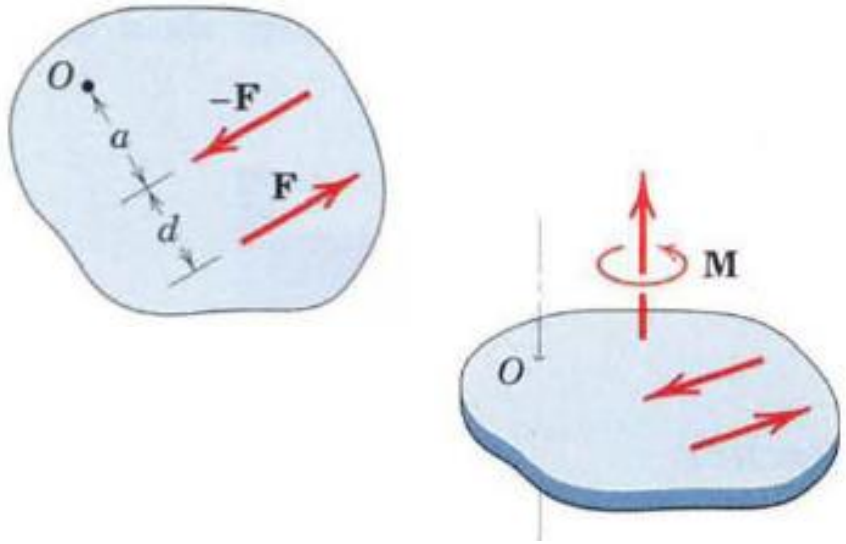
Couples

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics. Consider the action of two equal and opposite forces **F** and **- F** a distance *d* apart, as shown in Figure. These two force s cannot be combined into a single force because their sum in every direction is zero. Their on ly effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as a in their plane is the couple M. This couple has a magnitude



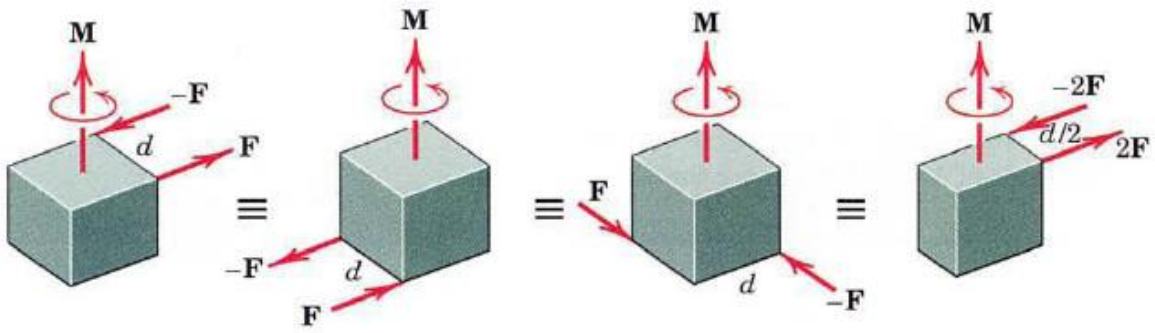
$$M = F(a + d) - Fa$$

Or

$$M = Fd$$

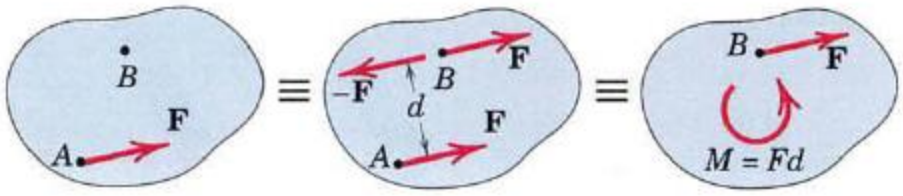
Equivalent Couples

Changing the values of **F** and **d** does not change a give n couple as long as the product **Fd** remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plan e. Figure shows four different configurations of the same couple **M**. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.



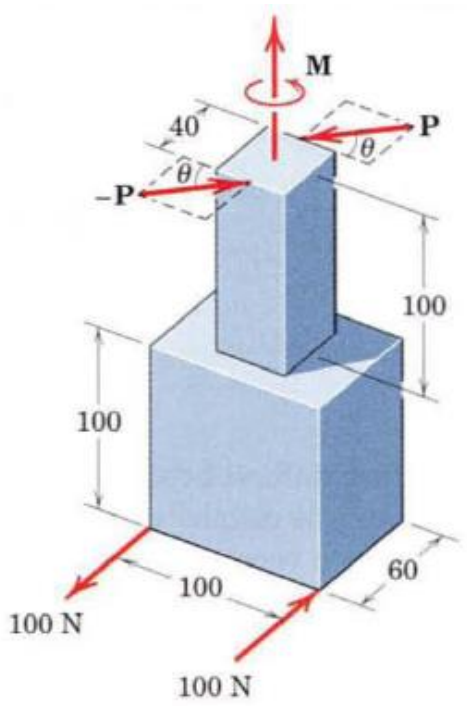
Force-Couple Systems

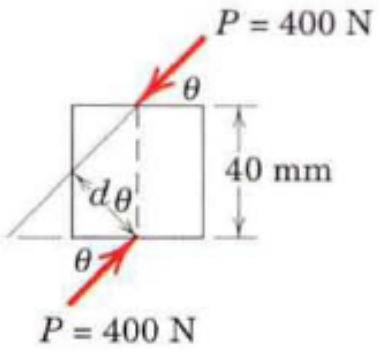
The replacement of a force by a force and a couple is illustrated in Figure, where the given force F acting at point A is replaced by an equal force F at some point B and the counterclockwise couple M = Fd.



Problem 1

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces P and - P , each of which has a magnitude of 400 N. Determine the proper angle θ.





Solution

M = Fd

M = 100(0.1) = 10N·m

The forces P and - P produce a counterclockwise couple

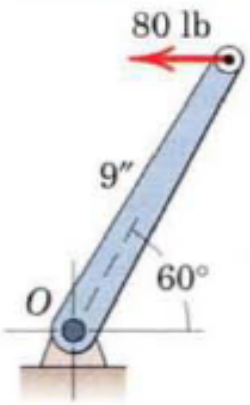
M = 400 (0.040) cos θ

10 = 400(0 .040) cos θ

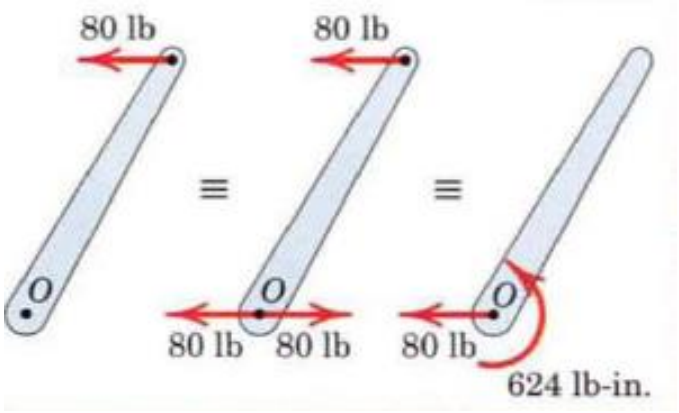
$\theta = \cos^{-1}\frac{10}{16} = 51.3$

Problem 2

Replace the horizontal 80· lb force acting on the lever by an equivalent system consisting of n force at O and a couple.



Solution



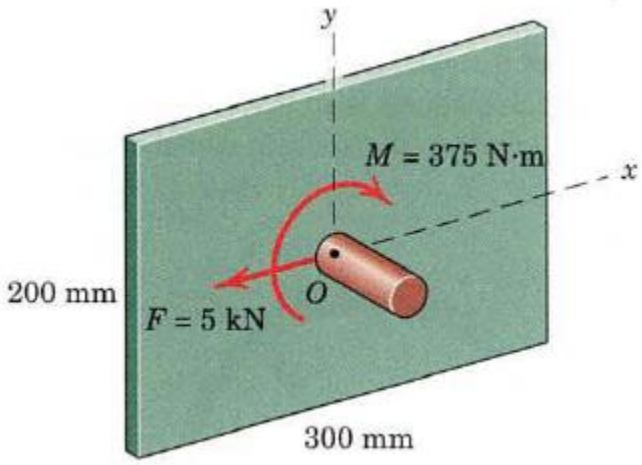
We apply two equal and opposite 80· lb forces at O and identify the counterclockwise couple

M = Fd

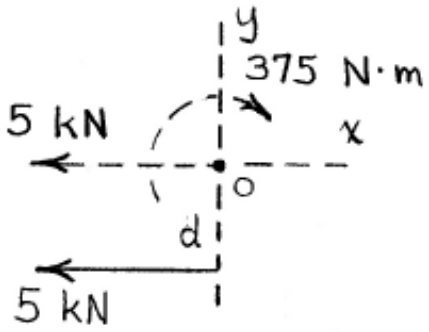
M = 80(9 sin 60) = 624 lb-in .

Problem 3

The indicated force- couple system is applied to a small shaft at the center of the rectangular plate. Replace this system by a single force and specify the coordinate of the point on the y-axis through which the line of action of this resultant force passes.



Solution



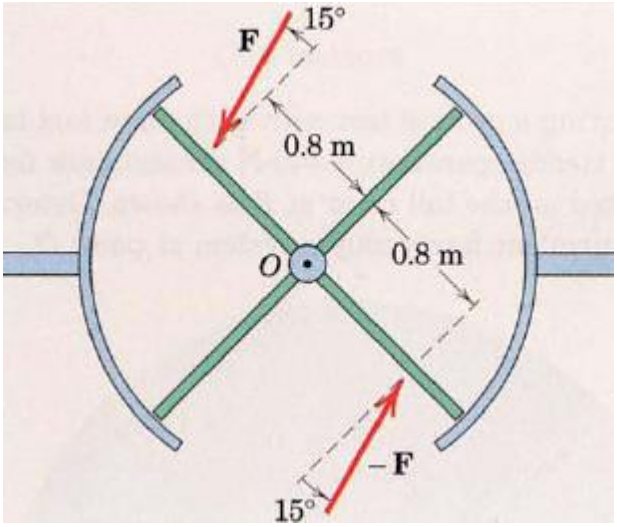
$$\circlearrowleft M = Fd: \quad 375 = 5000d$$

$$d = 0.075 \text{ m}$$

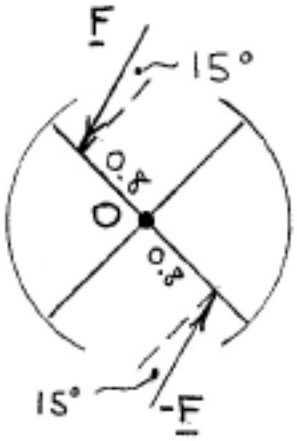
$$\therefore \quad y = -75 \text{ mm}$$

Problem 4

The top view of a revolving entrance door is shown. Two persons simultaneously approach the door and exert forces of equal magnitudes as shown. If the resulting moment about the door pivot axis at O is 25 N .m, determine the force magnitude F.



Solution



$$\curvearrowleft M_o = \Sigma Fd$$

$$25 = 2F(\cos 15°)(0.8)$$

$$F = \underline{16.18 \text{ N}}$$