

## Steady-State Conduction One Dimension

To examine the applications of Fourier's law of heat conduction to calculation of heat flow in some simple one-dimensional systems, we may take the following different cases:

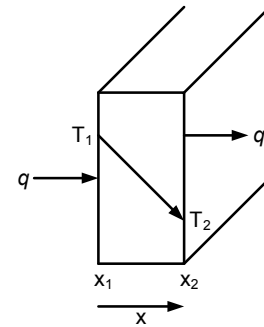
### 1- The plane wall

#### A) One material

Using Fourier's law

$$q = -kA \frac{dT}{dx} \quad \text{by } \int \Rightarrow$$

$$q = -kA \frac{T_2 - T_1}{x_2 - x_1}$$



$$\text{Flow} = \frac{\text{potential (Driving Force)}}{\text{Resistance}}$$

$$I = \frac{V}{R}$$

$$\therefore \frac{\Delta x}{kA} = \text{Thermal Resistance}$$

- When the thermal conductivity is considered constant

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$$q = -\frac{kA}{\Delta x} (T_2 - T_1)$$

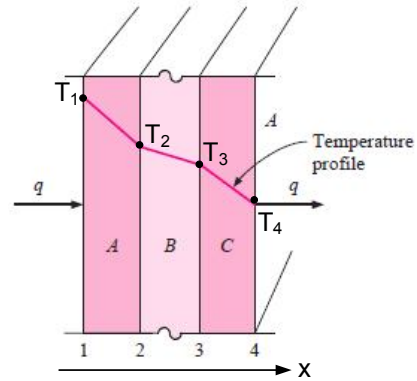
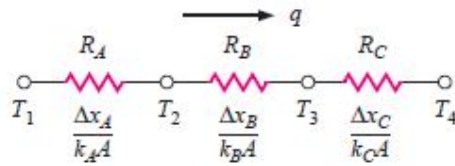
- When the thermal conductivity varies with temperature, the  $k$  can be described as

$$k = k_0(1 + \beta T)$$

$k_0$  and  $\beta$  are constants. The resultant equation for the heat flow is

$$q = -\frac{k_0 A}{\Delta x} \left[ (T_2 - T_1) + \frac{\beta}{2} (T_2^2 - T_1^2) \right]$$

**B) More than one material (Composite wall)**



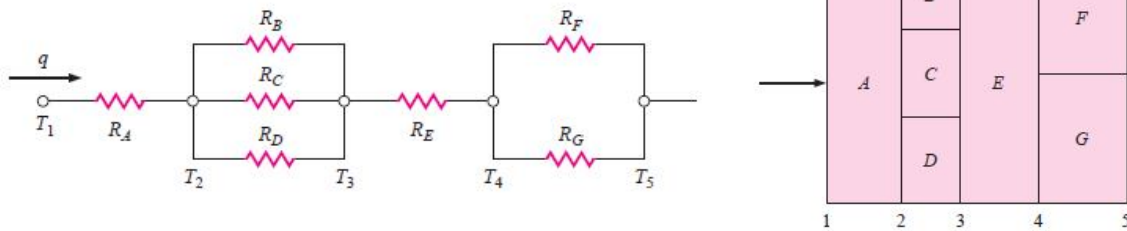
The heat flow must be the same through all sections, therefore,

$$q = -k_A A \frac{T_2 - T_1}{\Delta x_A} = -k_B A \frac{T_3 - T_2}{\Delta x_B} = -k_C A \frac{T_4 - T_3}{\Delta x_C}$$

Solving these three equations simultaneously, the heat flow is written:

$$q = \frac{T_1 - T_4}{\Delta x_A / k_A A + \Delta x_B / k_B A + \Delta x_C / k_C A}$$

For series and parallel one-dimensional heat transfer through a composite wall and electrical analog:



$$\frac{1}{R_1} = \frac{1}{R_A}; \quad \frac{1}{R_2} = \frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_D}; \quad \frac{1}{R_3} = \frac{1}{R_E}; \quad \frac{1}{R_4} = \frac{1}{R_F} + \frac{1}{R_G}$$

$$\therefore \frac{1}{R_{th}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

( $R_{th}$  is the thermal resistances)

Generally, one-dimensional heat-flow equation for this type of problem may be written

$$q = \frac{\Delta T_{overall}}{\sum R_{th}}$$

## 2- Radial systems

### A) Cylindrical

#### i- One material

Consider a long cylinder of inside radius  $r_i$ , outside radius  $r_o$ , and length  $L$ . The inner side temperature is  $T_i$ , The outer side is  $T_o$ , when the heat flows only in a radial direction.

The area for heat flow in the cylindrical system is

$$A_r = 2\pi rL$$

So that Fourier's law is written

$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -2\pi krL \frac{dT}{dr}$$

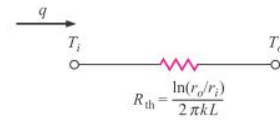
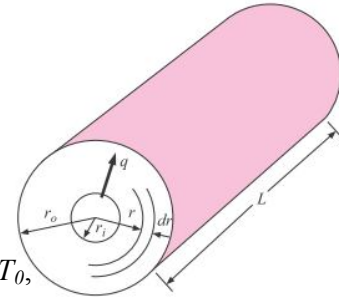
$$\frac{q}{2\pi kL} \int_{r_i}^{r_o} \frac{dr}{r} = - \int_{T_i}^{T_o} dT$$

The solution is

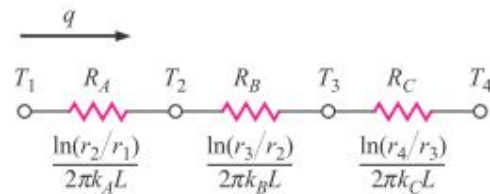
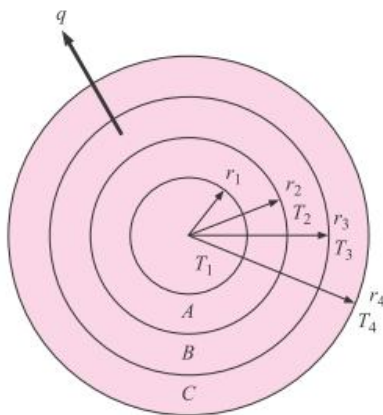
$$q = \frac{2\pi kL (T_i - T_o)}{\ln(r_o/r_i)}$$

and the thermal resistance in this case is

$$R_{th} = \frac{\ln(r_o/r_i)}{2\pi kL}$$



#### ii- Multi-Layer cylindrical wall

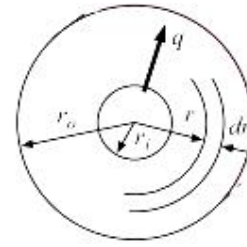


For the system shown, the solution is:

$$q = \frac{2\pi L (T_1 - T_4)}{\ln (r_2/r_1)/k_A + \ln (r_3/r_2)/k_B + \ln (r_4/r_3)/k_C}$$

### B) Spherical

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then



$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -4\pi k r^2 \frac{dT}{dr}$$

$$\frac{d}{dr} \int_{r_i}^{r_o} \frac{dr}{r^2} = - \int_{T_i}^{T_o} dT$$

$$q = \frac{4\pi k (T_i - T_o)}{1/r_i - 1/r_o}$$

The thermal resistance in spherical system is:

$$R_{th} = \frac{1}{4\pi k} \left( \frac{1}{r_i} - \frac{1}{r_o} \right)$$

**Example)** An outside wall of a building consists of 0.1m layer of common brick [k=0.69 W/m.K] and 25mm layer of fiber glass [k=0.05 W/m.K]. Calculate the heat flow with through the wall for a 45°C temperature differences.

### Solution

$$q = \frac{\Delta T}{\sum R_{th}} = \frac{\Delta T_{overall}}{\frac{\Delta x_b}{k_b A} + \frac{\Delta x_f}{k_f A}}$$

$$\Rightarrow q = \frac{45}{\frac{0.1}{0.69} + \frac{0.025}{0.05}} = 69.78 W/m^2$$