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8. THE IDEAL-GAS EQUATION OF STATE

A gas is made of molecules that move around with random motion. In a perfect gas, the molecules may collide but they have no tendency to stick together or repel each other. In reality, there is a slight force of attraction between gas molecules but this is so small that gas laws formulated for an ideal gas work quite well for a real gas.

Any equation that relates the pressure, temperature, and specific volume of a substance is called an equation of state. Property relations that involve other properties of a substance at equilibrium states are also referred to as equations of state. There are several equations of state, some simple and others very complex. The simplest and best-known equation of state for substances in the gas phase is the ideal-gas equation of state. This equation predicts the (P-υ-T) behavior of a gas quite accurately within some properly selected region.

Gas and vapor are often used as synonymous words. The vapor phase of a substance is customarily called a gas when it is above the critical temperature. Vapor usually implies a gas that is not far from a state of condensation. In 1662, Robert Boyle, an Englishman, observed during his experiments with a vacuum chamber that the pressure of gases is inversely proportional to their volume. In 1802, J. Charles and J. Gay-Lussac, Frenchmen, experimentally determined that at low pressures the volume of a gas is proportional to its temperature. That is,

$$PV = \mathrm{mRT} \qquad (18)$$

$$P\upsilon = \mathrm{RT} \qquad (19)$$

where

R: is called the gas constant and it is different for each gas. Table (1) shows the values of the gas constant for different gases.

P: is the absolute pressure.

T: is the absolute temperature (K).

V: is the volume (m^3)

υ: is the specific volume (m^3/kg).

Equations (18 and 19) is called the ideal-gas equation of state, or simply the ideal-gas relation, and a gas that obeys this relation is called an ideal gas.

$$R = \frac{R_u}{M} \qquad and \qquad M = \frac{m}{N} \qquad (20)$$

where R_u: is the universal gas constant.

M: is the molar mass (also called molecular weight) of the gas.

The constant (R_u) is the same for all substances, and its value is

$$R_u = 8.314 \text{ kJ/kmol.K}$$

$$R_u = 1545.37 \text{ ft.lbf/kmol.R}$$

Boyle's law states that the pressure of a given mass of an ideal gas is inversely proportional to its volume at a constant temperature. It is expressed as:

$$PV = C \qquad or \qquad P_1V_1 = P_2V_2 \Rightarrow \frac{P_1}{P_2} = \frac{V_2}{V_1} \qquad (21)$$

Charles's law states that the volume of an ideal gas at constant pressure is directly proportional to the absolute temperature. It is expressed as:

$$\frac{V}{T} = C \qquad or \qquad \frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad (22)$$

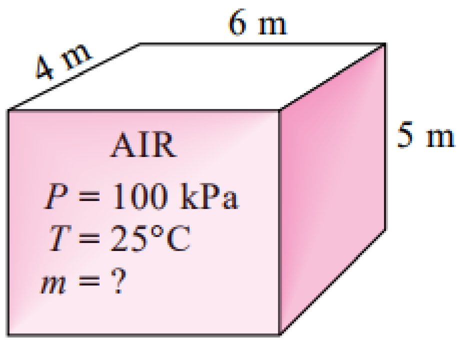
Gay-Lussac's law states that, for a given mass and constant volume of an ideal gas, the pressure exerted on the sides of its container is directly proportional to its absolute temperature. It is expressed as:

$$\frac{P}{T} = C \qquad or \qquad \frac{P_1}{T_1} = \frac{P_2}{T_2} \qquad (23)$$

Example (4): Determine the mass of the air in a room whose dimensions are (4 m ×5 m × 6 m) at (100 kPa) and (25 °C).

Solution:

From Table (1), the gas constant of air is



$R = 0.287\ kJ/kg.K$

$T = 25 + 273 = 298\ K$

$V = 4 * 5 * 6 = 120\ m^3$

$$PV = mRT \ \Rightarrow m = \frac{PV}{RT}$$

$$m = \frac{PV}{RT} = \frac{100 * 120}{0.287 * 298} = 140.3\ kg$$

Example (5): An amount of gas has a pressure of (350 kPa), a volume of (0.03 m^3) and a temperature of (35 °C). If (R = 0.29 kJ/kg.K), calculate the mass of the gas and the final temperature if the final pressure is (1.05 MPa) and the volume remains constant.

Solution:

The absolute temperature: $T_1 = 35 + 273 = 308\ K$

$P_1V_1 = mRT_1$

$$m = \frac{350 \times 0.03}{0.29 \times 308} \Rightarrow m = 0.12\ kg$$

Applying the equation of state between two conditions at constant volume:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{350}{308} = \frac{1.05 \times 10^3}{T_2} \Rightarrow T_2 = \frac{1.05 \times 10^3 * 308}{350}$$

$T_2 = 924$ K

Example (6): A tank has a volume of (0.5 m^3) and contains (10 kg) of an ideal gas having a molecular weight of (24). The temperature is (25 °C). What is the pressure of the gas?

Solution:

The absolute temperature: $T = 25 + 273 = 298\ K$

$$R = \frac{R_u}{M} = \frac{8.314}{24} = 0.35\ kJ/kg.K$$

$$PV = mRT$$

$$P \times 0.5 = 10 \times 0.35 \times 298$$

$$P = \frac{10 \times 0.35 \times 298}{0.5}$$

$$P = 2086\ kPa$$

Home Work (1):

1- A container of (0.2 m^3) contains nitrogen at a pressure of (1.013 bar) and at a temperature of (15 °C). (2 kg) of nitrogen was pumped by a special pump to the tank. Calculate the new gas pressure when the tank returns to its initial temperature. Nitrogen was considered an ideal gas, take R = 296.9 J / kg. K.

Ans. (1.87 bar)

2- Air in an internal combustion engine has (227°C), (1000 kPa) with a volume of (0.1 m^3). Now combustion heats it to (1500 K) in a constant volume process. What is the mass of air and how high does the pressure become?

Ans. (0.697 kg, 3000 kPa)

3- A rigid tank of 1 m3 contains nitrogen gas (molecular weight 28) at 600 kPa, 400 K. By mistake someone lets 0.5 kg flow out. If the final temperature is 375 K what is the final pressure?

Ans. (506.9 kPa)

4- A (1 m^3) rigid tank contains propane (molecular weight 44) at (100 kPa), (300 K) and connected by a valve to another tank of (0.5 m^3) with propane at (250 kPa), (400 K). The valve is opened and the two tanks come to a uniform state at (325 K). What is the final pressure?

Ans. (139.9 kPa)

5- (0.1) kg of ideal gas occupies a volume of (0.003 m^3) at a pressure of (7 bar) and a temperature (131 °C) when the gas was expand to a pressure of (1 bar), its final volume became (0.02 m^3). Calculate the final temperature.

Ans. (384.6 K)

6- Air is at (25 ° C) and (101.325 kPa). If the gas constant (R = 287 J / kg. k), find the specific volume and the molar mass of this gas, assuming it behaves as an ideal gas.

Ans. (0.8445 m^3/kg, 28.97 kg/kmol)

Table (1) Ideal-gas specific heats of various common gases at (300 K)

Gas	*Formula*	*Gas constant R* *kJ/(kg.K)*	c_p *kJ/(kg.K)*	c_v *kJ/(kg.K)*	γ
Air	–	0.2870	1.005	0.718	1.400
Argon	Ar	0.2081	0.5203	0.3122	1.667
Butane	C_4H_{10}	0.1433	1.7164	1.5734	1.091
Carbon dioxide	CO_2	0.1889	0.846	0.657	1.289
Carbon monoxide	CO	0.2968	1.040	0.744	1.400
Ethane	C_2H_6	0.2765	1.7662	1.4897	1.186
Ethylene	C_2H_4	0.2964	1.5482	1.2518	1.237
Helium	He	2.0769	5.1926	3.1156	1.667
Hydrogen	H_2	4.1240	14.307	10.183	1.405
Methane	CH_4	0.5182	2.2537	1.7354	1.299
Neon	Ne	0.4119	1.0299	0.6179	1.667
Nitrogen	N_2	0.2968	1.039	0.743	1.400
Octane	C_8H_{18}	0.0729	1.7113	1.6385	1.044
Oxygen	O_2	0.2598	0.918	0.658	1.395
Propane	C_3H_8	0.1885	1.6794	1.4909	1.126
Steam	H_2O	0.4615	1.8723	1.4108	1.327