كليــــــة
المستقبل الجامعة

قسم هندسة تقنيات
الأجهزة الطبيــــة

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اسم المادة : الميكانيك

عنوان المحاضرة: **Moment of Inertia**

رقم المحاضرة: 13

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Second Moments or Moments of Inertia

The second moment or moment of inertia of an element of area such as dA in Figure 1, with respect to any axis is defined as the product of the area of the element and the square of the distance from the axis to the element.

Rectangular and Polar Moments of Inertia

Consider the area A in the x-y plane, Figure 1. The moments of inertia of the element dA about the x- and y-axes are, by definition, $dI_x = y^2 dA$ and $dI_y = x^2 dA$ respectively.

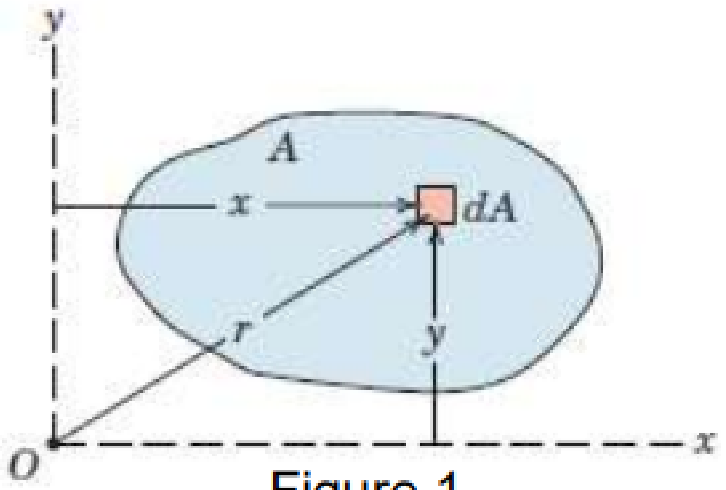


Figure 1

The sum of the second moments of all the elements of an area is defined as the moment of inertia of the area A, that is,

$$I_x = \int y^2 \, dA$$

$$I_y = \int x^2 \, dA$$

Where we carry out the integration over the entire area

The second moment of the element of area in Figure 1 with respect to an axis through O perpendicular to the plane of area is

$$dJ_o = r^2 dA = (x^2 + y^2) dA$$

The polar moment of inertia of the area is

$$J_o = \int (x^2 + y^2) \, dA = \int x^2 \, dA + \int y^2 \, dA = I_x + I_y$$

The Parallel-Axis Theorem for Areas

The parallel-axis theorem can be used to determine the moment of inertia of the area with respect to a parallel axis. The parallel-axis theorem (sometimes called the transfer formula) provides a convenient relationship between the moments of inertia of an area with respect to two parallel axes, one of which passes through the centroid of the area.

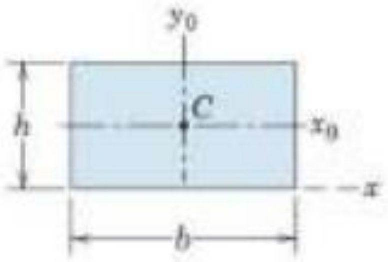
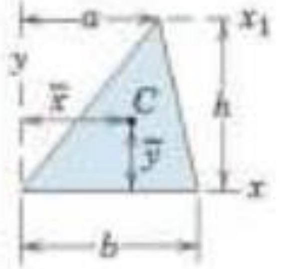
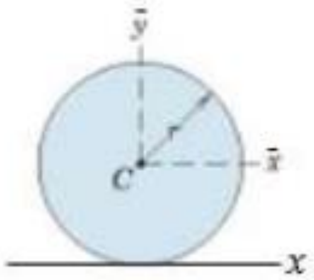
The parallel-axis theorem can be stated as follows: The moment of inertia of an area with respect to any axis is equal to the moment of Inertia with respect to a parallel axis through the centroid of the area plus the product of the area and the square of the distance between the two axes. Thus,

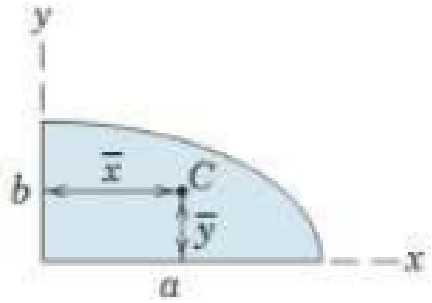
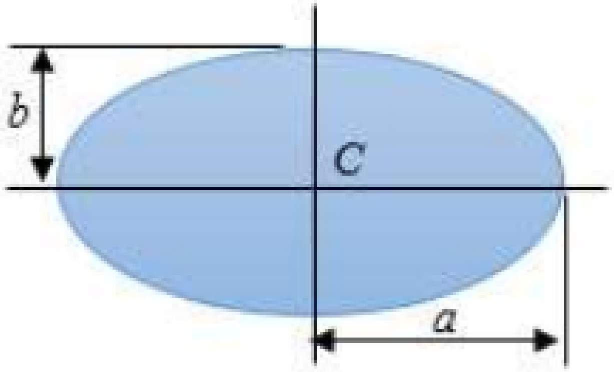
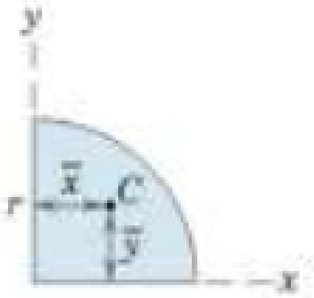
$$I_b = Ad^2 + I_c$$

Where I_c is the second moment of the area with respect to an axis through the centroid parallel to the axis b, A is the area, and d is the distance between the two axes. Similarly,

$$J_b = Ad^2 + J_c$$

Moments of Inertia of Composite Areas

<table>
<tr><th colspan="2">Geometric Area</th><th>Moment of Inertia</th></tr>
<tr><td>
</td><td>Rectangle</td><td>$I_x = \frac{bh^3}{3}$
$\bar{I}_x = \frac{bh^3}{12}$</td></tr>
<tr><td>
</td><td>Triangle</td><td>$I_x = \frac{bh^3}{12}$
$\bar{I}_x = \frac{bh^3}{36}$</td></tr>
<tr><td>
</td><td>Circle</td><td>$\bar{I}_x = \bar{I}_y = \frac{\pi r^4}{4}$
$I_x = \frac{5\pi r^4}{4}$</td></tr>
</table>

<table>
<tr>
<td>
</td>
<td>Area of Elliptical Quadrant</td>
<td>$I_x = \frac{\pi a b^3}{16}$, $\bar{I}_x = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) ab^3$
$I_y = \frac{\pi a^3 b}{16}$, $\bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) a^3 b$</td>
</tr>
<tr>
<td>
</td>
<td>Ellipse</td>
<td>$\bar{I}_x = \frac{\pi a b^3}{4}$</td>
</tr>
<tr>
<td>
</td>
<td>Quarter Circle</td>
<td>$\bar{I}_x = \bar{I}_y = \left(\frac{\pi}{16} - \frac{4}{9\pi}\right) r^4 = 0.0549 r^4$
$I_x = \frac{\pi r^4}{16} = 0.1963 r^4$</td>
</tr>
</table>

Second Moment of Area

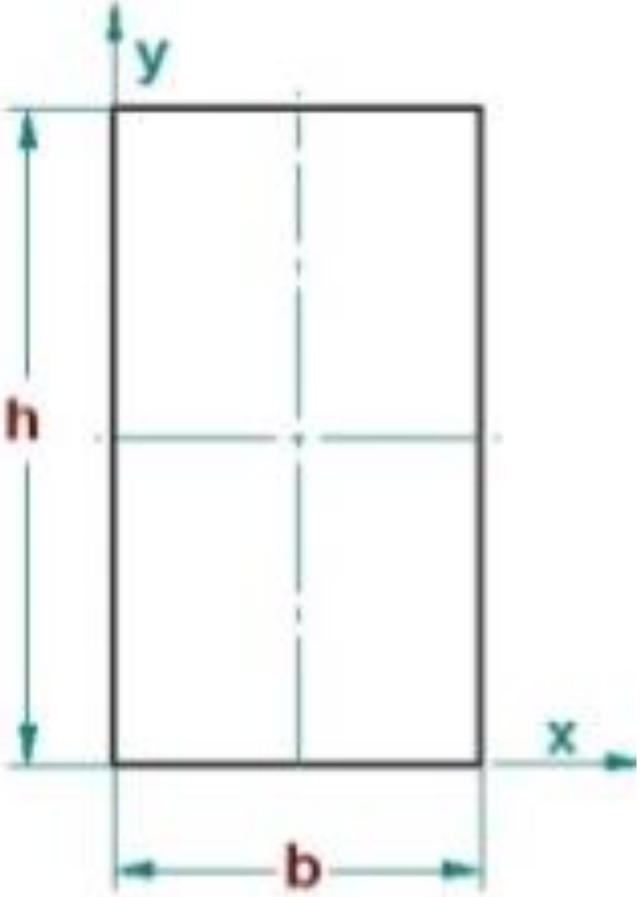
Second Moment of Area.
Ixx = Second moment of Area about the x-x axis

Q2: Find Ixx, where b = 6.9 mm and h = 13 mm.

Ixx $= bh^3/12 = 6.9*13\text{\^{}}3/12 = 1263.275\ mm^4$

Q3: (cont) Find Iyy, where b = 6.9 mm and h = 13 mm.

Iyy $= hb^3/12 = 13*6.9\text{\^{}}3/12 = 355.8848$ mm4



Q7: Find Ixx, D1 = 13 mm and Wall thickness = 1.7 mm

Do outside (solid shaft)

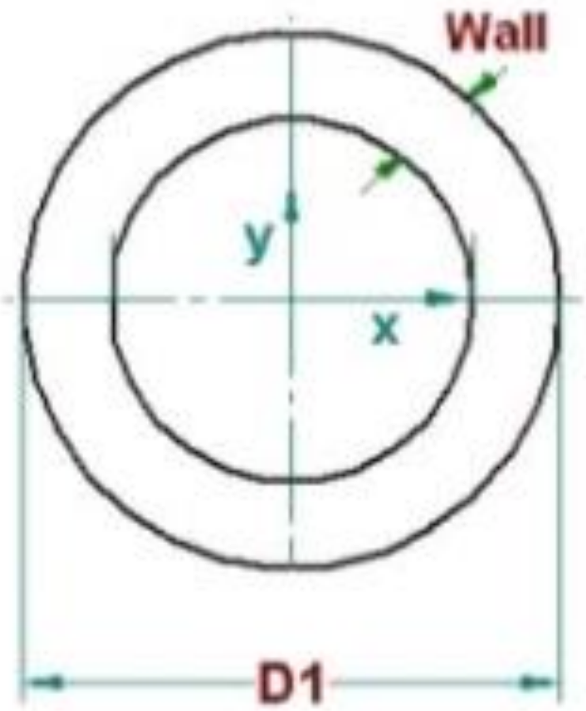
Ixx $= \pi * D^4/64$ = Pi*13^4/64 = 1401.985 mm4

Do inside (the hole)

ID = 13-2*1.7 = 9.6

Ixx $= \pi * D^4/64$ = Pi*9.6^4/64 = 416.922 mm4

Net Ixx = 1401.985 - 416.922 = 985.063 mm4



Parallel Axis Theroem

$I = I_c + Ad^2$

I = The second moment of area about non-centroidal plane

I_c = The second moment of area about its own centroid

A = Area of region

d = Distance from centroid to new plane

Q9: Find second moment of area Ixx about axis N-N where b = 18 , h = 4.9 and d = 6.2 mm.

$I_c = bh^3/12$

$= 18*4.9\text{^}3/12 = 176.4735\ mm^4$

$I = I_c + Ad^2$

$= 176.4735 + (18*4.9)*(6.2\text{^}2)$

= 3566.9 mm4