CHAPTER NINE

Free Convection

In preceding chapters we considered convectiontransfer in fluid flows that originate from an external forcing condition. For example, fluid motion may be induced by a fan or a pump, or it may result from propulsion of a solid through the fluid. In the presence of a temperature gradient, forced convection heat transfer will occur.

Now we consider situations for which there is no forced velocity, yet convection currents exist within the fluid. Such situations are referred to as free or natural convection, and they originate when a body force acts on a fluid in which there are density gradients. The net effect is a buoyancy force, which induces free convection currents. In the most common case, the density gradient is due to a temperature gradient, and the body force is due to the gravitational field. Since free convection flow velocities are generally much smaller than those associated with forced convection, the corresponding convection transfer rates are also smaller. There are many applications, Free convection strongly influences the operating temperatures of power generating and electronic devices. It plays a major role in a vast array of thermal manufacturing applications. Free convection is important in establishing temperature distributions within buildings and in determining heat losses or heat loads for heating, ventilating, and air conditioning systems.

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities: all lengths by a characteristic length (L), all velocities by an arbitrary reference velocity (u) (which, from the definition of Reynolds number, is taken to be ($u = Re_L \upsilon / L$), and temperature by a suitable temperature difference which is taken to be ($T_s - T_\infty$) as

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L} \qquad u^* \equiv \frac{u}{u_0}$$

$$v^* \equiv \frac{v}{u_0} \qquad T^* \equiv \frac{T - T_\infty}{T_s - T_\infty}$$

where asterisks are used to denote nondimensional variables. Substituting them into the momentum equation and simplifying give

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{g\beta(T_s-T_\infty)L}{u_0^2}T^*+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$$

The dimensionless parameter in the brackets represents the natural convection effects, and is called the **Grashof number (**Gr_L)

$$Gr_L \equiv \frac{g\beta(T_s-T_\infty)L^3}{\nu^2} \qquad (9.1)$$

where

g: is the gravitational acceleration (m/s^2).

β: is the coefficient of volume expansion (1/K).

($\beta = 1/T_f$ for ideal gases)

T_s: is the temperature of the surface (°C).

T_∞: is the temperature of the fluid sufficiently far from the surface (°C).

L: is the characteristic length of the geometry (m).

υ: is the kinematic viscosity of the fluid (m^2/s).

We mentioned in the preceding chapters that the flow regime in forced convection is governed by the dimensionless Reynolds number, which represents the ratio of inertial forces to viscous forces acting on the fluid. The flow regime in natural convection is governed by the dimensionless Grashof number, which represents the ratio of the buoyancy force to the viscous force acting on the fluid.

The role played by the Reynolds number in forced convection is played by the Grashof number in natural convection. As such, the Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection.

For vertical plates, for example, the critical Grashof number is observed to be about (10^9). Therefore, the flow regime on a vertical plate becomes turbulent at ($Gr > 10^9$). When a surface is subjected to external flow, the problem involves both natural and forced convection. The relative importance of each mode of heat transfer is determined by the value of the coefficient (Gr_L/Re_L^2):

1- Natural convection effects are negligible if $(Gr_L/Re_L^2 \ll 1)$.

2- Forced convection effects are negligible if $(Gr_L/Re_L^2 \gg 1)$.

3- Both effects are significant and must be considered if $(Gr_L/Re_L^2 \approx 1)$.

9.1 Natural Convection Over Surfaces

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid involved.

Although we understand the mechanism of natural convection well, the complexities of fluid motion make it very difficult to obtain simple analytical relations for heat transfer by solving the governing equations of motion and energy. Of the numerous such correlations of varying complexity and claimed accuracy available in the literature for any given geometry, we present here the ones that are best known and widely used.

The simple empirical correlations for the average Nusselt number Nu in natural convection are

$$\mathrm{Nu} = \frac{h\mathrm{L}}{k} = C(\mathrm{Gr}_L\mathrm{Pr})^n = C\mathrm{Ra}_L^n \qquad (9.2)$$

where Ra_L: is the **Rayleigh number,** which is the product of the Grashof and Prandtl numbers:

$$\mathrm{Ra}_L = \mathrm{Gr}_L\mathrm{Pr} = \frac{g\beta(T_s - T_\infty)L^3}{\upsilon^2}\mathrm{Pr} \qquad (9.3)$$

The values of the constants (C) and (n) depend on the geometry of the surface and the flow regime, which is characterized by the range of the Rayleigh number.

The value of (n) is usually

1- For laminar flow ($n = 1/4$).
2- For turbulent flow ($n = 1/3$).

The value of the constant is ($C < 1$).

There are various geometries we study some of it such as:

1- Vertical Plates

For a vertical flat plate, the characteristic length is the plate height (L).we give three relations for the average Nusselt number for an isothermal vertical plate. The first two relations are very simple. Despite its complexity, we suggest using the third one Eq. (9.6) recommended by Churchill and Chu since it is applicable over the entire range of Rayleigh number. This relation is most accurate in the range of $10^{-1} < Ra_L < 10^9$.

$$\mathrm{Nu} = 0.59\mathrm{Ra}_L^{1/4} \qquad (10^4 < Ra_L < 10^9) \qquad (9.4)$$

$$\mathrm{Nu} = 0.1\mathrm{Ra}_L^{1/3} \qquad (10^9 < Ra_L < 10^{13}) \qquad (9.5)$$

$$\mathrm{Nu} = \{0.825 + \frac{0.387\mathrm{Ra}_L^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\}^2 \qquad 10^{-1} < Ra_L < 10^9 \qquad (9.6)$$

All fluid properties are to be evaluated at the film temperature

$$T_f = \frac{T_s + T_\infty}{2}$$

When the average Nusselt number and thus the average convection coefficient is known, the rate of heat transfer by natural convection from a solid surface at a uniform temperature (T_s) to the surrounding fluid is expressed by Newton's law of cooling as

$$\mathrm{q_{conv}} = \bar{h}A_s(T_s - T_\infty) \qquad (9.7)$$

where A_s: is the heat transfer surface area (m^2).

$\bar{h}$: is the average heat transfer coefficient on the surface (W/m^2.K).

Example (9.1): Consider a vertical plate of a (0.6 m × 0.6 m) thin square plate in a room at (30 °C). One side of the plate is maintained at a temperature of (90 °C), while the other side is insulated. Determine the rate of heat transfer from the plate by natural convection.

Solution:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{90+30}{2} = 60\ °\text{C}$$

From (Table A-15) the properties of air at the film temperature are

$k = 0.02808\,W/m.°C$ $\qquad$ $\upsilon = 1.896 \times 10^5 \text{m}^2/\text{s}$

$Pr = 0.722$ $\qquad$ $\beta = \frac{1}{\text{T}_\text{f}} = \frac{1}{60+273} = \frac{1}{333} K^{-1}$

$$\text{Ra}_L = \frac{g\beta(T_s - T_\infty)L^3}{v^2}\text{Pr}$$

$$\text{Ra}_L = \frac{(9.81)[1/(333)](90-30)(0.6)^3}{(1.896\times 10^{-5})^2}(0.722)$$

$$\text{Ra}_L = 7.656 \times 10^8$$

$$Nu = \{0.825 + \frac{0.387\text{Ra}_L^{1/6}}{[1+(0.492/\text{Pr})^{9/16}]^{8/27}}\}^2$$

$$Nu = \left\{0.825 + \frac{0.387(7.656\times 10^8)^{1/6}}{1+(0.492/0.7202)^{9/16}]^{8/27}}\right\}^2$$

$$Nu = 113.4$$

$$\bar{h} = \frac{k}{L}\text{Nu} = \frac{0.02808}{0.6}(113.4)$$

$$\bar{h} = 5.306 \text{W/m}^2 \cdot° \text{C}$$

$$A_s = L^2 = (0.6)^2 = 0.36 \text{m}^2$$

$$\text{q} = \bar{h}A_s(T_s - T_\infty) = (5.306)(0.36)(90-30)$$

$$q = 115\ \text{W}$$