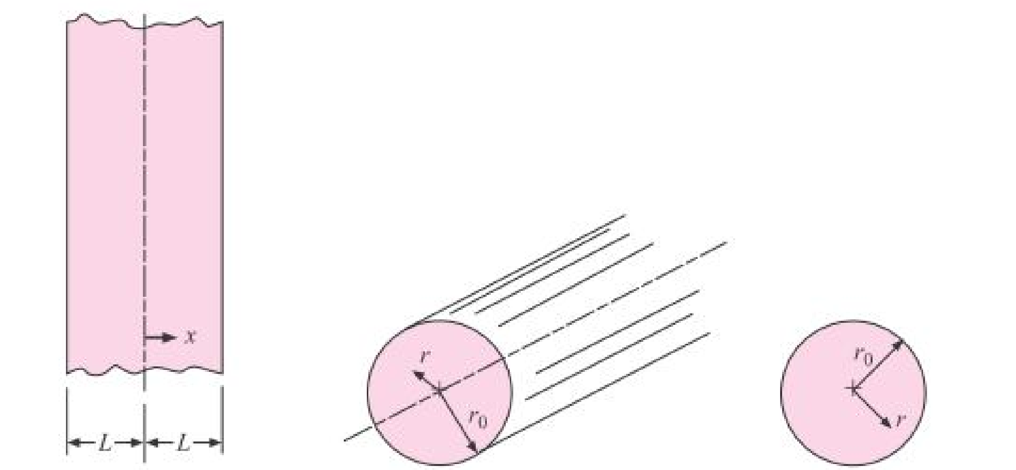
Heisler Charts

A group of curves are used with unsteady-state case when **Biot no**. is greater than **0.1**. The most cases that to be treated are

1- Infinite plate (plate where thickness is very small in comparison to other dimension).
2- Infinite cylinder (where the diameter is very small compared to length)
3- Sphere.

Figure 4-6 | Nomenclature for one-dimensional solids suddenly subjected to convection environment at T_∞: (*a*) infinite plate of thickness $2L$; (*b*) infinite cylinder of radius r_0; (*c*) sphere of radius r_0.



T_0 = centerline temperature (*a*) T_0 = centerline axis temperature (*b*) T_0 = center temperature (*c*)

The results of analysis for these geometries have been presented in graphical by Heister Charts (Figures 4.9 – 4.18).In these figures,

$T\infty$ = Environment temperature.
Ti = Initial temperature of the solid (t=0).

$$\theta = T(x,t) - T_\infty \quad or \quad T(r,t) - T_\infty$$
$$\theta_i = T_i - T_\infty$$
$$\theta_0 = T_0 - T_\infty$$

If a centerline temperature is desired, only one chart is required to obtain a value for θ_0 and then T_0, i.e **Figure 4.9** for infinite plate, **Figure 4.10** for infinite cylinder and **Figure 4.11** for sphere.

For off-center temperature, two charts are required to calculate the proc $\frac{\theta}{\theta_i} = \frac{\theta_0}{\theta_i}\frac{\theta}{\theta_0}$

i.e **Figure 4.9** and **Figure 4.12** for infinite plate, **Figure 4.10** and **Figure 4.13** for infinite cylinder and **Figure 4.11** and **Figure 4.14** for sphere.

The heat losses for the infinite plate, infinite cylinder, and sphere are given in **Figures 4-16, 4.17 and 4.18**, respectively, where Q_0 represents the initial internal energy content of the body in reference to the environment temperature, which is

$$Q_0 = \rho c V(T_i - T_\infty) = \rho c V \theta_i$$

In these figures Q is the actual heat lost by the body in time t.

The Biot and Fourier Numbers

The temperature profiles and heat flows may all be expressed in terms of two dimensionless parameters called the Biot and Fourier numbers:

$$Biot\ Number\ (Bi) = \frac{h\,s}{k}, \quad s = V/A$$

$$\text{Fourier number (Fo)} = \frac{\alpha\, t}{s^2} = \frac{k\, t}{\rho\, cp\, s^2}$$

All value of the Bi no. means that internal-conduction resistance is negligible in comparison with surface-convection resistance.

All value of Fo no. means that along period of time is required to heat or cool the body.

Applicability of the Heisler Charts

The Heisler charts is applicable in the case of ***Fo no***. is greater than ***0.2***.

$$\text{(Fo)} = \frac{\alpha\, t}{s^2} \rangle 0.2$$

Example: A steel plate (k=41 W/m.K, 5cm thick) is initially at a temperature of 425°C. its suddenly exposed on path side to a convention environment with h = 285 W/m^2.K and $T\infty$ = 65°C. Determine the centre line temperature and the temperature inside the body 1.25cm from the surface after 24min. α=1.172 ×10^{-5} m^2/s

Solution

$$2L = 5 \Rightarrow L = 2.5cm, \qquad t = 24 \times 60 = 1440 \sec$$

$$\frac{\alpha t}{L^2} = 25.7$$

$$\frac{k}{hL} = 5.75$$

$$From\ figure\ 4.9\ \frac{\theta_0}{\theta_i} = 0.015 \Rightarrow T_0 = 65 + 0.015(425 - 65) = 70.4°C \text{ (Centre temperature)}$$

$$From\ figure\ 4.12,\ at\ x/L = 0.5,\ \frac{\theta}{\theta_0} = 0.98$$

$$\therefore T = 65 + 0.98(70.4 - 65) = 70.3°C$$

Example: Along 6.5cm diameter solid cylinder steel (k=16.3 W/m.K, ρ=7817 kg/m^3, cp=460 J/kg.K). It's initially a uniform temperature *Ti* = 150°C. It's suddenly exposed to a convective environment at *T*∞ = 50°C and *h* = 285 W/m^2.K. Calculate the temperature at 1) the axis of cylinder, and 2) a 2.5cm radial distance after 5min of exposure to the cooling flow 3) determine the total energy transferred from the cylinder per meter length during the first five min. of cooling.

Solution

$$D = 6.5, r_0 = 3.25cm, \qquad t = 5 \times 60 = 300 \sec$$

$$\alpha = \frac{k}{\rho cp} = 0.4444 \times 10^{-5} m^2/s$$

$$\frac{k}{hr_0} = 1.76$$

$$\frac{\alpha t}{r_0^2} = 1.26$$

$$From\ figure\ 4.10\ \frac{\theta_0}{\theta_i} = 0.35 \Rightarrow$$

$$1)\ T_0 = 50 + 0.35(150 - 50) = 85°C$$

$$2)\ From\ figure\ 4.13,\ at\ r/r_0 = 0.77,\ \frac{\theta}{\theta_0} = 0.86 \Rightarrow T = 50 + 0.86(85 - 50) = 80°C$$

$$3)\ From\ figure\ 4.17,\ \frac{hr_0}{k} = 0.57, \quad \frac{h^2 \alpha t}{k^2} = 0.407 \Rightarrow \frac{Q}{Q_0} = 0.69$$

$$Q_0 = \rho V cp(T_i - T_\infty), \qquad V = \pi\, r_0 L$$

$$\frac{Q_0}{L} = 1193000 J/m$$

$$\therefore\ Q = 0.69 \times 1193000 = 823.170\ kJ/m$$

Example: A fused quartz sphere has a thermal diffusivity of 9.5×10^{-7} m^2/s. a diameter of 25mm and k=1.52 W/m.K. the sphere is initially at a uniform temperature Ti = 25°C. It's suddenly subjected to a convection environment at $T\infty$ = 200°C and h = 110 $W/m^2.K$. Calculate the temperature at the centre and at the radias of 6.4mm after 4min.

Solution

$$\frac{k}{hr_0} = 1.105$$

$$\frac{\alpha t}{r_0^2} = 1.46$$

$$From\ figure\ 4.11\ \frac{\theta_0}{\theta_i} = 0.048 \Rightarrow T_0 = 200 + 0.0485(25 - 200) = 191.6°C$$

2) *for r = 6.4mm,*

$$From\ figure\ 4.14,\quad \frac{\theta}{\theta_0} = 0.87 \Rightarrow T = 200 + 0.87(191.6 - 200) = 192.7°C$$

or

$$\frac{\theta}{\theta_i} = \frac{\theta}{\theta_0} \cdot \frac{\theta_0}{\theta_i} = 0.87 \times 0.048 = 0.04176 = \frac{T - T_\infty}{T_i - T_\infty} = \frac{T - 200}{25 - 200}$$

$\Rightarrow$

$$T = 192.7°C$$