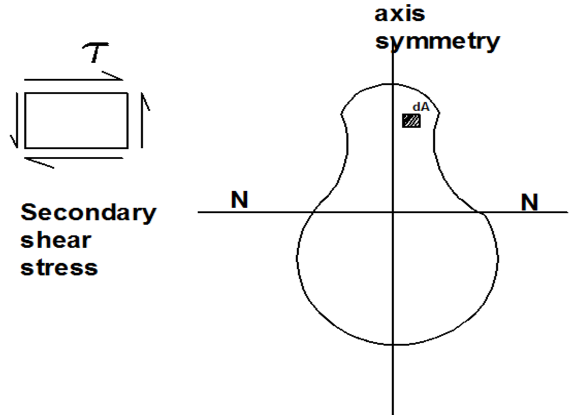
Chapter 6

Shear Stresses in beam

6-2 Introduction

Consider a beam under a load intensity w, Fig.6-18 let V be the shearing force and M be the bending moment. By considering the equilibrium of a small portion of beam of length dx. Then



$$w = -\frac{dv}{dx} \quad , \qquad V = \frac{dM}{dx}$$

let τ be the shear stress on an area dA

in a transverse section ,then

Fig. 6-18

$$v = \int \tau \, dA$$

The shear stress τ at a point on the transverse section cannot be found directly

but the equal and opposite secondary shear stress on a longitudinal section can be calculated.

At a distance y from the neutral axis let τ be the shear stress and b the width of section , also σ and σ+dσ be the bending stresses on a fiber of area dA and at a distance y_A from the neutral axis where $y_A > y$. The length of the portion of the beam is dx, then by considering the equilibrium of longitudinal; force

$$\tau(b.dx) = \int d\sigma . dA$$

but from the theory of bending of beams

$$\frac{M}{I} = \frac{\sigma}{y_A}$$

Then $\sigma = \frac{y_A}{I} M$

Also, $\sigma + d\sigma = \frac{y_A}{\mathrm{I}} (M + dM)$

And $d\sigma = \frac{y_A}{\mathrm{I}} dM$

By substituting

$$\tau.(b+dx) = \int \frac{y_A}{\mathrm{I}} dM.dA$$

$$= \frac{dM}{\mathrm{I}} \int y_A .\, dA$$

Thus $\tau = \frac{dM}{dx} . \frac{1}{b\mathrm{I}} \int y_A .\, dA$

But $\frac{dM}{dx} = V$ (shear force)

Then $\tau = \frac{V}{b\mathrm{I}} \int y_A .\, dA$, let $\int y_A .\, dA = A$

Where A is the area above the line y from the neutral axis and y' is the distance of the centroid of area A from the neutral axis .Then

$$\tau = \frac{VAy'}{b\mathrm{I}} \qquad (6\text{-}13)$$

6-8-1 Distribution of shear stresses on a rectangular section

From Fig. (6-19), the area from top edge to the line y will be

$$A = b\left(\frac{d}{2} - y\right) ,$$

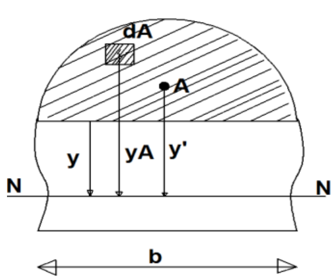


Fig. 6-19

and the distance of the centroid of area A from the neutral axis will be

$$y' = \frac{d}{2} - \frac{1}{2}\left(\frac{d}{2} - y\right)$$

$$y' = \frac{1}{2}\left(\frac{d}{2} + y\right)$$

Then, the shear stress at level y from the neutral axis will be

$$\tau = \frac{VAy'}{bI}$$

$$= \frac{Vb\left(\frac{d}{2} - y\right) \cdot \frac{1}{2}\left(\frac{d}{2} + y\right)}{b\frac{1}{12}bd^3}$$

Therefore

$$\tau = \frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

At the centroidal axis where y=0

$$\tau_{max} = \frac{3V}{2\,bd} \qquad (6\text{-}14)$$

which is 50% greater than obtained by dividing $\tau = \frac{V}{bd}$

6-8-2 Distribution of shear stress on I sections

Fig. (6-20), the distribution of the shear stress is given by

$$\tau = \frac{VAy'}{bI}$$

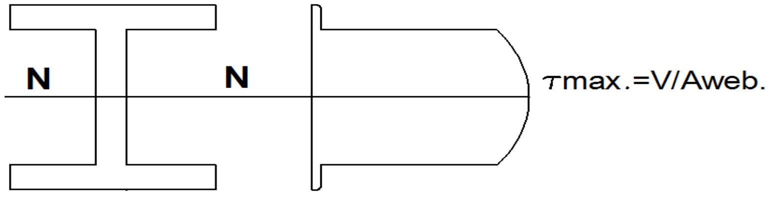


Fig. (6-20)

In I – sections the width of the web b is small when compared to the width of the flanges. Therefore, the shear stress is small in flanges and large in the web. The shear stress in the web is nearly constant and is given by

$$\tau_{max} = \frac{V}{A_{web}} \qquad (6\text{-}15)$$

6-8-3 Distribution of shear stresses on circular sections

In circular sections(Fig.6-21) of a diameter d. The moment of area can be calculated by considering an element of area of breadth b and depth dy at a distance y from the neutral axis.

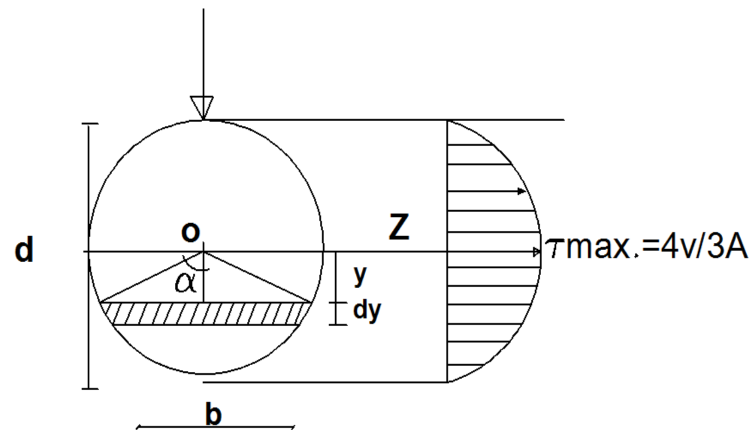


Fig. 6-21

From the previous Figure

$$b = 2 \cdot \frac{d}{2} \sin\alpha \ ,$$

$$= d \sin\alpha \quad \text{and}$$

$$y = \frac{d}{2} \cos\alpha$$

by the same meaning

$$dy = -\frac{d}{2} \sin\alpha \, d\alpha$$

$$dA = b.dy = -\frac{d^2}{2} \sin^2 \alpha \, d\alpha$$

The moment of area is $\int y dA$

$$= -\int_{\alpha}^{0} \frac{d^2}{4} \sin^2\alpha \cos\alpha \, d\alpha$$

$$= \frac{d^3}{12} . \sin^3\alpha$$

$$I = \frac{\pi d^4}{64} \quad \text{(here d is the diameter)}$$

The shear stress distribution can be obtained by

$$\tau = \frac{V \frac{d^3}{12} \sin\alpha}{d \sin\alpha \frac{\pi d^4}{64}} = \frac{16V \sin^2\alpha}{3\pi d^2}$$

for $\alpha = \frac{\pi}{2}$ (at the neutral axis) the shear stress has a maximum value

$$\tau_{max} = \frac{16V}{3\pi d^2} = \frac{4}{3} \frac{V}{\frac{\pi d^2}{4}}$$

Therefore,

$$\tau_{max} = \frac{4}{3} \frac{V}{A} \qquad (6\text{-}16)$$

Examples

1. Find the maximum shear stresses of simply supported beam (Fig. 6- 22) under a uniformly distributed load w= 2.5kN/m . Find the maximum shear stresses.

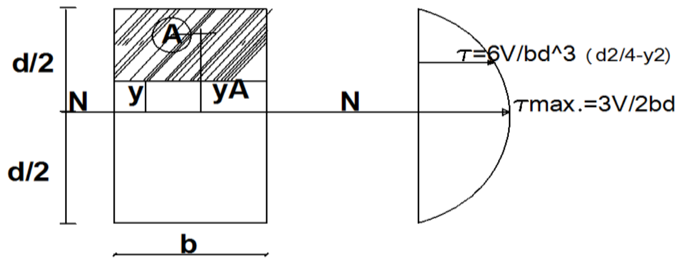


Fig. 6-22

Solution : the total load intensity is given by w = 2.5kN/m , then

$$V = \frac{wl}{2} = \frac{2.5x6}{2} = 7.5 \text{ kN}$$

From equation (6- 14)

$$\tau_{max} = \frac{3V}{2\,bd} \qquad \text{(for rectangular cross-section)}$$

$$= \frac{3}{2}\;\frac{7.5}{0.1x0.3}$$

$$= 0.375 \text{ MN/m}^2$$

This compares with a maximum bending stress of 7.5MN/m² which is twenty times greater.

2 . A beam of rectangular cross section is simply supported at the ends and subjected to the single concentrated forces shown in Fig. 6-23a . Determine the maximum shearing stress in the beam . Also, determine the shearing stress at a point on 3 cm below the top of the beam at a section 30 cm to the right of the left reaction.

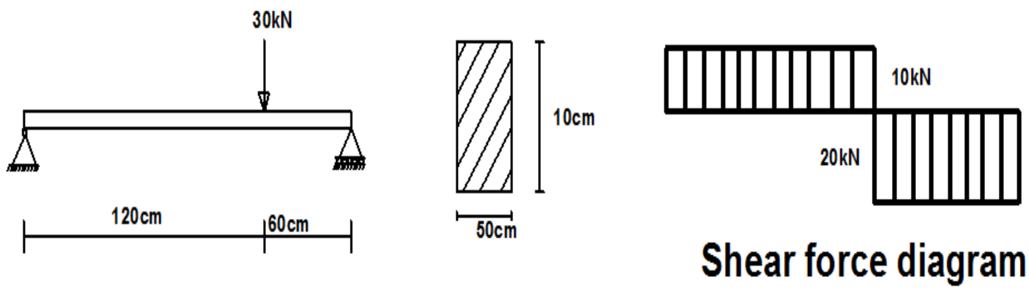


Fig. 6-23

Solution from statics the reactions are 10kN and 20kN as shown in Fig.6-23a. The shearing force diagram is Fig. 6-23b and the maximum value of shearing force is 20kN at the right of load 30kN.

$$\tau_{average} = \frac{20}{5x10} = 0.4 \text{ kN/ cm}^2$$

$$= 400 \text{N/cm}^2$$

The maximum shearing stress at the centroidal axis is 50% greater than that obtained by dividing the shearing force by the cross-sectional area

$$\tau = \frac{3}{2}\;\frac{V}{bd}$$

$= \frac{3}{2}$ x 400 = 600 N/ cm²

For y = 3 cm below the top fiber of the beam

$$\tau = \frac{6V}{bd^3}\left(\frac{d^2}{4} - y^2\right)$$

$$\tau = \frac{6x10}{5x10^3}\left(\frac{10^2}{4} - (3)^2\right)$$

= 0.192 kN/cm²

= 192 N/cm²

Exercises

6-11. A simply supported beam has I section of depth d=252 mm, breadth b= 203, flange thickness t_f =13.5 mm and web thickness t_w = 8.0 mm of a length 8 mm is subjected to the load system shown in Fig. 6-24 below. Find the maximum bending and shear stresses in the beam, together with the distribution of stresses over its cross section.

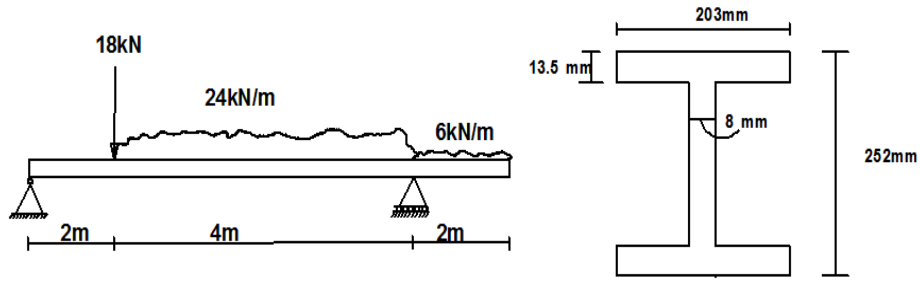


Fig. 6-24

6-12. A simply supported timber beam of modulus of elasticity 10 GPa and cross section as shown in Fig. 6-25 below. Find the maximum reading of strain gage of fiber a level 300 mm from the neutral axis also find the maximum bending stress. The beam has a circular hole in the center with a diameter of 250 mm.

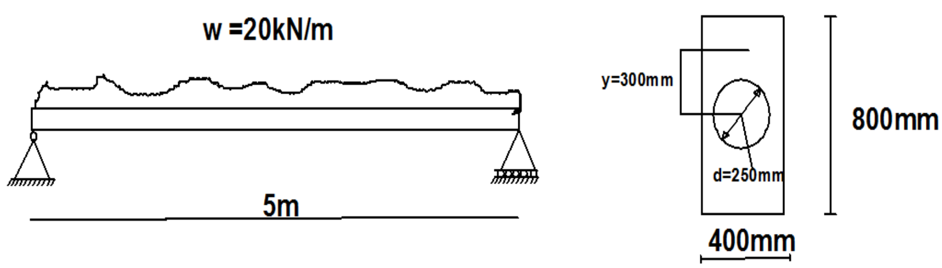


Fig. 6-25