Straight Aluminum Fin — EXAMPLE 2-9

An aluminum fin [$k = 200$ W/m · °C] 3.0 mm thick and 7.5 cm long protrudes from a wall, as in Figure 2-9. The base is maintained at 300°C, and the ambient temperature is 50°C with $h = 10$ W/m^2 · °C. Calculate the heat loss from the fin per unit depth of material.

Solution:

Let neglecting the heat lost from the end

$$q = \sqrt{hPkA}\,\theta_0 \tanh mL$$

$$P = 2(W + t) = 2(1 + 0.003) = 2.006m$$

$$A = W.t = 1 \times 0.003 = 0.003m^2$$

$$\theta_0 = T_0 - T_\infty = 300 - 50 = 250^0 C$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{10 \times 2.006}{200 \times 0.003}} \cong 5.774$$

$\therefore$

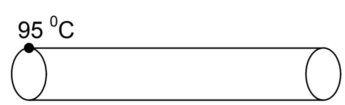
$$q = \sqrt{10 \times 2.006 \times 200 \times 0.003} \times 250 \times \tanh(5.774 \times 0.075)$$

$$q = 357\,W/m\ depth$$

Example: An experimental device that produce excess heat is supplied with pin fins to increase the rate of cooling consider a copper pin fin, 0.25cm in diameter that produces from a wall at 95°C into ambient air at 25°C. $h=10W/m^2.K$, $k=396\ W/mK$. Calculate the heat loss assuming that:

1- The fin is infinitely long.
2- The fin is 2.5cm long and heat is convected from the end.
3- How long would the fin have to be for the infinitely long solution to be corrected within 5%.

Solution:



1)

$$q = \sqrt{hPkA}\,\theta_0$$

$$P = \pi.D, \quad A = \frac{\pi.D^2}{4}$$

$$q = \sqrt{10 \times \pi \times 0.0025 \times 396 \times \pi / 4 \times (0.0025)^2} \times (95 - 25) = 0.865W$$

2)

$$q = \sqrt{hPkA}\,\theta_0 \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} \quad =0.14W$$

3) For only 5% error

$$\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL} \geq 0.95$$

Which gives L≥28.3cm

Fin Effectiveness

The effectiveness of the fin in transferring heat is given by the *fin efficiency* (η_f).

$$\eta_f = \frac{\text{actual heat transferred}}{\text{heat that would be transferred if entire fin area were at base temperature}} = \frac{q_{fin}}{q_{\max}}$$

$$q_{\max} = h.A_f.\theta_0 , \quad A_f = P.L , A_f = surface\ area\ of\ the\ fin$$

$$For\ case\ 2:$$

$$q_{fin} = \sqrt{hPkA}\,\theta_0 \tanh mL \quad \Rightarrow$$

$$\eta_f = \frac{\sqrt{hPkA}\,\theta_0 \tanh mL}{hPL\theta_0} = \frac{\tanh mL}{mL}$$

Fin Performance (Є)

$$\varepsilon = \frac{q\ with\ fin}{q\ without\ fin} = \frac{\eta_f A_f h\theta_0}{hA_b\theta_0}$$

$$A_f = surface\ area\ of\ the\ fin = P.L$$

$$A_b = base\ area\ of\ the\ fin = A$$

Corrected Length (Lc)

Lc is used in all equations that apply for the case of the fin with an insulated tip (**case 2**).

$$Lc = L + \frac{t}{2} \quad \text{(For general)}$$

$$Am = L.t$$

Example: The outer surface of an oil heater at a uniform temperature of 150°C is to be filled with straight rectangular fins having a uniform thermal conductivity of 25 W/m.K. The ambient air temperature is 20°C and the heat transfer coefficient is 570 W/m^2.K. Determine the thickness and fin efficiency if the length of each fin is 10mm and each to remove 900W per meter length of primary surface.

Solution.

$$T_0 = 150°\text{C},\ \ k = 25W/m.K,\ \ h = 570W/m^2.K,\ \ L = 10mm,\ \ T_\infty = 20°\text{C}$$

$$q = \sqrt{\text{hPkA}}\,\theta_0 \tanh mLc$$
$$P = 2(w+t) = 2(t+1),\quad A = wt,\ \ w = 1$$
$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{570\times 2(t+1)}{25\times t}} = 6.75\sqrt{\frac{t+1}{t}}$$
$$q = \sqrt{570\times 2(t+1)\times 25\times t}\,(150-20)\tanh\left(0.0675\sqrt{\frac{t+1}{t}}\right)$$
$$900 = 21964.5\sqrt{t^2+t}\,\tanh\left(0.0675\sqrt{\frac{t+1}{t}}\right)$$
$$\textit{by trial and error} \Rightarrow t \cong 2.07mm$$
$$\eta_f = \frac{\tanh mLc}{mLc} = 60.7\%$$

Example 2: A very long 1cm diameter copper rod (k =377 W/m.K) is exposed to an environment at 22°C. The base temperature is maintained at 150°C and the heat transfer coefficient between the rod and surrounding air is 11 W/m^2.K. Determine the heat transfer rate from the rod to the surrounding air.

Solution.

$$q = \sqrt{hPkA}\,\theta_0$$
$$q = \sqrt{h(\pi D)\times k\times \pi D^2/4}\,(T_0 - T_\infty)$$
$$q = 12.95W$$

Example 3: Repeat example 2 for finite length of 2, 4, 8, …..128cm, a surrounding heat loss at the end (**case 3**).

Solution.

For case 3

$$q = \sqrt{hPkA}\,\theta_0 \; \frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{h(\pi D)}{k(\frac{\pi D^2}{4})}} = \sqrt{\frac{4h}{kD}} = 3.416$$

At $L=2cm,\ mL=0.06832,\ \sinh mL=0.06837,\ \cosh mL=1.00233 \quad \Rightarrow \quad q=0.993W$