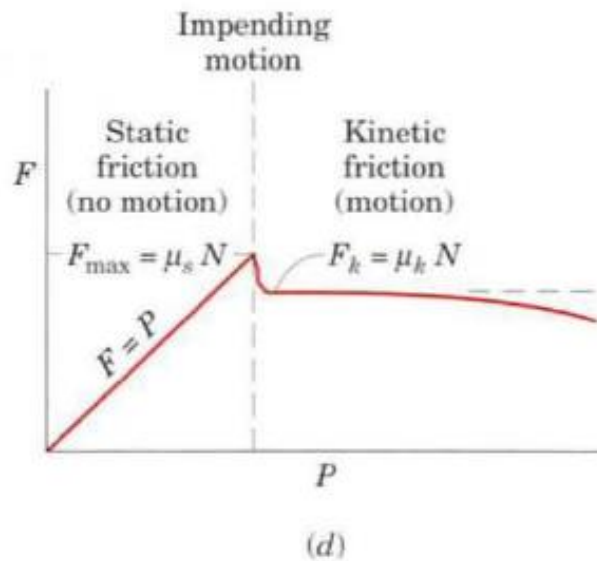
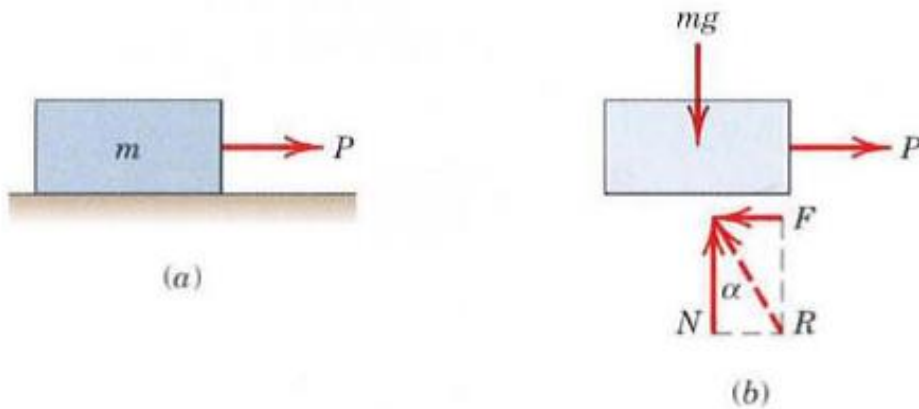


FRICTION

Consider a solid block of mass m resting on a horizontal surface, as shown in Figures we assume that the contacting surfaces have some roughness.





Static Friction

The region in Figure d up to the point of slippage or impending motion is called the range of static friction, and in this range the value of the friction force is determined by the equations of equilibrium.

$$F_{\max} = \mu_s N$$

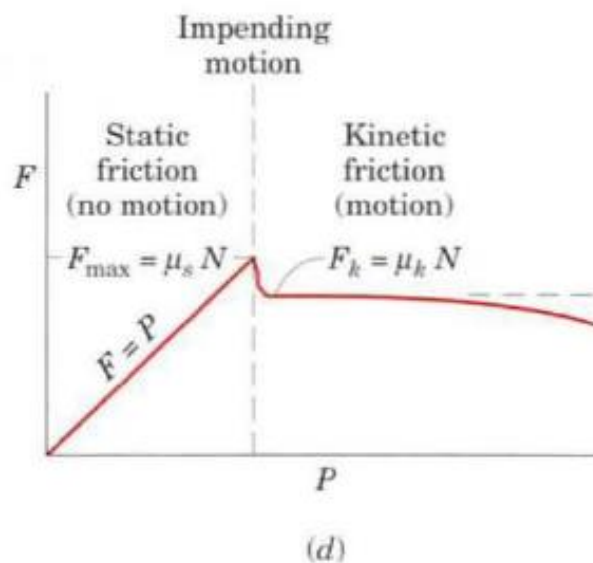
Where μ_s , is the proportionality constant, called the coefficient of static friction.

$$F < \mu_s N$$

Kinetic Friction

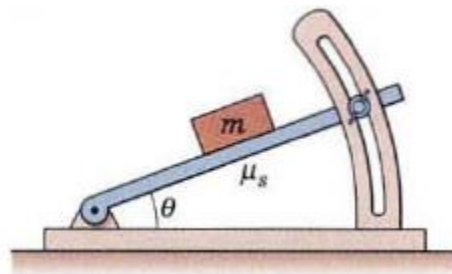
After slippage occurs, a condition of kinetic friction accompanies the ensuing motion. Kinetic friction force is usually somewhat less than the maximum static friction force. The kinetic friction force F_k is also proportional to the normal force. Thus,

$$F_k = \mu_k N$$

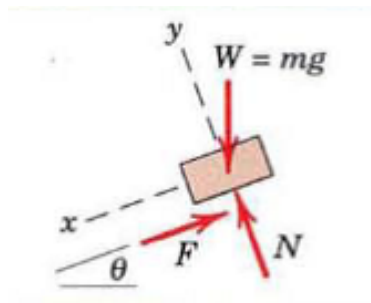


Problem 1

Determine the maximum angle θ which the adjustable incline may have with the horizontal before the block of mass m begins to slip. The coefficient of static friction between the block and the inclined surface is μ_s .



Solution



Equilibrium in the x- and y-directions requires

$$\Sigma F_x = 0 \quad mg \sin \theta - F = 0 \quad F = mg \sin \theta$$

$$\Sigma F_y = 0 \quad - mg \cos \theta + N = 0 \quad N = mg \cos \theta$$

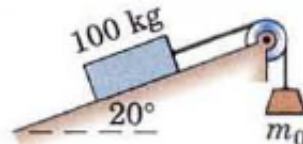
$$F/N = \tan \theta$$

$$F = F_{\max} = \mu_s N$$

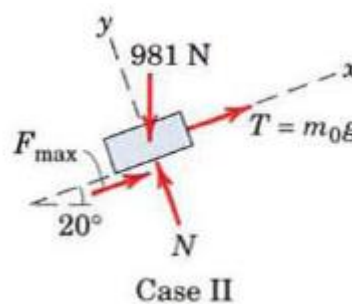
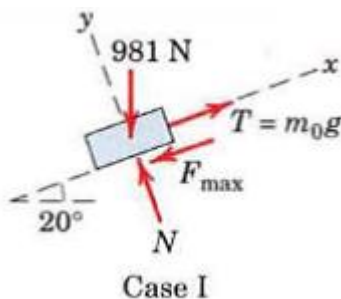
$$\mu_s = \tan \theta_{\max} \quad \text{or} \quad \theta_{\max} = \tan^{-1} \mu_s$$

Problem 2

Determine the range of values which the mass m_0 may have so that the 100-kg block shown in the figure will neither start moving up the plane nor slip down the plane. The coefficient of static friction for the contact surfaces is 0.30



Solution



$$mg = 100(9.81) = 981 \text{ N} ,$$

$$\Sigma F_y = 0 \quad , \quad N - 981 \cos 20^\circ = 0 \quad N = 922 \text{ N}$$

$$F_{\max} = \mu_s N \quad , \quad F_{\max} = 0.30(922) = 277 \text{ N}$$

The block for Case I in the figure

$$\Sigma F_x = 0 \quad , \quad m_0(9.81) - 277 - 981 \sin 20^\circ = 0 \quad , \quad m_0 = 62.4 \text{ kg}$$

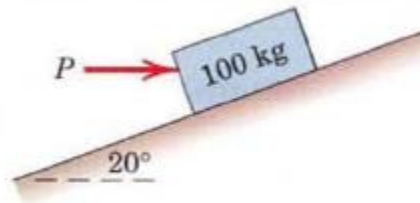
The block for Case II in the figure

$$\Sigma F_x = 0 \quad m_0(9.81) + 277 - 981 \sin 20^\circ = 0 \quad m_0 = 6.01 \text{ kg}$$

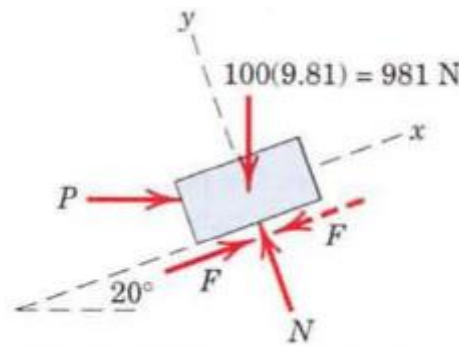
Thus, m_0 may have any value from 6.01 to 62.4 kg, and the block will remain at rest.

Problem 3

Determine the magnitude and direction of the friction force acting on the 100-kg block shown if, first, $P = 500 \text{ N}$ and, second, $P = 100 \text{ N}$. The coefficient of static friction is 0.20, and the coefficient of kinetic friction is 0.17. The forces are applied with the block initially at rest.



Solution



$$\Sigma F_x = 0 \quad P \cos 20^\circ + F - 981 \sin 20^\circ = 0$$

$$\Sigma F_y = 0 \quad N - P \sin 20^\circ - 981 \cos 20^\circ = 0$$

Case I. $P = 500 \text{ N}$

$$F = -134.3 \text{ N} \quad , \quad N = 1093 \text{ N}$$

$$F_{\max} = \mu_s N \quad F_{\max} = 0.20(1093) = 219 \text{ N}$$

Case II. $P = 100 \text{ N}$

$$F = 242 \text{ N} \quad , \quad N = 956 \text{ N}$$

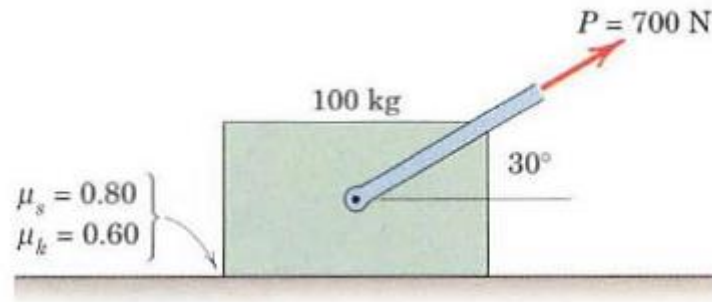
$$F_{\max} = \mu_s N \quad F_{\max} = 0.20(956) = 191.2 \text{ N}$$

It follows that 242 N of friction cannot be supported. Therefore, equilibrium cannot exist, and we obtain the correct value of the friction force by using the kinetic coefficient of friction accompanying the motion down the plane.

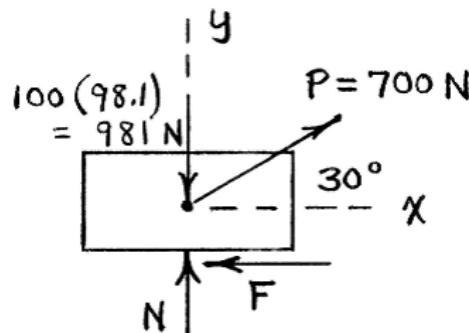
$$F_k = \mu_k N \quad F_k = 0.17(956) = 162.5 \text{ N}$$

Problem 4

The 700-N force is applied to the 100-kg block, which is stationary before the force is applied. Determine the magnitude and direction of the friction force F exerted by the horizontal surface on the block.



Solution



$$\sum F_x = 0 : 700 \cos 30^\circ - F = 0, \quad F = 606 \text{ N}$$

$$\sum F_y = 0 : N - 981 + 700 \sin 30^\circ = 0, \quad N = 631 \text{ N}$$

$$F_{\max} = \mu_s N = 0.8 (631) = 505 \text{ N} < F = 606 \text{ N}$$

Assumption invalid, motion occurs.

$$F = \mu_k N = 0.6 (631) = \underline{379 \text{ N}}$$