Al- Mustaqbal
University College

BME 322
Signals and Systems for BME

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- Impulse Response -

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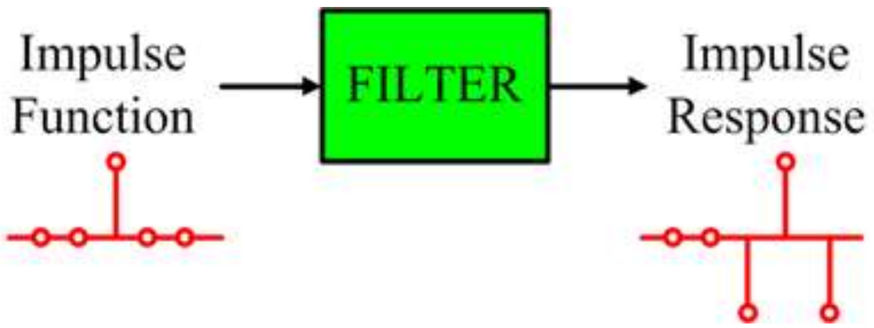
Impulse Response

- An impulse response for a filter is a the response of the filter to an impulse.



Impulse Response

- The impulse response for a filter is designated as h[n].

- The impulse response can be calculated from the difference equation by replacing the input x[n] and output of the filter by δ[n] and h[n] respectively.

Transfer Function

- Transfer function, H(z) of digital filters is the ratio of output to input in the z domain.

$$H[z] = \frac{Y[z]}{X[z]}$$

Transfer Function

Term-by-term transformation of a general difference equation.

$$\sum_{k=0}^{N} {}_{k}y\,a\,[n-k] \sum_{k=0}^{M} + {}_{k}b\,x[n-k]$$

$$H[z] = \frac{\sum_{k=0}^{M} b_k z^{-1}}{\sum_{k=0}^{N} a_k z^{-1}}$$

Example 1

Determine the transfer function of a digital filter described by the difference equation.

$$2y[n] + y[n-1] + 0.9y[n-2] = x[n-1] + x[n-4]$$

Example 1 (solution)

Taking z transforms term by term:.

$$2Y[z] + z^{-1}Y[z] \; + 0.9z^{-2}Y[z]$$
$$= z^{-1}X\ [z] + z^{-4}X\ [z]$$

$$H\ [z] = \frac{z^{-1} + z^{-4}}{2 + z^{-1} + 0.9z^{-2}}$$

Finite Impulse Response (FIR) Filters

- FIR filters are nonrecursive filters.
- The input-output relation of the FIR filters in time domain:

$$y[n] = \sum_{k=0}^{M} b_k x[n-k]$$

b_k are the filter coefficients

Finite Impulse Response (FIR) Filters

- FIR filters have a finite-duration impulse response.

- FIR filters take the number of samples equals to the number of past inputs for the impulse response to become zero.

Finite Impulse Response (FIR) Filters

- This FIR filter has the effect of averaging every N samples in the input signal.

- Any filter with this type of impulse response is called as a moving average filter.

Example 2

- A FIR filter has a set of filter coefficients {bk} = {3, -1, 2, 1}. Determine the difference equation for the filter.

Example 2 (solution)

The length of the filter is 4 with M = 3.

$$y[n] = 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$$

Example 3

Determine the first six samples in the impulse response for the FIR filter.

$$y[n] = 0.25(x[n] + x[n-1] + x[n-2] + x[n-3])$$

Example 3 (solution)

Substituting δ[n] for $x\ [n]$ and h[n] for $y\ [n]$.

$$h\ [n] = 0.25(\delta\ [n] + \delta\ [n-1]\ + \delta\ [n-2] + \delta\ [n-3]\ )$$

$$h\ [0] = 0.25(\delta\ [0] + \delta\ [-1]\ + \delta\ [-2] + [\delta\ -3]\ )$$
$$= 0.25(1.0 + 0.0 + 0.0 + 0.0) = 0.25$$

$$h\ [1] = 0.25(\delta\ [1]\ + \delta\ [0]\ + \delta\ [-1] + \delta\ [-2]\ )$$
$$= 0.25(0.0 + 1.0 + 0.0 + 0.0) = 0.25$$

Example 3 (solution)

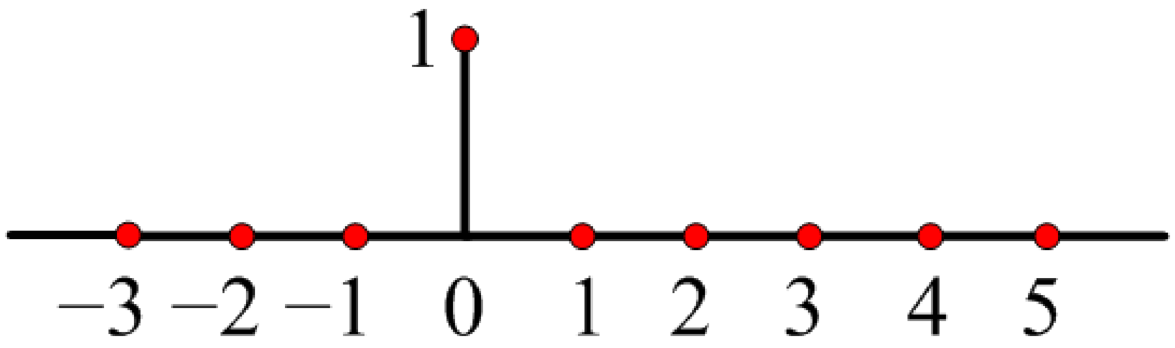
$$h[2] = 0.25(\delta[2] + \delta[1] + \delta[0] + \delta[-1])$$
$$= 0.25(0.0 + 0.0 + 1.0 + 0.0) = 0.25$$

$$h[3] = 0.25(\delta[3] + \delta[2] + \delta[1] + \delta[0])$$
$$= 0.25(0.0 + 0.0 + 0.0 + 1.0) = 0.25$$

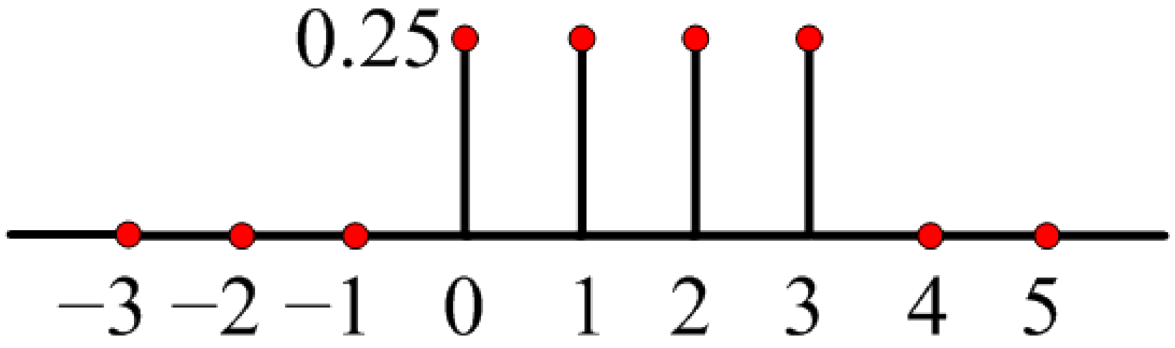
$$h[4] = 0.25(\delta[4] + \delta[3] + \delta[2] + \delta[1])$$
$$= 0.25(0.0 + 0.0 + 0.0 + 0.0) = 0.0$$

$$h[5] = 0.25(\delta[5] + \delta[4] + \delta[3] + \delta[2])$$
$$= 0.25(0.0 + 0.0 + 0.0 + 0.0) = 0.0$$

Example 3 (solution)



Impulse Function



Impulse Response

Advantages of FIR

- They can have exactly linear phase.
- They are always stable.
- The design methods are generally linear.
- They can be realized efficiently in hardware.
- The filter startup transients have finite duration

Disadvantages of FIR

- The primary disadvantage of FIR filters is that they often require a much higher filter order than IIR filters to achieve a given level of performance.

- Correspondingly, the delay of these filters is often much greater than for an equal performance IIR filter.

Direct-form FIR structures

- FIR filters of order M is characterized by M + 1 coefficients which require M + 1 multipliers, and M two-input adder.

- For FIR filters in which the multiplier coefficients are precisely the coefficients of the transfers function are called direct-form structures

- FIR filters transfer function:

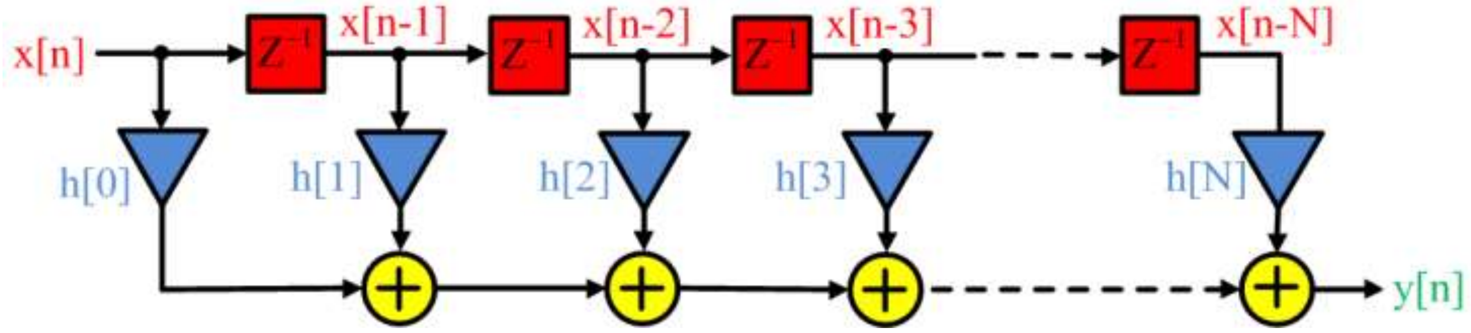
$$H[z] = \frac{Y[z]}{X[z]} = \sum_{k=0}^{M} h_k z^{-k}$$

- which is a polynomial in z^{-1} of degree M.

Direct-form FIR structures

- Expanding the filters transfer function:

$$y[n] = h[0]x[n] + h[1]x[n-1] + h[2]x[n-2] + h[3]x[n-3] + \ldots + h[N]x[n-N]$$



Example 4

- Based on the transfer function, realize the digital filter using the direct form.

$$H(z) = (1 - 2z^{-1})(1 + z^{-1} - 4z^{-2})$$

Example 4 (solution)

- Since the transfer function has only the numerator part or zeroes, therefore this is an FIR filter.

$$H(z)=\frac{Y[z]}{X[z]}=(1-2z^{-1})(1+z^{-1}-4z^{-2})$$

Example 4 (solution)

$$Y(z) = X(z) - z^{-1}X(z) - 6z^{-2}X(z) + 8z^{-3}X(z)$$

$$y(n) = x(n) - x(n-1) - 6x(n-2) + 8x(n-3)$$