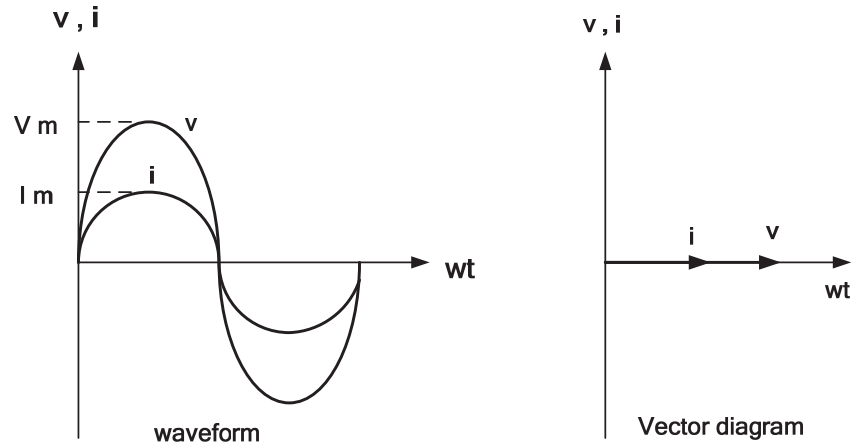
$$i = \frac{v}{R} = \frac{V_m \sin \omega t}{R}$$

$$i = \frac{V_m}{R} \sin \omega t$$

$$\text{Let } \frac{V_m}{R} = I_m$$

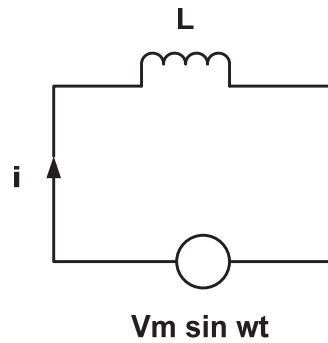
$$i = I_m \sin \omega t \quad \text{----- (2)}$$

From equation (1) and (2) , it is clear that alternating voltage and current are in phase with each other as shown below .



The instantaneous value of voltage and current are always either both positive or negative , therefore the power is always positive .

2. Pure inductance :



$$v = V_m \sin wt \text{ ----- (1)}$$

$$v = L \frac{di}{dt}$$

$$di = \frac{v}{L} dt$$



$$\int di = \int \frac{V}{L} dt$$

$$i = \int \frac{V_m \sin \omega t}{L} dt$$

$$i = \frac{V_m}{L} \int \sin \omega t dt$$

$$i = \frac{V_m}{\omega L} \int \sin \omega t \omega dt$$

$$i = \frac{V_m}{\omega L} (-\cos \omega t)$$

$$i = \frac{V_m}{\omega L} \sin (\omega t - \pi / 2)$$

Let $I_m = \frac{V_m}{X_L}$

$$i = I_m \sin (\omega t - \pi / 2) \text{ ----- (2)}$$

Where

$$\omega L = X_L = 2\pi f L$$

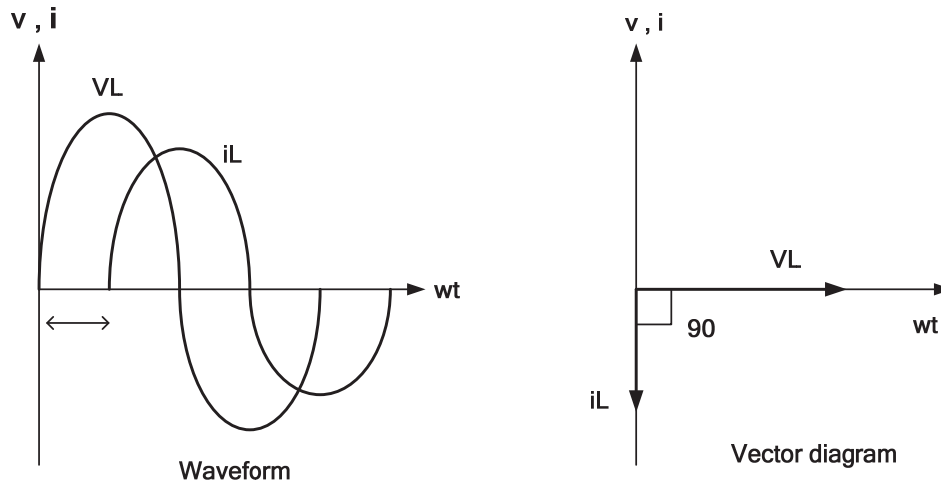
X_L is inductive reactance (Ω)

f is the frequency of the circuit (HZ)

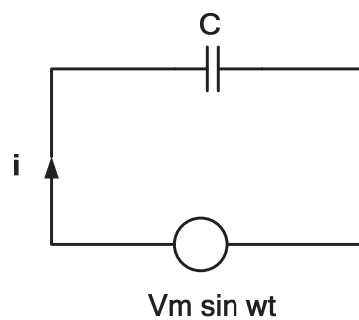
L is the inductance (H)



From equation (1) and (2) , it is clear that the alternating voltage leads the current by 90° in pure inductive circuit as shown below .



3. Pure capacitance :



$$v = V_m \sin wt \quad \text{----- (1)}$$

$$i = \frac{dq}{dt} = \frac{d Cv}{dt}$$



$$i = C \frac{d (V_m \sin \omega t)}{dt}$$

$$i = C V_m \frac{d}{dt} \sin \omega t = C V_m (\cos \omega t) \omega$$

$$i = V_m \omega C (\sin \omega t + \pi / 2)$$

$$i = \frac{V_m}{1 / \omega C} \sin (\omega t + \pi / 2)$$

Let $I_m = \frac{V_m}{1 / \omega C} = \frac{V_m}{X_C}$

$$i = I_m \sin (\omega t + \pi / 2) \text{ ----- (2)}$$

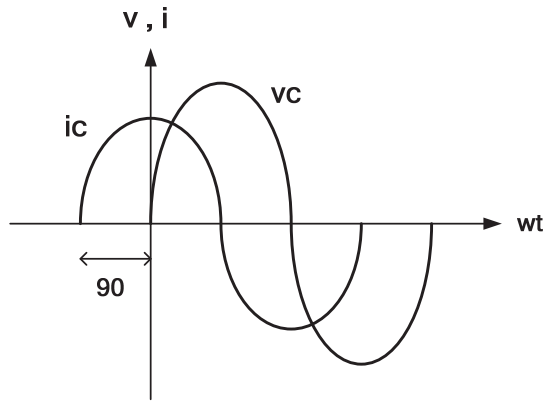
where

X_C is the capacitive reactance (Ω)
 $X_C = \frac{1}{2\pi f C}$

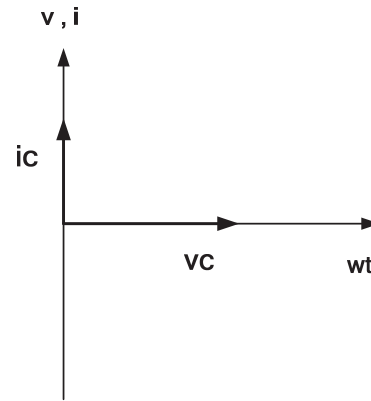
f is the frequency of the circuit (HZ)

C is the capacitance (F)

From equation (1) and (2) , it is clear that the alternating voltage lags the current by 90° as shown below , then draw the vector diagram.



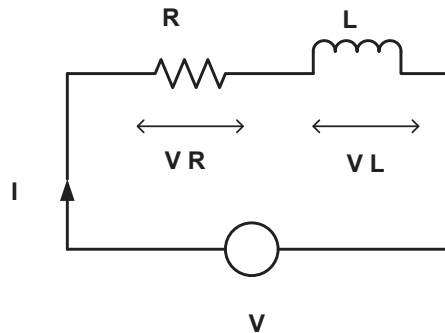
Waveform



Vector diagram

Series A.C circuits :

1. Resistance and inductance in series :



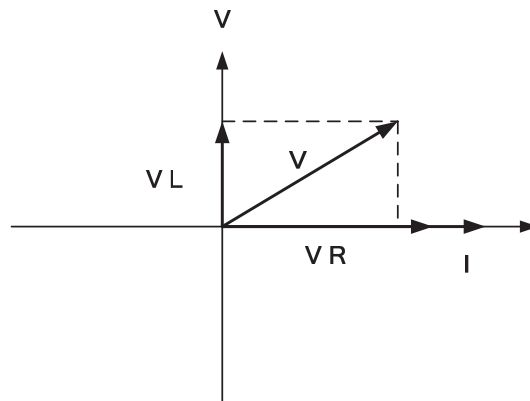
V = r.m.s value of applied voltage .

I = r.m.s value of current .

$V_R = I \times R$ (voltage drop across R , it is in phase with I) .

$V_L = I \times X_L$ (voltage drop across L , it leads I by 90°) .

* in series A.C circuits , plot the current on x – axis , then plot the voltages .



From the above figure ,

$$V = \sqrt{V_R^2 + V_L^2}$$

$$V = \sqrt{R^2 + X_L^2} I$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}}$$

$$Z = \sqrt{R^2 + X_L^2} \quad \text{Where } Z \text{ is the impedance of the circuit } (\Omega)$$

It is clear that the applied voltage leads the current by angle θ . This angle depends on the value of R and L .

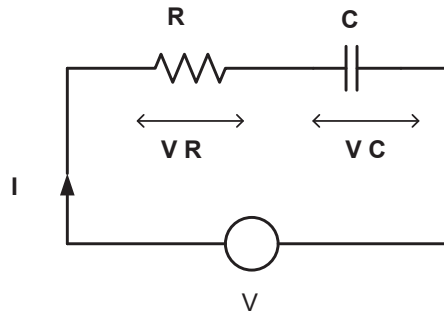
$$\tan \theta = \frac{V_L}{V_R} = \frac{X_L}{R}$$

$$\theta = \tan^{-1} \frac{X_L}{R}$$

$$\text{Power factor (p.f)} = \cos \theta$$



2. Resistance and capacitance in series :

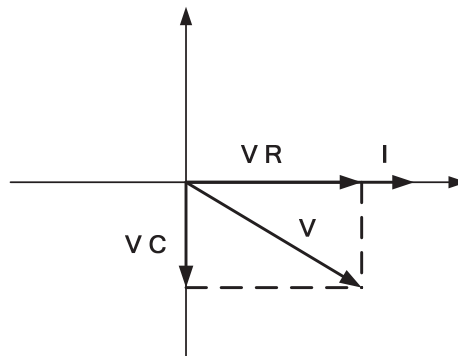


V = r.m.s value of applied voltage .

I = r.m.s value of current .

$V_R = I \times R$ (voltage drop across R , it is in phase with I) .

$V_C = I \times X_C$ (voltage drop across C , it lags I by 90°) .



From the above figure ,

$$V = \sqrt{V_R^2 + (-V_C)^2} = \sqrt{(IR)^2 + (I X_C)^2}$$

$$V = I \sqrt{R^2 + X_C^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_C^2}} \quad , \quad I = \frac{V}{Z}$$



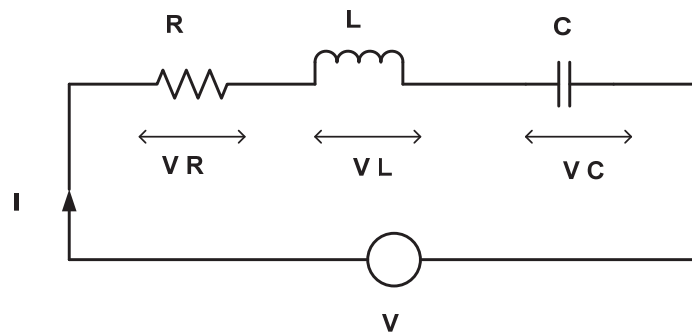
$$Z = \sqrt{R^2 + X_C^2} \quad \text{Where } Z \text{ is the impedance of the circuit } (\Omega).$$

It is clear that the applied voltage lags the current by angle θ . This angle depends on the value of R and C.

$$\tan \theta = \frac{-V_C}{V_R} = \frac{-X_C}{R}$$

$$\theta = \tan^{-1} \frac{-X_C}{R}$$

3. Resistance , inductance and capacitance in series :



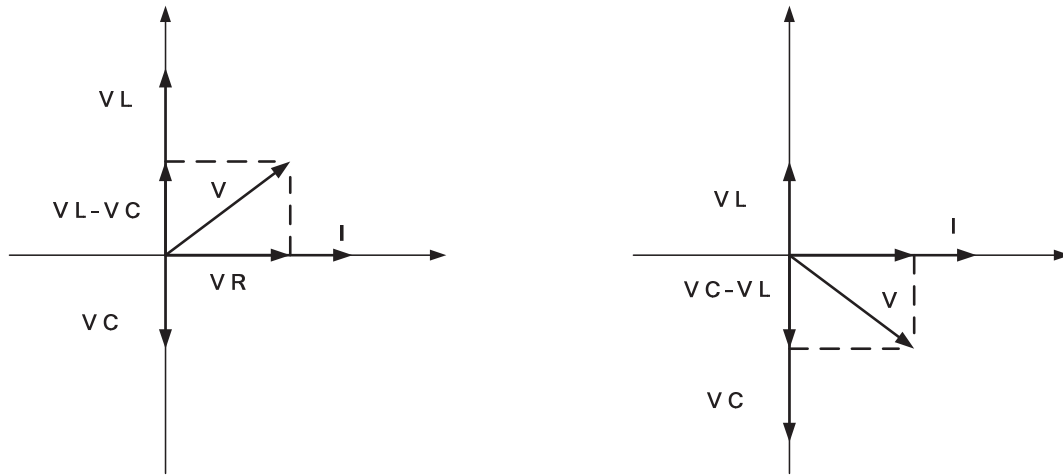
V = r.m.s value of applied voltage .

I = r.m.s value of current .

$V_R = I \times R$ (voltage drop across R , it is in phase with I) .

$V_L = I \times X_L$ (voltage drop across L , it leads I by 90°) .

$V_C = I \times X_C$ (voltage drop across C , it lags I by 90°) .



From the above figure ,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$V = \sqrt{(IR)^2 + (I X_L - I X_C)^2}$$

$$V = I \sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}, \quad I = V / Z$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

It is clear that the equation of the current in R - L - C in series is given by :

1. $i = I_m \sin (wt - \theta)$, when current lags voltage i.e. when $X_L > X_C$ as shown in the above figure .



$$\theta = \tan^{-1} \frac{V_L - V_C}{V_R} = \tan^{-1} \frac{X_L - X_C}{R}$$

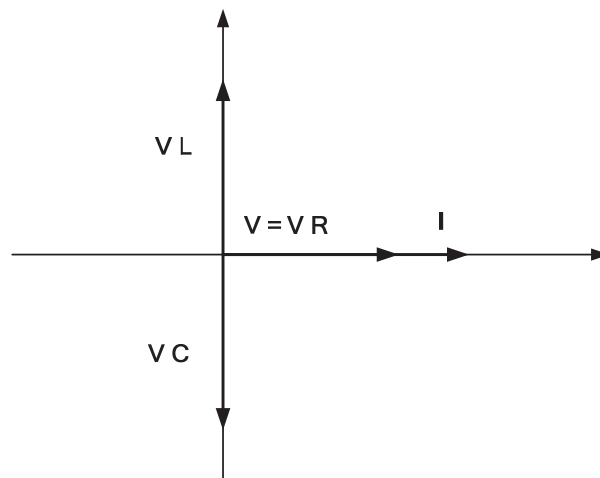
$$\text{p.f} = \cos \theta = \frac{V_R}{V} = \cos \frac{R}{Z}$$

2. $i = I_m \sin (\omega t + \theta)$, when current leads voltage i.e. when $X_C > X_L$ as shown in the above figure.

$$\theta = \tan^{-1} \frac{-(X_C - X_L)}{R}$$

$$\text{p.f} = \cos \theta = \frac{R}{Z}$$

3. $i = I_m \sin \omega t$, when current and voltage are in phase i.e. when $X_L = X_C$ (resonance), in this case $Z = R$ and $\theta = 0$, as shown in figure below .





Following points about series resonance must be noted :

1. $X_L = X_C$, $V_L = V_C$.
2. $Z = R$.
3. The current is maximum and is equal to V / R .
4. Power factor is unity i.e $\cos \theta = 1$.
5. Resonance frequency f_0 is calculated as follows :

$$f_0 = \frac{1}{2\pi \sqrt{LC}}$$

Example : A resistance of 20Ω , an inductance of 0.2 H and a capacitor of $100 \mu\text{F}$ are connected in series across 220 volt , 50 HZ mains . Determine the followings :

1. Voltage across R , L and C .
2. Power factor of the circuit .

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.2 = 62.8 \Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = 31.8 \Omega$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(20)^2 + (62.8 - 31.8)^2} = 36.8 \Omega$$

$$I = \frac{V}{Z} = \frac{220}{36.8} = 5.97 \text{ A}$$

$$V_R = I \times R = 5.97 \times 20 = 119.4 \text{ v}$$



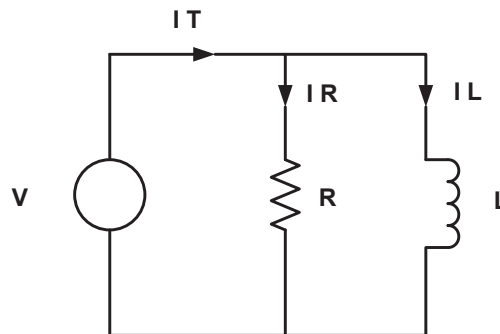
$$V_L = I \times X_L = 5.97 \times 62.8 = 374.9 \text{ v}$$

$$V_C = I \times X_C = 5.97 \times 31.8 = 189.8 \text{ v}$$

$$\text{p.f} = \frac{R}{Z} = \frac{20}{36.8} = 0.54$$

Parallel A.C circuits :

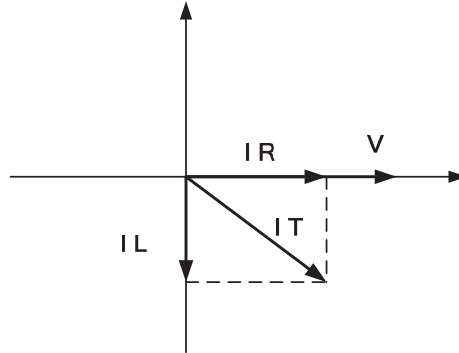
1. Resistance and inductance in parallel :



$$I_R = \frac{V}{R} \quad (\text{in phase with } V)$$

$$I_L = \frac{V}{X_L} \quad (\text{Lags } V \text{ by } 90^\circ)$$

* in parallel A.C circuits , plot the voltage on x – axis , then plot the branch currents .



From the above figure ,

$$I_T = \sqrt{I_R^2 + (-I_L^2)}$$

$$I_T = \sqrt{(V/R)^2 + (V/X_L)^2}$$

$$I_T = V \sqrt{(1/R)^2 + (1/X_L)^2}$$

$$\frac{I_T}{V} = \sqrt{(1/R)^2 + (1/X_L)^2} = Y$$

Where Y is the admittance of the circuit , its unit is siemens .

$$Y = \frac{1}{Z}$$

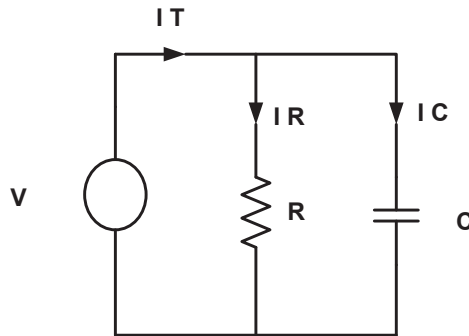
From Fig. 52

$$\tan \theta = \frac{-I_L}{I_R} = \frac{-R}{X_L}$$



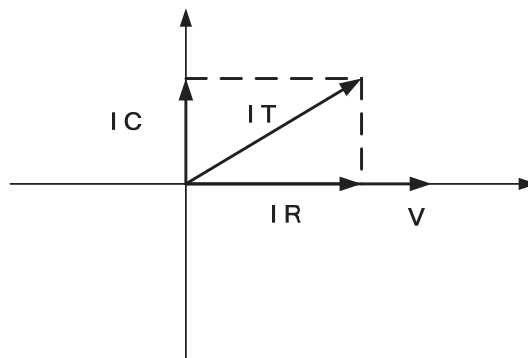
$$\theta = \tan^{-1} \frac{-R}{X_L}$$

2. Resistance and capacitance in parallel :



$$I_R = \frac{V}{R} \quad (\text{in phase with } V)$$

$$I_C = \frac{V}{X_C} \quad (\text{Leads } V \text{ by } 90^\circ)$$





From the above figure ,

$$I_T = \sqrt{I_R^2 + (I_C)^2}$$

$$I_T = \sqrt{(V/R)^2 + (V/X_C)^2}$$

$$I_T = V \sqrt{(1/R)^2 + (1/X_C)^2}$$

$$\frac{I_T}{V} = \sqrt{(1/R)^2 + (1/X_C)^2} = Y$$

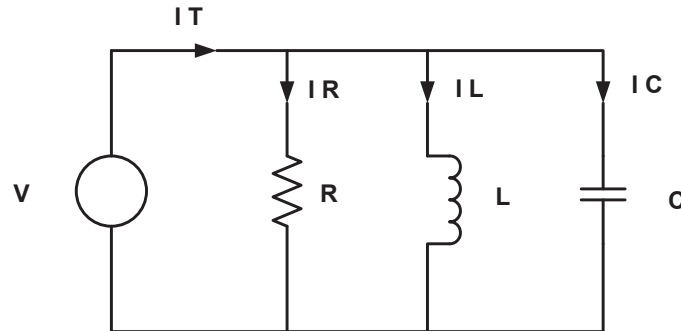
From the above figure ,

$$\tan \theta = \frac{I_C}{I_R} = \frac{R}{X_C}$$

$$\theta = \tan^{-1} \frac{R}{X_C}$$



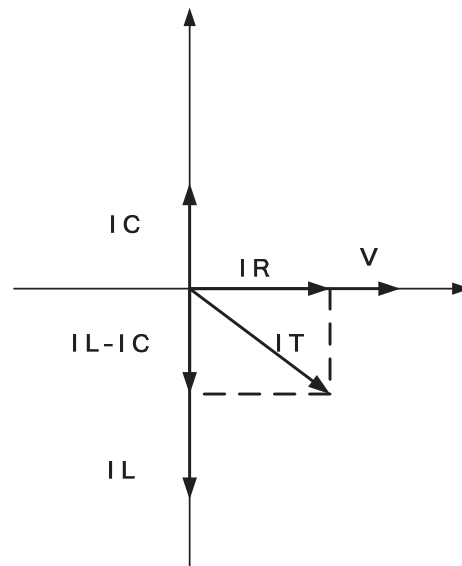
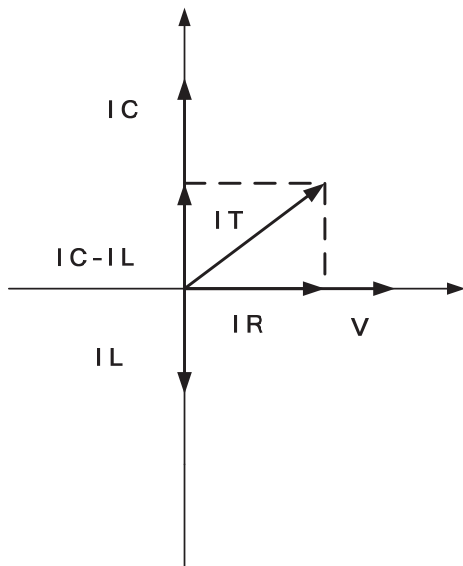
3. Resistance , inductance and capacitance in parallel :



$$I_R = \frac{V}{R} \quad (\text{in phase with } V)$$

$$I_L = \frac{V}{X_L} \quad (\text{Lags } V \text{ by } 90^\circ)$$

$$I_C = \frac{V}{X_C} \quad (\text{Leads } V \text{ by } 90^\circ)$$



From the above figure ,

$$I_T = \sqrt{I_R^2 + (I_C - I_L)^2}$$

$$I_T = \sqrt{(V/R)^2 + (V/X_C - V/X_L)^2}$$

$$I_T = V \sqrt{(1/R)^2 + (1/X_C - 1/X_L)^2}$$

$$\frac{I_T}{V} = \sqrt{(1/R)^2 + (1/X_C - 1/X_L)^2} = Y$$

There are three different cases :

1. When $I_C > I_L$, the circuit is capacitive , I_T leads V by θ as shown in Fig. 55 .

$$\tan \theta = \frac{I_C - I_L}{I_R}$$

$$\theta = \tan^{-1} \frac{1/X_C - 1/X_L}{1/R}$$

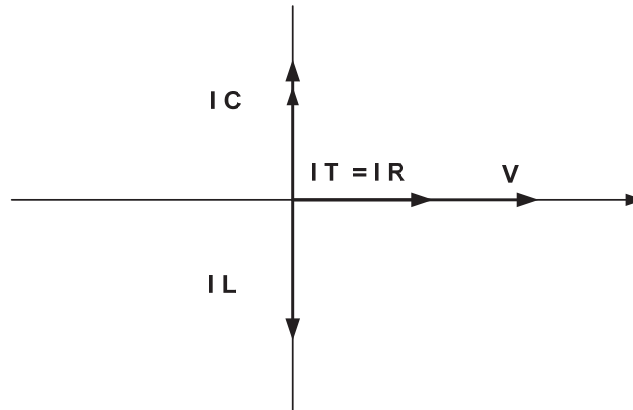
2. When $I_C < I_L$, the circuit is inductive , I_T lags V by θ as shown in Fig. 56 .

$$\tan \theta = - \frac{I_L - I_C}{I_R}$$

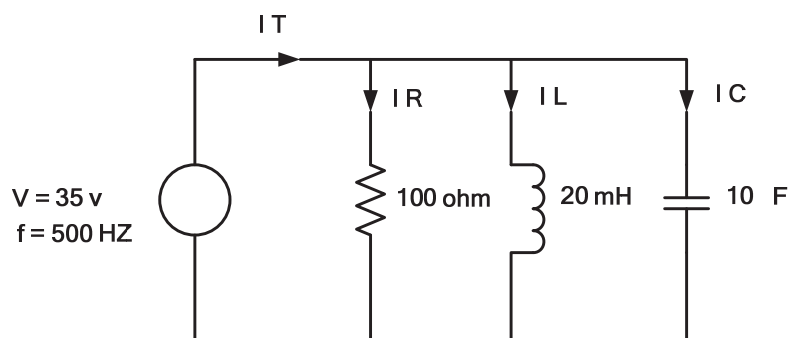
$$\theta = - \frac{1/X_L - 1/X_C}{1/R}$$



3. When $I_C = I_L$, in this case the circuit in resonance , $I_T = I_R$, I_T and V are in phase i.e $\theta = 0$ and the power factor is unity as shown below .



Example : For the circuit shown in figure shown below , find total current , all branch currents , phase angle , admittance and impedance .



$$I_R = \frac{V}{R} = \frac{35}{100} = 350 \text{ mA}$$

$$X_L = 2\pi f L = 2\pi \times 500 \times 20 \times 10^{-3} = 62.8 \Omega$$



$$I_L = \frac{V}{X_L} = \frac{35}{62.8} = 557 \text{ mA}$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 500 \times 10 \times 10^{-6}} = 31.8 \Omega$$

$$I_C = \frac{V}{X_C} = \frac{35}{31.8} = 1.1 \text{ A}$$

$$I_T = \sqrt{I_R^2 + (I_C - I_L)^2} = \sqrt{(0.35)^2 + (1.1 - 0.557)^2} = 0.645$$

$$\tan \theta = \frac{I_C - I_L}{I_R}$$

$$\theta = \tan^{-1} \frac{1.1 - 0.557}{0.35} = 57.17^\circ$$

$$Y = \sqrt{(1/R)^2 + (1/X_C - 1/X_L)^2}$$

$$Y = \sqrt{(1/100)^2 + (1/31.8 - 1/62.8)^2} = 18.4 \text{ ms}$$

$$Z = \frac{1}{Y} = \frac{1}{18.4 \times 10^{-3}} = 54.3 \Omega$$



D.C motors

Basic structure of D.C motor :

The structure of D.C motor has two major components , stator and rotor separated by the air cap .

Stator : This part of the motor does not move and normally is the outer frame of the motor .

Rotor : This part of the motor is free to move and normally is the inner part of the motor and contain a commutator .

The conductors placed in the slots of the stator and rotor are interconnected to form winding .The winding in which voltage is induced called the armature winding .The winding through which a current is passed to produce the primary source of flux in the motor called the field winding .The field winding is placed on the stator and the armature winding on the rotor.

D.C motor action :

Its action is based on the principle that ,when a current- carrying conductor is placed in a magnetic field , a mechanical force (f) is generated whose magnitude is given by :

$$f = B l i \text{ Newton}$$

where

- B** flux density
- l** length of conductor
- i** current pass through the conductor

When its field magnets are excited and its armature conductors are supplied with a current from the main supply , a mechanical force is

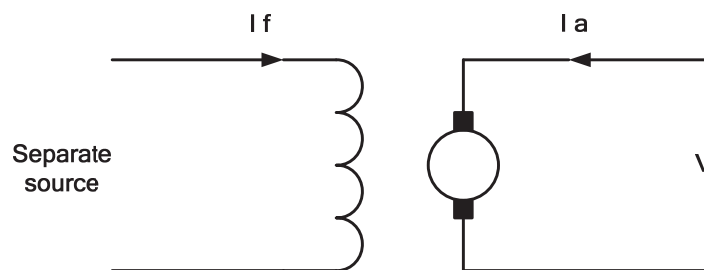


generated tending to rotate the armature .This force produce a driving torque which sets the armature rotating .

Classification of D.C motors :

The field and armature winding can be interconnected in various ways to provide a wide variety of performance characteristics Therefore there are many types of D.C motor .

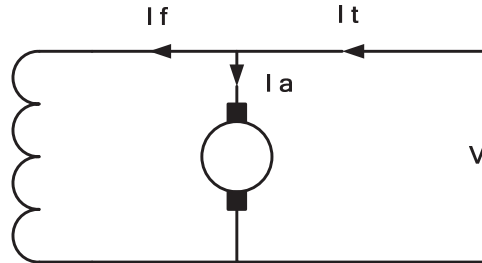
1.Separately excited D.C motor :The field winding is excited from a separate source as shown in figure below.



2.Self excited D.C motor:

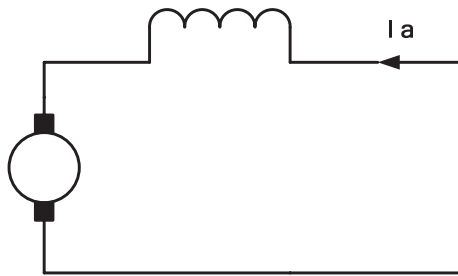
a- Shunt D.C motor :The field winding is parallel connected to the armature winding as shown in figure below .

$$I_t = I_a + I_f$$



b- Series D.C motor : The field winding is connected in series with the armature winding as shown in figure below.

$$I_t = I_a = I_f$$



c – Compound D.C motor : In this type both shunt and series field winding are connected to the armature winding .