CHAPTER SIX
External Flow

In this chapter we focus on the problem of computing heat and mass transfer rates to or from a surface in external flow. Examples include fluid motion over a flat plate (parallel to the free stream velocity) and flow over curved surfaces such as a sphere or cylinder. Whenever a solid body is exposed to a moving fluid having a temperature deferent from that of the body, energy is carried or convicted away by the fluid and is given as

$$Q = \bar{h}A(T_s - T_\infty) \qquad (6.1)$$

the heat flux is

$$q = \bar{h}(T_s - T_\infty) \qquad (6.2)$$

Where $(\bar{h})$ is the average convection heat transfer coefficient $(W/m^2.°C)$

There are two types of convection:

a- Forced convection: if the flow motion is achieved by some external mean.
b- Free convection: if the flow motion is achieved by density changes arising from the heating process.

6.1 Laminar and Turbulent Flow

Consider the flow over a flat plate as shown in Figure (6.1). Beginning at the leading edge of the plate, a region develops where the influence of viscous forces is felt. These viscous forces are described in terms of shear stress between the fluid layers. The region of flow that develops from the leading edge of the plate in which the effects of viscosity are observed is called the boundary layer.

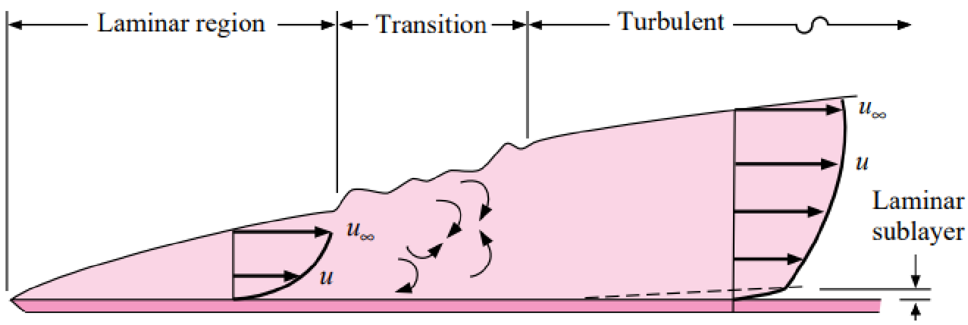

Figure (6.1) Sketch Showing Different Boundary Layer Flow Regimes on a Flat Plate.

Initially, the boundary layer development is laminar, but at some critical distance from the leading edge, depending on the flow field and fluid properties, small disturbances in the flow begin to become amplified, and a transition process takes place until the flow becomes turbulent. The turbulent flow region may be pictured as a random churning action with chunks of fluid moving to and fro in all directions.

6.2 The Flow Across Flat Plate

The characteristic length for a flat plate is taken to be (L). Thus, the Reynolds number is defined as:

$$Re_x = \frac{\rho u x}{\mu} = \frac{u x}{\upsilon} \qquad (6.3)$$

$$Re_L = \frac{\rho u L}{\mu} \qquad (6.4)$$

Where

Re: is the Reynolds number

ρ: is the density (kg/m^3)

u: is the velocity (m/s)

μ: is the is dynamic viscosity ($kg/m.s$)

υ: is the kinematic viscosity (m^2/s)

$if\ Re_L < 5 * 10^5\ laminar\ flow$

$if\ Re_L > 5 * 10^5\ turbulent\ flow$

6.2.1 For constant wall temperature:

The local Nusselt number at a location (x) for **laminar flow over a flat plate** can be shown by solving the differential energy equation to be

$$Laminar:\ Nu_x = \frac{h_x x}{k} = 0.332\ Pr^{1/3} Re_x^{1/2} \qquad (6.5)$$

For $Re_x < 5 \times 10^5 \quad and \quad 0.6 < Pr < 50$

The local friction coefficient is then

$$C_{f,x} = 0.664\, Re_x^{-1/2} \qquad (6.6)$$

Average Nusselt number

$$\overline{Nu} = Nu_L = \frac{\bar{h}L}{k} = 0.664\, Pr^{1/3} Re_L^{1/2} \qquad (6.7)$$

$$\bar{h} = h_L = 2h_x$$

The average friction coefficient is then

$$\overline{C_f} = 1.328\, Re_L^{-1/2} \qquad (6.8)$$

The corresponding relation for turbulent flow is

$$\text{Turbulent:}\ Nu_x = \frac{h_x x}{k} = 0.0296\, Pr^{4/5} Re_x^{1/3} \qquad (6.9)$$

For $5 \times 10^5 < Re_x < 10^7$

The local friction coefficient is then

$$C_{f,x} = 0.0592\, Re_x^{-1/5} \qquad (6.10)$$

Average Nusselt number in the turbulent region

$$\overline{Nu} = Pr^{1/3}(0.037 Re_L^{0.8} - 871) \qquad (6.11)$$

The average friction coefficient is then

$$\overline{C_f} = 0.074\, Re_L^{-1/5} - 1742\, Re_L^{-1} \qquad (6.12)$$

6.2.2 For Constant Heat Flux:

When a flat plate is subjected to uniform heat flux instead of uniform temperature, the local Nusselt number is given by

$$Laminar:\ Nu_x = 0.453\, Pr^{1/3} Re_x^{1/2} \qquad (6.13)$$

For $Re_x < 5 \times 10^5 \quad and \quad 0.6 < Pr < 50$

The average Nusselt number is given by

$$\overline{Nu} = 0.68\, Pr^{1/3} Re_L^{1/2} \qquad (6.14)$$

$$Turbulent: Nu_x = 0.0308\, Pr^{1/3} Re_x^{1/2} \qquad (6.15)$$

For $5 \times 10^5 < Re_x < 10^7$

The heat flow is

$$q = \bar{h} A (T_w - T_\infty) \qquad (6.16)$$

The foregoing analysis was based on the assumption that the fluid properties were constant throughout the flow. When there is an appreciable variation between a wall and free-stream conditions, it is recommended that the properties be evaluated at the so-called film temperature (T_f) unless otherwise noted.

$$T_f = \frac{T_w + T_\infty}{2} \qquad (6.17)$$

For example, air properties at (15 °C) from the table (A-15) are:

$\rho = 1.225\, kg/m^3$ $\qquad$ $K = 0.02476\, W/m.k$

$\mu = 1.802 \times 10^{-5} kg/m.s$ $\qquad$ $Pr = 0.7323$

If the value not found in the table we must find it by using the Interpolation method as shown following

Temperature (°C)	Thermal conductivity (W/m.K)
50	0.02735
55	K
60	0.02808

To find thermal conductivity (K) at (55 °C)

$$\frac{55-50}{60-50} = \frac{K - 0.02735}{0.02808 - 0.02735} \Rightarrow K = 0.027715\, W/m.K$$

For the convenience of the reader we have summarized the heat transfer equations in Table (6.1) along with the restrictions that apply.

Table (6.1) Summary of equations for flow over flat plates.

Flow regime	Restrictions	Equation
Laminar, local	$T_w = const,\ Re_x < 5\times10^5,$ $0.6 < Pr < 50$	$Nu_x = 0.332\ Pr^{1/3} Re_x^{1/2}$ $C_{f,x} = 0.664\ Re_x^{-1/2}$
Laminar, average		$\overline{Nu} = 0.664\ Pr^{1/3} Re_L^{1/2}$ $\overline{C_f} = 1.328\ Re_L^{-1/2}$
Turbulent, local	$T_w = const$ $5\times10^5 < Re_x < 10^7$	$Nu_x = 0.0296\ Pr^{4/5} Re_x^{1/3}$ $C_{f,x} = 0.0592\ Re_x^{-1/5}$
Turbulent, average		$\overline{Nu} = Pr^{1/3}(0.037Re_L^{0.8} - 871)$ $\overline{C_f} = 0.074\ Re_L^{-1/5} - 1742\ Re_L^{-1}$

Laminar, local Laminar, average	$q_w = const, Re_x < 5 \times 10^5,$ $0.6 < Pr < 50$	$Nu_x = 0.453\, Pr^{1/3} Re_x^{1/2}$ $\overline{Nu} = 0.68\, Pr^{1/3} Re_L^{1/2}$
Turbulent, local	$q_w = const$ $5 \times 10^5 < Re_x < 10^7$	$Nu_x = 0.0308\, Pr^{1/3} Re_x^{1/2}$

The Reynolds number at which the flow becomes turbulent is called the critical Reynolds number ($Re_{x,c}$). Transition criterion for a flat plate in parallel flow:

$$Re_{x,c} = \frac{\rho u x_c}{\mu} \qquad (6.18)$$

Where (x_c): is the location at which transition to turbulence begins

Example (6.1): Air at (7 kPa) and (35 ºC) flows across a (30 cm) square flat plate at (7.5 m/s). The plate is maintained at (65 ºC). Estimate the heat lost from the plate.

Solution:

$$T_f = \frac{T_w + T_\infty}{2} = \frac{35 + 65}{2} = 50\ ℃$$

$$\rho = \frac{P}{RT} = \frac{7000}{287 * 323} = 0.0755\ kg/m^3$$

From table (A-15) the properties are

$$\mu = 1.963 \times 10^{-5}\ kg/m.s\,, \qquad k = 0.02735\ W/m.℃ \qquad Pr = 0.7228$$

$$Re_L = \frac{\rho u L}{\mu} = \frac{0.0755 * 7.5 * 0.3}{1.963 \times 10^{-5}} = 8653.85 < 5 \times 10^5$$

The flow is laminar

$$\overline{Nu} = 0.664\, Pr^{1/3} Re_L^{1/2}$$

$$\overline{Nu} = 0.664\,(0.7228)^{1/3}(8653.85)^{1/2} = 55.43$$

$$\overline{Nu} = \frac{\bar{h}L}{k} \Rightarrow \bar{h} = \frac{k}{L}\overline{Nu}$$

$$\bar{h} = \frac{0.02735}{0.3} * 55.43 = 5.05\ W/m^2.℃$$

$$q = \bar{h}A(T_w - T_\infty)$$

$$q = 5.05 * (0.3)^2(65 - 35)$$

$$q = 13.56\ W$$

Example (6.2): Air at (20 ºC) and (1 atm) flows over a flat plate at (35 m/s). The plate is (75 cm) long and is maintained at (60 ºC). Assuming unit depth in the z-direction, calculate the heat transfer from the plate.

Solution:

$$T_f = \frac{T_w + T_\infty}{2} = \frac{20 + 60}{2} = 40\ ℃ + 273 = 313\ K$$

$$\rho = \frac{P}{RT} = \frac{1.01325 * 10^5}{287 * 313} = 1.128\ kg/m^3$$

From table (A-15) the properties are

$$\mu = 1.918 \times 10^{-5}\ kg/m.s \qquad k = 0.02662\ W/m\ .℃$$

$$Pr = 0.7255$$

$$Re_L = \frac{\rho u L}{\mu} = \frac{1.128 * 35 * 0.75}{1.918 \times 10^{-5}} = 1.55 \times 10^6 > 5 \times 10^5$$

The flow is turbulent

$$\overline{Nu} = Pr^{1/3}(0.037 Re_L^{0.8} - 871)$$

$$\overline{Nu} = (0.7255)^{1/3}(0.037(1.55 \times 10^6)^{0.8} - 871)$$

$$\overline{Nu} = 2180$$

$$\overline{Nu} = \frac{\bar{h} L}{k} \Rightarrow \bar{h} = \frac{k}{L}\overline{Nu}$$

$$\bar{h} = \frac{0.02662}{0.75} * 2180$$

$$\bar{h} = 77.38\ W/m^2.℃$$

$$q = \bar{h} A (T_w - T_\infty)$$

$$q = 77.38 * 0.75(60 - 20)$$

$$q = 2321.26\ W$$

Example (6.3): A blackened plate is exposed to the sun so that a constant heat flux of ($800\ W/m^2$) is absorbed. The backside of the plate is insulated so that all the energy absorbed is dissipated to an airstream that blows across the plate at conditions of (25 °C), (1 atm), and (3 m/s). The plate is (25 cm) square. Estimate the local heat transfer coefficient. Take $\upsilon = 20.76 \times 10^{-6}\ m^2/s,\ \ k = 0.03003\ W/m\,.°C\ and\ Pr = 0.697$

Solution:

$$Re_L = \frac{uL}{\upsilon} = \frac{3 * 0.25}{20.76\ \times\ 10^{-6}}$$

$$Re_L = 36127 < 5 \times 10^5$$

The flow is laminar

$$Nu_x = 0.453\ Pr^{1/3} Re_x^{1/2}$$

At $x = 25\ cm$

$$Nu_x = 0.453(0.697)^{1/3}(36127)^{\frac{1}{2}}$$

$$Nu_x = 76.35$$

$$Nu_x = \frac{h_x L}{k} \Rightarrow h_x = \frac{k}{L} Nu_x$$

$$h_x = \frac{0.03003}{0.25} * 76.35$$

$$h_x = 9.17 W/m^2.°C$$

6.3 Flow Across Cylinders And Spheres

The characteristic length for a circular cylinder or sphere is taken to be the external diameter (D). y Thus, the Reynolds number is defined as:

$$Re_D = \frac{\rho u D}{\mu} = \frac{uD}{\upsilon} \qquad (6.19)$$

Flow over cylinder

the equation is recommended for all (Re_D) and$Pr \geq 0.2$ has the form

$$\overline{Nu_D} = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}} \left[1 + (\frac{Re_D}{282000})^{5/8}\right]^{4/5} \qquad (6.20)$$

where all properties are evaluated at the film temperature (T_f).

Flow over sphere

the equation is recommended for $(3.5 \leq Re_D \leq 80000), and\ 0.7 \leq Pr \leq 380$ has the form

$$\overline{Nu_D} = 2 + \left(0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}\right) Pr^{0.4} \left(\frac{\mu_\infty}{\mu_w}\right)^{1/4} \qquad (6.21)$$

where all properties are evaluated at the free-stream temperature (T_∞) except (μ_w) is evaluated at the surface temperature ($T_w = T_s$).

The critical Reynolds number for flow across a circular cylinder or sphere is about $(Re_{cr} = 2 \times 10^5)$.

Example (6.4): A fine wire having a diameter of (3.94×10^{-5} m) is placed in a (1 atm) airstream at (25 °C) having a flow velocity of (50 m/s) perpendicular to the wire. An electric current is passed through the wire, raising its surface temperature to (50 °C). Calculate the heat loss per unit length.

Solution:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{25+50}{2} = 37.5\ ℃$$

From table (A-15) the properties are

$$\upsilon = 16.7 \times 10^{-6}\ m^2/s \qquad k = 0.02704\ W/m\,.℃$$

$$Pr = 0.706$$

$$Re_D = \frac{\rho u D}{\mu} = \frac{uD}{\upsilon} = \frac{50 * 3.94 * 10^{-5}}{16.7 \times 10^{-6}} = 118$$

$$\overline{Nu_D} = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1+(0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1+\left(\frac{Re_D}{282000}\right)^{5/8}\right]^{4/5}$$

$$\overline{Nu_D} = 0.3 + \frac{0.62 * (118)^{1/2} * (0.706)^{1/3}}{[1+(0.4/0.706)^{2/3}]^{1/4}}\left[1+(\frac{118}{282000})^{5/8}\right]^{4/5}$$

$$\overline{Nu_D} = 5.593$$

$$\overline{Nu_D} = \frac{\bar{h}D}{k} \Rightarrow \bar{h} = \frac{k}{D}\overline{Nu_D}$$

$$\bar{h} = \frac{0.02704}{3.94 * 10^{-5}} * 5.593$$

$$\bar{h} = 3838\ W/m^2.℃$$

$$q = \bar{h}A(T_w - T_\infty)$$

$$q/L = 3838 * \pi * 3.94 * 10^{-5}(50-25)$$

$$q/L = 11.88\ W/m$$

Home Work (6):

1- Air at (30 °C) and (1 atm) flows over a flat plate at a speed of (2 m/s). Assume that the plate is heated over its entire length to a temperature of (60 °C). Assume unit depth in the z-direction. Calculate the heat transferred in:

(a) The first (20 cm) of the plate.

(b) The first (40 cm) of the plate.

2- A fan that can provide air speeds up to (50 m/s) is to be used in a low-speed wind tunnel with atmospheric air at (25 °C). If one wishes to use the wind tunnel to study flat plate boundary layer behavior up to Reynolds numbers of ($Re_x = 10^8$), what is the minimum plate length that should be used? At what distance from the leading edge would transition occur if the critical Reynolds number were ($Re_{x,c} = 5 \times 10^5$)?

3- An object of irregular shape has a characteristic length of ($L = 1\ m$) and is maintained at a uniform surface temperature of ($T_s = 400\ K$). When placed in atmospheric air at a temperature of ($T = 300\ K$) and moving with a velocity of ($u = 100\ m/s$), the average heat flux from the surface to the air is ($20{,}000\ W/m^2$). If a second object of the same shape, but with a characteristic length of ($L = 5\ m$), is maintained at a surface temperature of ($T_s = 400\ K$) and is placed in atmospheric air at ($T = 300\ K$), what will the value of the average convection coefficient be if the air velocity is ($V\ =\ 20\ m/s$)?

4- Engine oil at (60 °C) flows over the upper surface of a (5 m) long flat plate whose temperature is (20 °C) with a velocity of (2 m/s). Determine the rate of heat transfer per unit width of the entire plate. The properties of oil at film temperature are ($\rho = 876\ kg/m^3$, $k = 0.144\ W/m.°C, \upsilon = 242 \times 10^{-6}\ m^2/s\ and\ Pr = 2870$).

5- Air at (20 °C) and (1 atm) flows over a flat plate at (8 m/s). The plate dimension is (1.5 m × 6 m) and is maintained at (220 °C). Determine the rate heat transfer from the plate if the air flows parallel to the (1.5 m) side.

6- Air at (90 °C) and atmospheric pressure flow over a horizontal flat plate at (60 m/s). The plate is (60 cm) square and is maintained at a uniform temperature of (10 °C). What is the total heat transfer?

7- Air at (1 atm) and (27 °C) blows across a (12 mm) diameter sphere at a free-stream velocity of (4 m/s). A small heater inside the sphere maintains the surface temperature at (77 °C). Calculate the heat lost by the sphere.