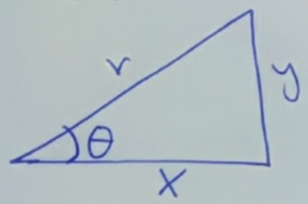
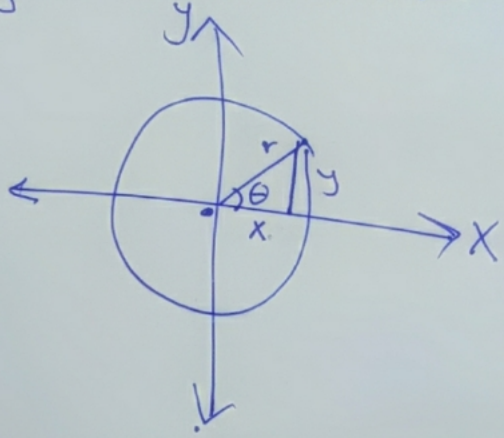
# المحاضرة السادسة

## الدوال المثلثية

## Trigonometric Functions:

مثلث فيثاغورس




الدوال المثلثية تعتمد على ثلاث معادلات وهي معادلات Sin و Cos و Tan

* اذن نعرف المعادلات المثلثية بواسطة المعادلات الاتية :-

① $\sin\theta = \dfrac{\text{مقابل}}{\text{الوتر}} = \dfrac{y}{r} = \dfrac{1}{\csc\theta}$

② $\cos\theta = \dfrac{\text{المجاور}}{\text{الوتر}} = \dfrac{x}{r} = \dfrac{1}{\sec\theta}$

③ $\tan\theta = \dfrac{y}{x} = \dfrac{1}{\cot\theta} = \dfrac{\sin\theta}{\cos\theta}$

$\tan\theta$ = وتر = $\dfrac{\text{مقابل}}{\text{المجاور}}$

(1) * معكوس Sin هو Csc

(2) * معكوس Cos هو Sec

(3) * معكوس Tan هو Cot

Following Properties of these Functions:

1. $\sin^2\theta + \cos^2\theta = 1$
2. $1 + \tan^2\theta = \sec^2\theta$
3. $1 + \cot^2\theta = \csc^2\theta$
4. $\cos(\theta \mp \beta) = \cos\theta . \cos\beta \mp \sin\theta . \sin\beta$
5. $\tan(\theta \mp \beta) = \dfrac{\tan\theta \mp \tan\beta}{1 \mp \tan\theta . \tan\beta}$

6. $\sin\cancel{(\theta \mp \beta) = \sin\theta . \cos\beta \mp \sin}\theta . \sin$

6. $\sin(\theta \mp \beta) = \sin\theta . \cos\beta \mp \cos\theta . \sin\beta$
7. $\sin 2\theta = 2\sin\theta . \cos\theta$
8. $\cos 2\theta = \cos^2\theta - \sin^2\theta$
9. $\cos^2\theta = \dfrac{1 + \cos 2\theta}{2}$
10. $\sin^2\theta = \dfrac{1 - \cos 2\theta}{2}$
11. $\sin(\theta \mp \frac{\pi}{2}) = \mp \cos\theta$
12. $\cos(\theta \mp \frac{\pi}{2}) = \mp \sin\theta$
13. $\boxed{\sin(-\theta) = -\sin\theta}$, $\boxed{\cos(-\theta) = \cos\theta}$
14. $\tan(-\theta) = -\tan\theta$

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(15) $\sin\theta \cdot \sin\beta = \frac{1}{2}[\cos(\theta-\beta) - \cos(\theta+\beta)]$

$\cos\theta \cdot \cos\beta = \frac{1}{2}[\cos(\theta-\beta) + \cos(\theta+\beta)]$

$\sin\theta \cdot \cos\beta = \frac{1}{2}[\sin(\theta-\beta) + \sin(\theta+\beta)]$

(16) $\sin\theta + \sin\beta = 2\sin\frac{\theta+\beta}{2} \cdot \cos\frac{\theta-\beta}{2}$

$\sin\theta - \sin\beta = 2\cos\frac{\theta+\beta}{2} \cdot \sin\frac{\theta-\beta}{2}$

(17) $\cos\theta + \cos\beta = 2\cos\frac{\theta+\beta}{2} \cdot \cos\frac{\theta-\beta}{2}$

$\cos\theta - \cos\beta = -2\sin\frac{\theta+\beta}{2} \cdot \sin\frac{\theta-\beta}{2}$

ex(1) Solve the following equations for values of $\theta$ from $\boxed{0^\circ \text{ to } 360^\circ}$ inclusive:

(a) $\boxed{\tan\theta} = 2\sin\theta$

لم يتم تحويله

$\tan\theta$ ← $\frac{\sin\theta}{\cos\theta} = 2\sin\theta$

$\sin\theta = \cos\theta(2\sin\theta)$

$\sin\theta - 2\cos\theta\sin\theta = 0$

$\sin\theta(1 - 2\cos\theta) = 0$

either $\sin\theta = 0$, $\theta = 0^\circ, 180^\circ, 360^\circ$

or $\cos\theta = \frac{1}{2} \Rightarrow \theta = 60^\circ, 300^\circ$

(b) $1+\cos\theta = 2\cdot\boxed{\sin^2\theta}$

بيقول

$1+\cos\theta = 2(1-\cos^2\theta)$

$1+\cos\theta = 2 - 2\cos^2\theta$

$1+\cos\theta - 2 + 2\cos^2\theta = 0$

$\cos\theta - 1 + 2\cos^2\theta = 0$

$2\cos^2\theta + \cos\theta - 1 = 0$

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~~$(2\cos 2\theta + 1)(2\cos\theta + 1)$~~

$(2\cos\theta - 1)(\cos\theta + 1) = 0$

either $\cos\theta = \frac{1}{2}$

$2\cos\theta - 1 = 0 \Rightarrow 2\cos\theta = 1$

$\boxed{\cos\theta = \frac{1}{2}}$ $\Rightarrow \theta = 60^\circ, 300^\circ$

or $\cos\theta + 1 = 0$

$\cos\theta = -1 \Rightarrow \theta = 180^\circ$

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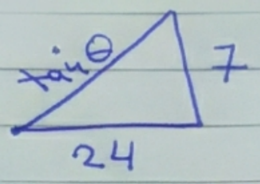
التاريخ / / ٢٠ الموضوع :

EX) 2) If $\tan\theta = 7/24$, find without using tables, the values of $\boxed{\sec\theta}$ and $\boxed{\sin\theta}$

$$\tan\theta = \frac{y}{x} = \frac{7}{24}$$




$$r = \sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{\text{المقابل}}{\text{المجاور}} = \frac{7}{24}$$

$$r = \sqrt{7^2 + 24^2} = 25$$

$$\boxed{\sec\theta =} \frac{r}{x} = \frac{25}{24}$$

$$\boxed{\sin\theta = \frac{y}{r} \quad \frac{7}{25}}$$

EX) 3 Prove the following identities.

a) $\csc\theta + \tan\theta \cdot \sec\theta = \csc\theta \cdot \sec^2\theta$

L.H.S.

$$\csc\theta + \tan\theta \cdot \sec\theta = \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\cos\theta}$$

$$= \frac{1}{\sin\theta} + \frac{\sin\theta}{\cos^2\theta}$$

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المادة : ...... التاريخ / / ٢٠

$(\cos^2\theta + \sin^2\theta = 1)$

$$= \frac{\cos^2\theta + \sin^2\theta}{\sin\theta \cdot \cos^2\theta}$$

$$= \frac{1}{\sin\theta * \cos^2\theta} = \frac{1}{\sin\theta} \cdot \frac{1}{\cos^2\theta} = \csc\theta \cdot \sec^2\theta$$

R.H.S

**b)** $\cos^4\theta - \sin^4\theta = \cos^2\theta - \sin^2\theta$

L.H.S. $= \cos^4\theta - \sin^4\theta$

$$= \underbrace{(\cos^2\theta - \sin^2\theta)}_{\text{R.H.S}} \cdot \underbrace{(\cos^2\theta + \sin^2\theta)}_{1}$$

**(c)** $\dfrac{\sec\theta - \csc\theta}{\tan\theta - \cot\theta} = \dfrac{\tan\theta + \cot\theta}{\sec\theta + \csc\theta}$

L.H.S.

$$= \frac{\frac{1}{\cos\theta} - \frac{1}{\sin\theta}}{\frac{\sin\theta}{\cos\theta} - \frac{\cos\theta}{\sin\theta}} = \frac{1}{\sin\theta + \cos\theta}$$

EX) 3 Simplify $\frac{1}{\sqrt{X^2-a^2}}$ when $X = a.\csc\theta$

Sol) $\frac{1}{\sqrt{X^2-a^2}} = \frac{1}{\sqrt{a^2\csc^2\theta - a^2}}$

$$= \frac{1}{a\sqrt{\underbrace{\csc^2\theta - 1}_{\cot^2\theta}}} = \frac{1}{a\sqrt{\cot^2\theta}} = \frac{1}{a}\tan\theta.$$

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ex) 4) Eliminate $\theta$ From the equations:

1) $X = a\sin\theta$ and $y = b\tan\theta$

① $X = a.\sin\theta$

$\sin\theta = \frac{X}{a} \Rightarrow \csc\theta = \frac{a}{X}$

② $y = b\tan\theta$

$\tan\theta = \frac{y}{b} \Rightarrow \cot\theta = \frac{b}{y}$

$\csc^2 = \cot^2\theta + 1$

$\frac{a^2}{X^2} = \frac{b^2}{y^2} + 1$

EX) 5) If $\tan^2\theta - 2\tan^2\beta = 1$,

Show that $2\cos^2\theta - \cos^2\beta = 0$

$$\tan^2\theta - 2\tan^2\beta = 1$$

$$=(\sec^2\theta - 1 - 2(\sec^2\beta - 1)) = 1$$

$$= \sec^2\theta \underbrace{-1 + 2}_{+1} - 2\sec^2\beta = 1 \quad -1$$

$$= \sec^2\theta - 2\sec^2\beta = 0$$

$$\frac{1}{\cos^2\theta} - \frac{2}{\cos^2\beta} = 0$$

$$\frac{\cos^2\beta - 2\cos^2\theta}{\cos^2\theta\cos^2\beta} = 0$$

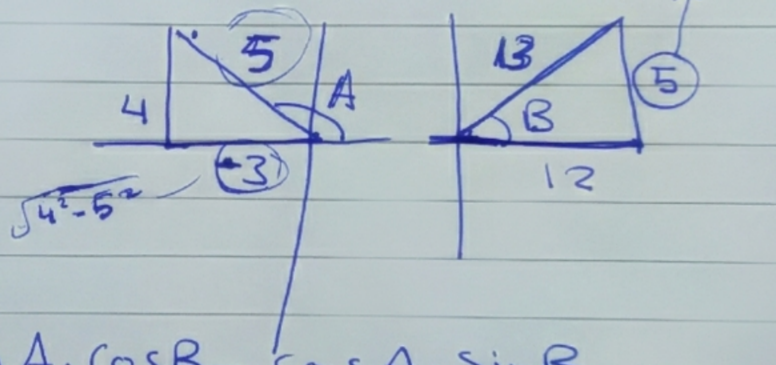
$$[\cos^2\beta - 2\cos^2\theta = 0] \times -1$$

$$\boxed{2\cos^2\theta - \cos^2\beta = 0}$$

ex) 6 IF $\sin A = \frac{4}{5}$ and $\cos B = 12/13$, where A is obtuse and B is acute Find, without table

(a) $\sin(A-B)$

$\sqrt{13^2 - 12^2}$




$$\sin(A-B) = \sin A \cdot \cos B - \cos A \cdot \sin B$$

$$= \frac{4}{5} \cdot \frac{12}{13} + \frac{3}{5} \cdot \frac{5}{13} = \frac{63}{65}$$

(b) $$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$$

$$= \frac{-\frac{4}{3} - \frac{5}{12}}{1 + \left(-\frac{4}{3} \cdot \frac{5}{12}\right)} = \frac{-63}{16}$$