Al- Mustaqbal
University College

BME 322
Signals and Systems for BME

- 8 -

- Z-TRANSFORM -

Dr. Zaidoon AL-Shammari

Lecturer / Researcher

Department of Biomedical Engineering

Al-Mustaqbal University College, Babylon - IRAQ

zaidoon.waleed@mustaqbal-college.edu.iq

Z-TRANSFORM

$$X(z) = \sum_{k=0}^{\infty} x(k) z^{-k}$$

Learning Outcomes

Students are able to:

1.calculate z-transform for a discrete signal sequence $x[n]$ to the frequency domain $X(z)$.

2.calculate inverse z-transform to recover time domain discrete signal sequence $x[n]$ from the frequency domain $X(z)$.

Definition

- Z-transform converts a discrete-time signal x[n] from time domain into a frequency domain X(z), where z is a complex variable.

$$Z\{x[n]\} = X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- The set of z-values for which the summation converges is called the region for convergence (ROC) for the transform.

Bilateral vs. Unilateral

- Two sided or bilateral z-transform

$$Z\{x[n]\} = X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

- Unilateral z-transform

$$Z\{x[n]\} = X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

Example 1

Find the z-transform X(z) of the finite-length sequence signal x[n].

n	$n \le -1$	0	1	2	3	4	5	$n > 5$
x[n]	0	2	4	6	4	2	1	0

Example 1 (solution)

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

$$X(z) = 2(z^{-0}) + 4(z^{-1}) + 6(z^{-2}) + 4(z^{-3}) + 2(z^{-4}) + 1(z^{-5})$$

$$X(z) = 2 + 4z^{-1} + 6z^{-2} + 4z^{-3} + 2z^{-4} + z^{-5}$$

Region of convergence is all z except $z = 0$.

Example 2

Al- Mustaqbal
University College

Find the z-transform $X(z)$ of the signal $x[n] = \delta[n]$.

The signal is nonzero at only a single place, n = 0. Thus,

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = 1(z^{-0}) = 1$$

Region of convergence includes all z.

Example 3

Find the z-transform $X(z)$ of the signal $x[n] = \delta[n-1]$.

The signal is nonzero only at n = 1. Therefore,

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n} = 1(z^{-1}) = z^{-1}$$

Region of convergence is all z except z = 0.

Example 4

Find $X(z)$ if $x[n] = u[n]$.

$$X(z) = \sum_{0}^{\infty} x[n]z^{-n} = \sum_{0}^{\infty} u[n]z^{-n}$$

$$X(z) = \sum_{0}^{\infty} z^{-n} = 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + \ldots$$

This is a geometric series of the form $a + ar + ar^2 + \ldots$with $a = 1$ and $r = z^{-1}$.

Example 4

The sum of an infinite geometry series is given by as long as $|r| < 1$.

$$S_{\infty} = \frac{a}{1 - r}$$

Therefore

$$X(z) = \frac{1}{1 - z^{-1}} = \frac{z}{z - 1}$$

Provided that $|z^{-1}| < 1$.

Region of convergence (ROC) is $|z| > 1$.

Example 5

A signal $x[n] = 2\delta[n] + \delta[n-1] + 0.5\,\delta[n-2]$. Find the z-transform, $X(z)$ of the signal.

$$X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$$

$$X(z) = x[0](z^{-0}) + x[1](z^{-1}) + x[2](z^{-2})$$

$$= 2(z^{-0}) + 1(z^{-1}) + 0.5(z^{-2})$$

$$= 2 + z^{-1} + 0.5z^{-2}$$

Region of convergence is all z except z = 0.

Example 6

Find the z-transform, $X(z)$ of the signal $x[n] = (-0.5)^n u[n]$.

Since $u[n] = 1$ for $n \geq 0$,

$$X(z) = \sum_{n=0}^{\infty} x[n] z^{-n}$$

$$X(z) = \sum_{0}^{\infty} (-0.5)^n z^{-n} = \sum_{0}^{\infty} (-0.5z^{-1})^n$$

Example 6

Al- Mustaqbal
University College

$$X(z) = \sum_{0}^{\infty} (-0.5z^{-1})^n$$

$$X(z) = (-0.5z^{-1})^0 + (-0.5z^{-1})^1 + (-0.5z^{-1})^2 + (-0.5z^{-1})^3 + \dots$$

$$X(z) = 1 - 0.5z^{-1} + 0.25z^{-2} - 0.125z^{-3} + \dots$$

This is a geometric series of the form $a + ar + ar^2 + \dots$ with $a = 1$ and $r = 0.5z^{-1}$. Therefore,

$$X(z) = \frac{1}{1 + 0.5z^{-1}} = \frac{z}{z + 0.5}$$

Provided that $|-0.5z^{-1}| < 1$.

Region of convergence (ROC) is $|z| > 0.5$.