كلية المستقبل الجامعة
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اشتقاق الدوال المثلثية

Trigonometric Functions.

اذا كانت (u) اي دالة يمكن اشتقاقها

IF $\underline{u}$ is any differentiable function of $\underline{x}$, then.

① $\frac{d}{dx} \sin u = \cos u \cdot \frac{du}{dx}$

② $\frac{d}{dx} \cos u = -\sin u \cdot \frac{du}{dx}$

③ $\frac{d}{dx} \tan u = \sec^2 u \cdot \frac{du}{dx}$

④ $\frac{d}{dx} \cot u = \csc^2 u \cdot \frac{du}{dx}$

⑤ $\frac{d}{dx} \sec u = \sec u \cdot \tan u \cdot \frac{du}{dx}$

⑥ $\frac{d}{dx} \csc u = -\csc u \cdot \cot u \cdot \frac{du}{dx}$

EX) Find $\frac{dy}{dX}$ for the following functions:

① $y = \tan(3X^2)$

Ans

$$\frac{dy}{dX} = \sec^2(3X^2) \cdot \underline{6X}$$

مشتقة الدالة $(3X^2)$

$$\frac{dy}{dX} = 6X \cdot \sec^2(3X^2)$$

② $y = (\csc X + \cot x)^2$

$$\frac{dy}{dX} = 2(\csc X + \cot X)(-\csc X \cdot \cot X - \csc^2 X)$$

$$= -2\csc X \cdot (\csc X + \cot X)^2$$

③ $y = 2\sin\frac{x}{2} - x\cos\frac{x}{2}$

مشتقه ضرب دالتين

$$\frac{dy}{dx} = \left(2\cos\frac{x}{2}\right) * \frac{1}{2} - \left[x \cdot \left(-\sin\frac{x}{2} \cdot \frac{1}{2}\right) + \left(\cos\frac{x}{2} \cdot 1\right)\right]$$

$$\frac{dy}{dx} = \frac{x}{2} \cdot \sin\frac{x}{2}$$

④ $y = \tan^2(\cos x)$

مشتقه دالة الأسية

$$\frac{dy}{dx} = 2\underbrace{(\tan(\cos x))}_{\text{داخل القوس نفسه}} \cdot \underbrace{\sec^2(\cos x)}_{\text{مشتقه اول القوس}} \cdot \underbrace{(-\sin x)}_{\text{مشتقه الدالة}}$$

$$\frac{dy}{dx} = +2 \cdot \sin x \cdot \tan(\cos x) \cdot \sec^2(\cos x)$$

(3)

(5) $X + \tan(xy) = 0$ اشتقاق ضمني

Ans

$$1 + \sec^2(\underbrace{xy}_{\downarrow}) \cdot \left(x \cdot \frac{dy}{dx} + y\right) = 0$$

مشتقه ضرب دالتين

$$1 + x\sec^2(xy)\frac{dy}{dx} + y\sec^2(xy) = 0$$

$$1 + y\sec^2(xy) = -x\sec^2(xy)\frac{dy}{dx}$$

$$\frac{dy}{dx} = -\frac{1 + y\sec^2(xy)}{x\sec^2(xy)}$$

(6) $y = \sec^4 x - \tan^4 x$

$$\frac{dy}{dx} = 4\sec^3 x \cdot \sec x \tan x - 4\tan^3 x \cdot \sec^2 x$$

$$= 4\tan x \sec^2 x(\underbrace{\sec^2 x - \tan^2 x}_{1})$$

$$\frac{dy}{dx} = 4\tan x \sec^2 x$$

(4)

EX) Prove that:

$$\frac{d}{dx}\tan u = \sec^2 u \cdot \frac{du}{dx}$$

$$L.H.S = \frac{d}{dx}\tan u = \frac{d}{dx}\frac{\sin u}{\cos u}$$

$$= \frac{\cos u \cdot \cos u \cdot \frac{du}{dx} - \sin u(-\sin u)\frac{du}{dx}}{\cos^2 u}$$

$$= \frac{\cos^2 u \frac{du}{dx} + \sin^2 u \frac{du}{dx}}{\cos^2 u}$$

$$\cos^2 u + \sin^2 u = 1$$

$$= \frac{(\cos^2 u + \sin^2 u)\frac{du}{dx}}{\cos^2 u}$$

$$= \frac{1}{\cos^2 u} \cdot \frac{du}{dx} = \sec^2 u \cdot \frac{du}{dx} = R.H.S$$

EX) Prove that :

$$\frac{d}{dx}\sec u = \sec u \cdot \tan x \frac{du}{dx}$$

L.H.S $\quad \frac{d}{dx}\sec u = \frac{d}{dx}\frac{1}{\cos u}$

$$\frac{d}{dx}\ \frac{\cos u * 0 - 1 * -\sin u \frac{du}{dx}}{\cos^2 u}$$

$$\frac{d}{dx}\ \frac{\sin u}{\cos^2 u} \cdot \frac{du}{dx}$$

$$\frac{1}{\cos u} \cdot \frac{\sin u}{\cos u} \cdot \frac{du}{dx}$$

$$= \sec u \cdot \tan u \cdot \frac{du}{dx} = R.H.S$$

ex) Find $\frac{dy}{dx}$ For the Function?

$$y = \sin^3 2x$$

<u>Ans</u>

$$\frac{dy}{dx} = 3\,\underbrace{\sin^2 2x}_{\text{(القوس) ٢-١}} \cdot \underbrace{\cos 2x}_{\text{مشتقه داخل القوس}} \cdot 2$$

الاس

مشتقه زاويه ادواله بعد sin.

$$\frac{dy}{dx} = 6\sin^2 2x \cdot \cos 2x$$

ex) Find $\frac{dy}{dx}$ For the

$$y = \sqrt{x \tan x}$$

$$y = (x \tan x)^{\frac{1}{2}}$$

<u>Ans</u>

$$\frac{dy}{dx} = \frac{1}{2}(\underbrace{x \tan x}_{\text{مشتقه جزئين البتين}})^{-\frac{1}{2}} \cdot (x \cdot \sec^2 x + \tan x \cdot 1)$$

$$\frac{dy}{dx} = \frac{1}{2}(x \tan x)^{-\frac{1}{2}} \cdot (x \cdot \sec^2 x + \tan x)$$

تمرين
Homework

Q) Find the $\frac{dy}{dX}$ for the

$$y = \cos 2X^2$$

Q_2) Prove that

$$\frac{d}{dX} \sec(X) = \sec X \cdot \tan X$$

(8)