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BME 322
Signals and Systems for BME

- 7 -

- Infinite Impulse Response -

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- IIR filters are recursive filters.
- Difference equation for IIR filters

$$y[n] = -\sum_{k=0}^{N} a_k y[n-k] + \sum_{k=0}^{M} b_k x[n-k]$$

a_k and b_k are the filter coefficients

Infinite Impulse Response (IIR) Filters

- IIR filters outputs depends on N past outputs and M past inputs.

- The impulse response samples getting smaller and smaller but they never settle to zero.

Example 1

- Determine the first six samples in the impulse response for the FIR filter.

$$y[n] - 0.4y[n-1] = x[n] - x[n-1]$$

Example 1 (solution)

- Substituting δ[n] for $x\ [n]$ and h[n] for $y\ [n]$.

$$h\ [n] - 0.4h\ [n-1] = \delta\ [n] - \delta\ [n-1]$$

$$h\ [n] = 0.4h\ [n-1\ ] + \delta\ [n] - \delta\ [n-1]$$

$$h\ [0] = 0.4h\ [-1] + \delta\ [0] - \delta\ [n-1]$$

$$= 0.4\ \ (0.0) + 1.0 - 0.0 = 1.0$$

Example 1 (solution)

$$h[1] = 0.4h[0] + \delta[1] - \delta[0]$$
$$= 0.4(1.0) + 0.0 - 1.0 = -0.6$$

$$h[2] = 0.4h[1] + \delta[2] - \delta[1]$$
$$= 0.4(-0.6) + 0.0 - 0.0 = -0.24$$

$$h[3] = 0.4h[2] + \delta[3] - \delta[2]$$
$$= 0.4(-0.24) + 0.0 - 0.0 = -0.096$$

Example 1 (solution)

$$h[4] = 0.4h[3] + \delta[4] - \delta[3]$$
$$= 0.4(-0.96) + 0.0 - 0.0 = -0.0384$$

$$h[5] = 0.4h[4] + \delta[5] - \delta[4]$$
$$= 0.4(-0.0384) + 0.0 - 0.0 = -0.01536$$

Direct-form IIR structures

- M^{th} order IIR filters are characterized by 2N + 1 coefficients and, require 2N + 1 multipliers and 2N two-input adders.

- For IIR filters in which the multiplier coefficients are precisely the coefficients of the transfers function are called direct-form structures.

Direct-form-I IIR structures

- Consider the transfer function for N-th order IIR filter:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{P_0 + P_1 z^{-1} + \cdots + P_N z^{-N}}{1 + d_1 z^{-1} + \cdots + d_N z^{-N}}$$

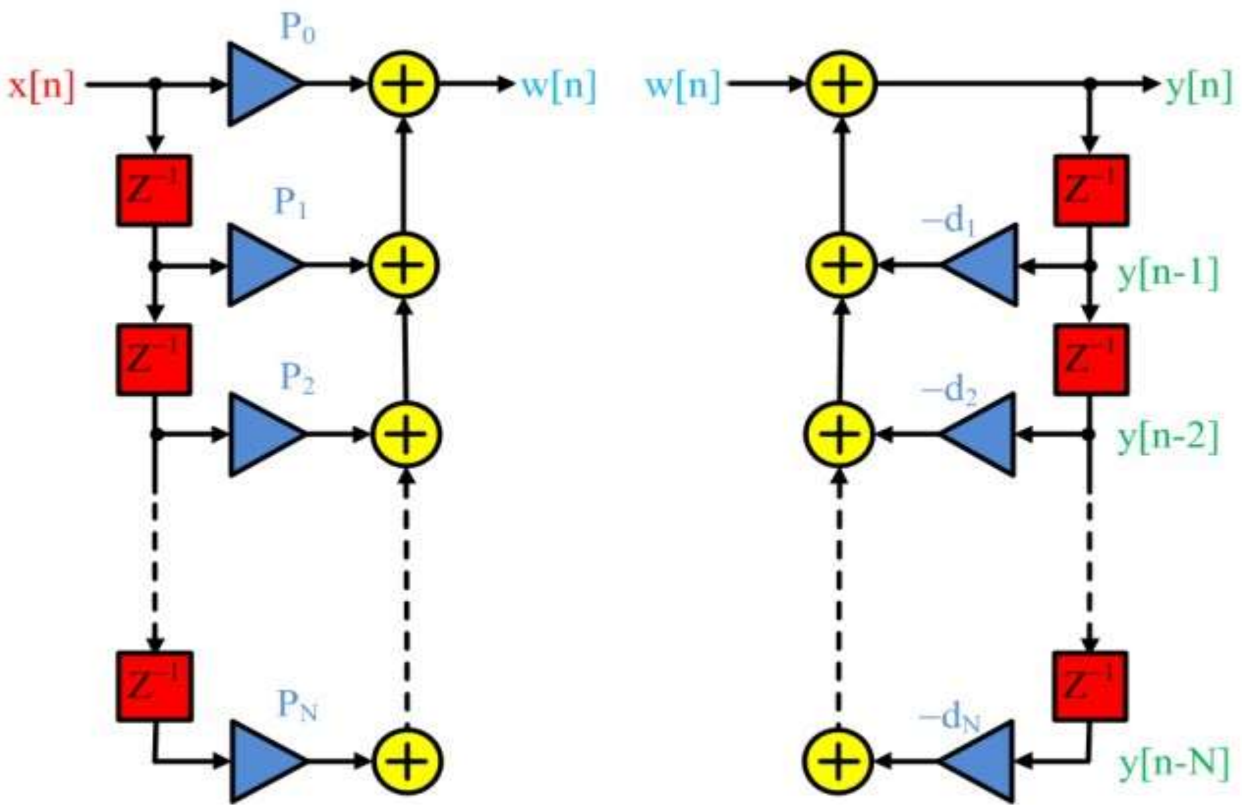
$$H_1(z) = \frac{W(z)}{X(z)}$$

$$= P_Z = P_0 + P_1 z^{-1} + \cdots + P_N z^{-N}$$

Direct-form-I IIR structures

$$H_2(z) = \frac{Y(z)}{W(z)} = \frac{1}{D(z)}$$

$$= \frac{1}{1 + d_1 z^{-1} + \cdots + d_N z^{-N}}$$



Example 2

Realize the infinite impulse response (IIR) filter using the direct form-I from the transfer function:

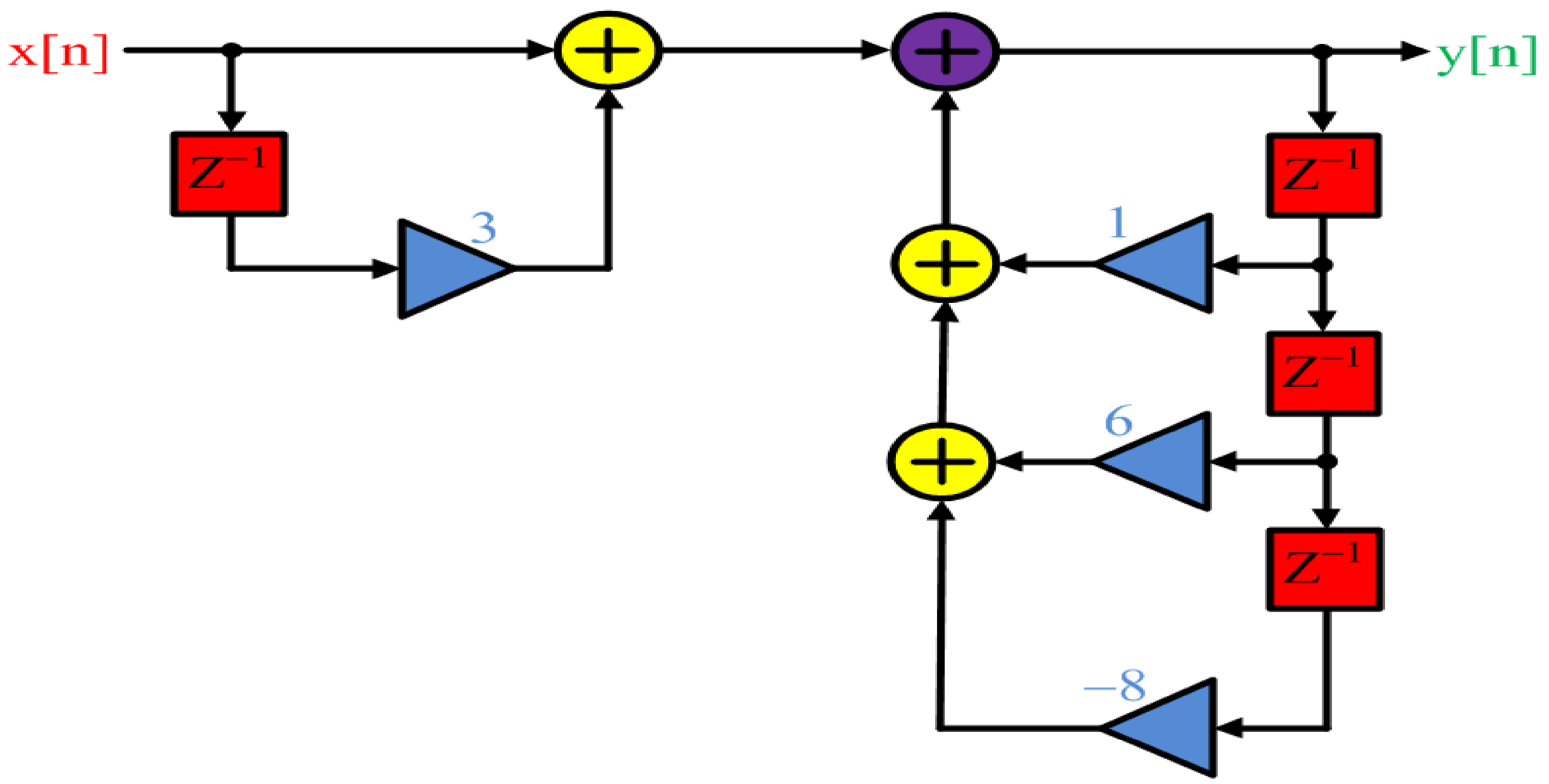
$$H(z) = \frac{1 + 3z^{-1}}{(1 - 2z^{-1})(1 + z^{-1} - 4z^{-2})}$$

Example 2 (solution)

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 3z^{-1}}{1 - z^{-1} - 6z^{-2} + 8z^{-3}}$$

$$Y(z) = z^{-1}Y(z) + 6z^{-2}Y(z) - 8z^{-3}\,Y(z) +$$
$$X(z) + 3z^{-1}\,X(z)$$

Example 2 (solution)



Example 3

Realize the infinite impulse response (IIR) filter using the direct form-II from the transfer function:

$$y(n) + ay(n-1) = bx(n) + cx(n-1)$$

Direct-form-II IIR structures

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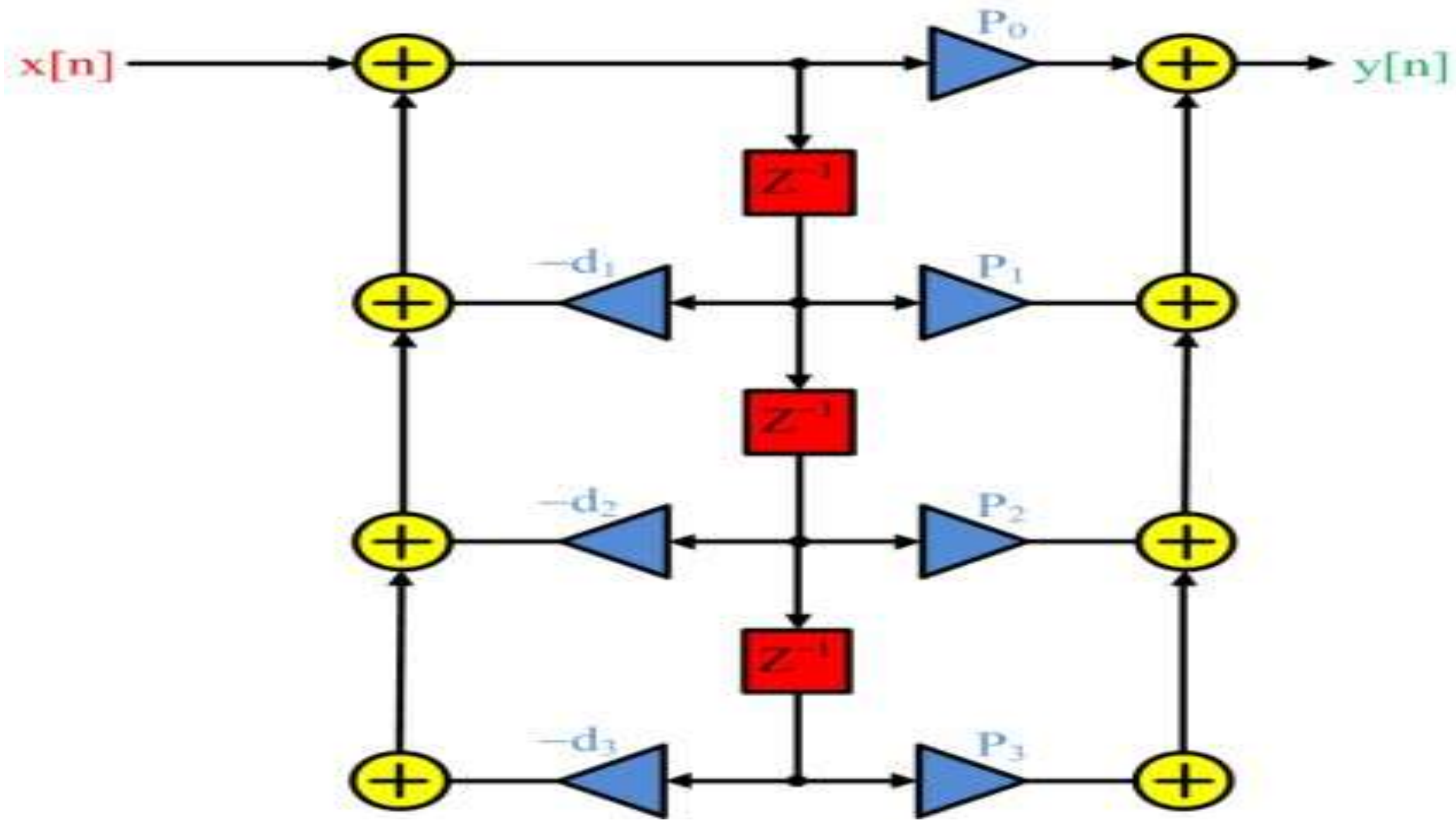

Example 3 (solution)

Rearranging the difference equation:

$$y(n) = -ay(n-1) + bx(n) + cx(n-1)$$