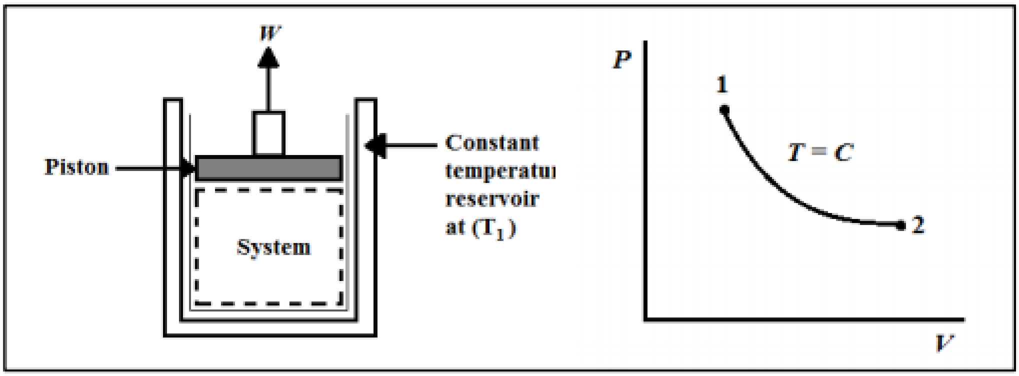
5.3 Constant Temperature (Isothermal) Process: an isothermal process is shown in the figure below. It consists of a constant temperature reservoir at temperature (T_1) surrounding a piston-cylinder arrangement. Assume that a perfect gas is at any instant, at the temperature of the system (T_1), is contained inside the cylinder. At the thermal equilibrium state, the temperature of the system and the surroundings are the same. Hence, there is no transfer of heat across the boundary. If the piston now moves slightly downward, expansion of the gas takes place increasing its volume by (dV) and consequently the pressure and temperature of the system drop by an amount of (dP) and (dT) respectively. Therefore, heat will flow from the surroundings until the system reaches the original temperature (T_1). The isothermal process will be possible only when the process is quasi-static. The (P-V) diagram of the isothermal expansion process is shown in the figure below.



Applying the first law of thermodynamics:

$$Q - W = \Delta U = mC_v(T_2 - T_1)$$

Since ($T_2 = T_1$)for an isothermal process

then:

$$Q = W = \int_1^2 PdV \qquad (19)$$

For an isothermal process, from Boyles's law, we have:

$$PV = C \rightarrow P = C/V \qquad (20)$$

Substituting equation (20) in (19), we get

$$Q = W = C\int_1^2 dV/V = C\ln\left(\frac{V_2}{V_1}\right)$$

Since $P_1V_1 = P_2V_2 = mRT = C$

then:

$$Q = W = P_1V_1 \ln\left(\frac{V_2}{V_1}\right) \qquad (21)$$

or

$$Q = W = mRT \ln\left(\frac{V_2}{V_1}\right) \qquad (22)$$

Example (7): Air enters a compressor at (10^5 Pa) and (25 °C) having a volume of (1.8 m^3 /kg), is compressed to (5×10^5 Pa) isothermally. Determine:

1) Work done. 2) Change in internal energy. 3) Heat transferred.

Solution:

1) The work done by an isothermal process is:

$$w = P_1 v_1 \ln\left(\frac{V_2}{V_1}\right)$$

Since for an isothermal process

$$\frac{V_2}{V_1} = \frac{P_1}{P_2}$$

then

$$w = P_1 v_1 \ln\left(\frac{P_1}{P_2}\right) = 10^5 * 1.8 \ln\left(\frac{10^5}{5 * 10^5}\right)$$

$$w = -289.7\ kJ/kg$$

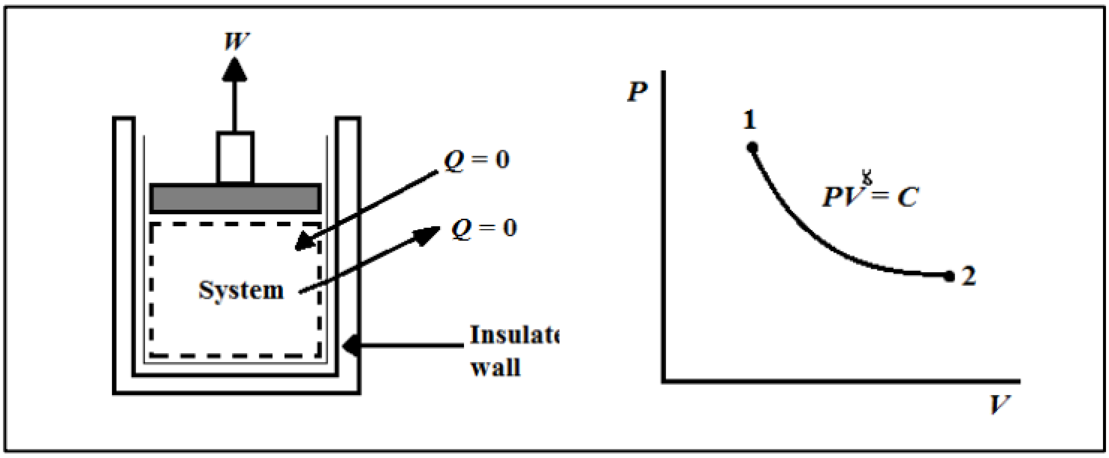
2) The change in internal energy for an isothermal process is:

$$\Delta u = 0$$

3) For an isothermal process:

$$q = w = -289.7\ kJ/kg$$

5.4 Adiabatic Process: an adiabatic process is one in which the system undergoes no heat transfer with the surroundings, but the boundary of the system moves giving displacement work. The arrangement for the adiabatic process is shown in the figure below. It consists of a piston-cylinder arrangement where the cylinder is insulated from all sides to prevent heat transfer. Since (ΔQ = 0), therefore (ΔW) is only due to (ΔU). The (P-V) diagram for an adiabatic process is shown in the figure below.



Applying the first law of thermodynamics:

$\delta Q - \delta W = dU$

For an adiabatic process $\delta Q = 0$

Also $\delta W = PdV$ and $dU = mC_v dT$

So

$0 - PdV = mC_v dT$

$PdV + mC_v dT = 0$ (1)

From the equation of state: $PV = mRT$

Differentiating both sides, we get:

$PdV + VdP = mRdT$

From the equation of state: $PV = mRT$

Differentiating both sides, we get:

$PdV + VdP = mRdT$

$$PdV + VdP = mRdT$$

$$dT = \frac{PdV + VdP}{mR} \qquad (2)$$

Substituting Eq. (2) in Eq. (1), we get:

$$PdV + \frac{mC_v(PdV + VdP)}{mR} = 0$$

Multiplying both sides by R, we get:

$$RPdV + C_v(PdV + VdP) = 0$$

Since $R = C_p - C_v$, then:

$$(C_p - C_v)PdV + C_v(PdV + VdP) = 0$$

$$C_pPdV - C_vPdV + C_vPdV + C_vVdP = 0$$

$$C_p\,PdV + C_v\,VdP = 0$$

Dividing by C_vPV, we get:

$$\left(\frac{C_p}{C_v}\right).\frac{dV}{V} + \frac{dP}{P} = 0$$

Since $\gamma = \frac{C_p}{C_v}$, then:

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

Integrating, we get:

$$\ln P + \gamma \ln V = C$$

$$\ln(PV^\gamma) = C$$

$$PV^\gamma = C$$

For a unit mass:

$$Pv^\gamma = C$$

Now

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

For a perfect gas:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{T_2}{T_1} = \frac{P_2}{P_1} \cdot \frac{V_2}{V_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \cdot \frac{V_2}{V_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

So

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

Also

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

The work done is derived as follows:

$$W = \int_1^2 PdV$$

Since $PV^{\gamma} = C \rightarrow P = \frac{C}{V^{\gamma}}$, then:

$$W = C\int_1^2 \frac{dV}{V^{\gamma}} = C\int_1^2 V^{-\gamma}\, dV$$

$$= C\left|\frac{V^{-\gamma+1}}{-\gamma+1}\right|_1^2 = \left[\frac{C}{1-\gamma}\right]\left[V_2^{1-\gamma} - V_1^{1-\gamma}\right]$$

Since $C = P_1V_1^{\gamma} = P_2V_2^{\gamma}$, then:

$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

Since $PV = mRT$, then:

$$W = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

For a unit mass:

$$w = \frac{P_1v_1 - P_2v_2}{\gamma - 1}$$

or

$$w = \frac{R(T_1 - T_2)}{\gamma - 1}$$

Example (8): Air at (1.02 bar) and (22 °C), initially occupying a cylinder volume of (0.015 m^3), is compressed reversibly and adiabatically by a piston to a pressure of (6.8 bar). Calculate: 1) The final temperature. 2) The final volume. 3) The work done. 4) The heat transferred to or from the cylinder walls.

Solution:

The absolute temperature is: $T_1 = 22 + 273 = 295$ K

To find the mass of the air:

$$P_1V_1 = mRT_1 \rightarrow m = \frac{P_1V_1}{RT_1} = \frac{1.02 \times 10^5 \times 0.015}{0.287 \times 295 \times 10^3} = 0.018 \text{ kg}$$

1) The final temperature can be calculated as:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \rightarrow T_2 = 295 \times \left(\frac{6.8}{1.02}\right)^{\frac{1.4-1}{1.4}}$$

$$\boldsymbol{T_2 = 507.25\ \mathrm{K}}$$

2) The final volume can be calculated as:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} \rightarrow V_2 = V_1 \times \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} \rightarrow V_2 = 0.015 \times \left(\frac{1.02}{6.8}\right)^{\frac{1}{1.4}}$$

$$\boldsymbol{V_2 = 0.00387\ \mathrm{m^3}}$$

3) The work done is:

$$W = \frac{mR(T_1 - T_2)}{\gamma - 1} = \frac{0.018 \times 0.287 \times (295 - 507.25)}{1.4 - 1}$$

$$\boldsymbol{W = -2.741\ \mathrm{kJ}}$$

4) The heat transferred for an adiabatic process:

$$\boldsymbol{Q = 0}$$

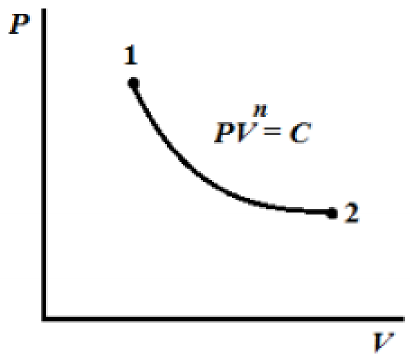
5.5 Polytropic Process: During actual expansion and compression processes of gases, pressure and volume are often related by ($PV^n = C$), where n and C are constants. A process of this kind is called a polytropic process. The (P-V) diagram for such a process is shown below. As mentioned, the general equation for polytropic processes is expressed as: $PV^n = C$

From the above equation, we can derive the following equations in the same method as in adiabatic processes:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{n}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{n-1}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$$



From Eq. (1), the work done is derived in the same method earlier and expressed as:

$$W = \frac{P_1V_1 - P_2V_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

For a unit mass:

$$w = \frac{P_1v_1 - P_2v_2}{n-1} = \frac{R(T_1 - T_2)}{n-1}$$

The heat transfer for polytropic processes does not equal zero and can be calculated from the following equation:

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right)W$$

For a unit mass:

$$q = \left(\frac{\gamma - n}{\gamma - 1}\right)w$$

Example (9): (1 kg) of air at (1.02 bar) and (17 °C) is compressed reversibly according to a law ($\boldsymbol{PV}^{1.3} = \boldsymbol{C}$), to a pressure of (5.5 bar). Calculate the work done on the air and the heat flow to or from the cylinder walls during the compression.

Solution:

The absolute temperature is: $T_1 = 17 + 273 = 290$ K

First we find the final temperature:

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} \rightarrow T_2 = 290 \times \left(\frac{5.5}{1.02}\right)^{\frac{1.3-1}{1.3}} = 427.83 \text{ K}$$

The work done is calculated as:

$$W = \frac{mR(T_1 - T_2)}{n-1} = \frac{1 \times 0.287 \times (290 - 427.83)}{1.3 - 1}$$

$$\boldsymbol{W = -131.86 \text{ kJ}}$$

The heat transfer is:

$$Q = \left(\frac{\gamma - n}{\gamma - 1}\right) W = \left(\frac{1.4 - 1.3}{1.4 - 1}\right) \times -131.86$$

$$\boldsymbol{Q = -32.97\ \text{kJ}}$$

SIGNIFICANCE OF THE FIRST LAW

The first law of thermodynamics leads directly to the non-flow energy equation and embodies four important concepts, as follows:

1- Heat and work are mutually convertible one into the other as they are both modes of energy transfer.
2- The existence of a type of energy (internal energy) that depends on the thermodynamic state of a system.
3- The possibility of measuring a difference in internal energy between thermodynamic states by making measurements of heat transfer and work.
4- The fact that energy is conserved whenever the thermodynamic state of a closed system changes.

SUMMARY

The non-flow process is the one in which there is no mass interaction across the system boundaries during the occurrence of the process such as heating and cooling of a fluid inside a closed container, compression and expansion of a fluid in a piston-cylinder arrangement, etc. For non-flow processes the first law can be written as:

$$Q - W = \Delta U + \Delta KE + \Delta PE$$

For non-flow processes the kinetic and potential energies are very small and can be neglected, so the energy equation becomes:

$$Q - W = U_2 - U_1$$

$$q - w = u_2 - u_1 \qquad \text{per (kg)}$$

where: state (1) refers to the initial state and state (2) refers to the final state. For reversible processes:

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$$W = \int_1^2 PdV$$

• For adiabatic processes (no heat transfer) $Q = 0$

• For constant volume processes $W = 0$

•For constant temperature processes $\Delta U = 0$

The following table contains the governing equations, displacement work equation and heat interaction equation for different non-flow thermodynamic processes:

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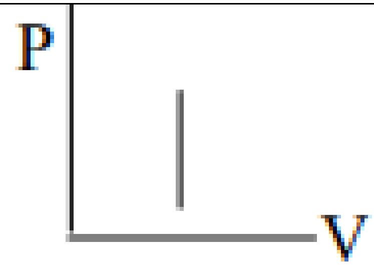
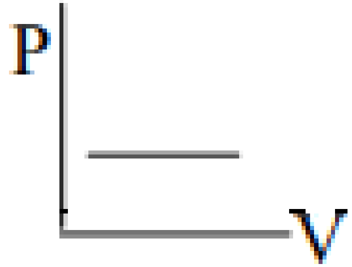
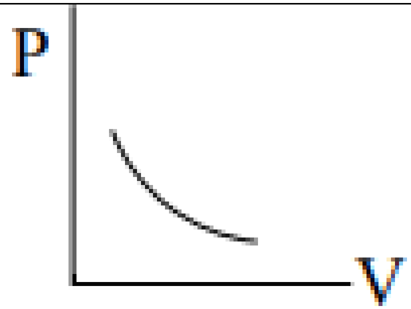
Process	**Governing equations**	**Work $(W = \int_1^2 PdV)$**	**Heat interaction**	**Figure**
Constant volume (Isochoric)	$V = Constan$ $\frac{T_1}{T_2} = \frac{P_1}{P_2}$	$W = 0$	$Q = mC_v(T_2 - T_1)$	
Constant pressure (Isobaric)	$P = Constan$ $\frac{T_1}{T_2} = \frac{V_1}{V_2}$	$W = P(V_2 - V_1)$	$Q = mC_p(T_2 - T_1)$	
Constant temperature (Isothermal)	$T = Constan$ $P_1V_1 = P_2V_2$ $\frac{V_2}{V_1} = \frac{P_1}{P_2}$	$W = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$ $W = mRT\ln\left(\frac{V_2}{V_1}\right)$	$Q = P_1V_1 \ln\left(\frac{V_2}{V_1}\right)$ $Q = mRT\ln\left(\frac{V_2}{V_1}\right)$	

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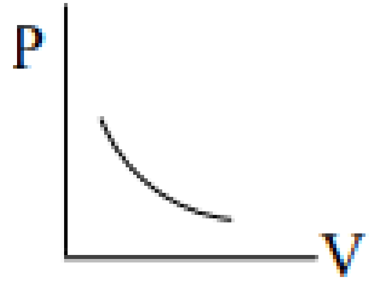
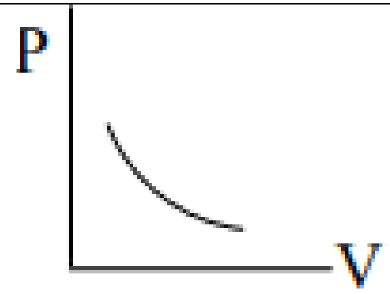
Adiabatic	$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma}$ $\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$ $\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$	$W = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$ $W = \frac{mR(T_1 - T_2)}{\gamma - 1}$	$Q = 0$	
Polytropic	$\left(\frac{P_2}{P_1}\right) = \left(\frac{V_1}{V_2}\right)^{n}$ $\left(\frac{T_2}{T_1}\right) = \left(\frac{V_1}{V_2}\right)^{n-1}$ $\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$	$W = \frac{P_1V_1 - P_2V_2}{n - 1}$ $W = \frac{mR(T_1 - T_2)}{n - 1}$	$Q = \left(\frac{\gamma - n}{\gamma - 1}\right) W$	

NOTE:

• The heat **transferred to the system** is positive (+).

• The heat **transferred from the system** is negative (-).

• The work **transferred from the system** is positive (+).

• The work **transferred to the system** is negative (-).

HOMEWORK (2)

1- In an air motor cylinder the compressed air has an internal energy of (450 kJ/kg) at the beginning of the expansion and internal energy of (220 kJ/kg) after expansion. If the work done by the air during the expansion is (120 kJ/kg), calculate the heat flow to or from the cylinder.

Ans. (-110 kJ/kg)

2- (2 kg) of gas, occupying (0.7 m^3) has an initial temperature of (15 °C). It was then heated at constant volume until its temperature became (135 °C). How much heat was transferred to the gas and what is its final pressure? Take (C_v = 0.72 kJ/kg.K) and (R = 0.29 kJ/kg.K).

Ans. (158.4 kJ, 338.1 kPa)

3- A mass of air whose pressure, volume and temperature are (275 kPa), (0.09 m^3) and (185 °C), respectively has its state changed at constant pressure until its temperature becomes (15 °C). How much heat is transferred from the gas and how much work is done on the gas during the process?

Ans. (-32.5 kJ, -9.1 kJ)

4- A quantity of air occupies a volume of 0.3 m3 at a pressure of (100 kPa) and a temperature of (20 °C). The air is compressed isothermally to a pressure of (500 kPa). Draw the (P-V) diagram of the process and determine:
1) The heat received or rejected (stating which) during the compression process.
2) The mass of the air.
3) The final volume of the air.

Ans. (-48.3 kJ, 0.36 kg, 0.06 m^3)

5- (0.05 kg) of carbon dioxide (molecular weight 44), occupying a volume of (0.03 m^3) at (1.025 bar), is compressed in a perfectly thermally insulated cylinder, until the pressure is (6.15 bar). Calculate the final temperature, the work done on the gas and the heat flow to or from the cylinder walls. Assume carbon dioxide to be a perfect gas and take $\gamma = 1.3$.

Ans. (492 K, -5.25 kJ, 0 kJ)

6- A cylinder contains (0.07 kg) of fluid having a pressure of (1 bar), a volume of (0.06 m^3) and specific internal energy of (200 kJ/kg). After a polytropic compression process, the pressure and volume of the gas become (9 bar) and (0.0111 m^3) respectively and the internal energy becomes 370 kJ/kg. Draw the (P-V) diagram of the process and determine:
1) The amount of work required for compression.
2) The quantity and direction of heat transferred during the compression process.

Ans. (-13.3 kJ, -1.4 kJ)

7- Air at a pressure of (1.06 bar) and a temperature of (15 °C), is compressed isothermally to (14 bar) and is then expanded adiabatically to the original pressure. Draw the (P-V) diagram of the processes then calculate:
1) The final temperature and specific volume of the gas.
2) The net work done.
3) The heat transferred to or from the surroundings.

Ans. (137.8 K, 0.37 m^3 /kg, -105.5 kJ/kg, -213.3 kJ/kg)

8- From the first law of thermodynamic for a closed system drive an expression the relation of Specific heat at constant volume.

9- From the first law of thermodynamic for a closed system drive an expression the relation of Specific heat at constant pressure.