كلية المستقبل الجامعة
قسم هندسة تقنيات البناء والانشاءات

Composite beams

المرحلة الثانية

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Chapter 6 Stresses in Beam (Advanced Topics)

6.1 Introduction

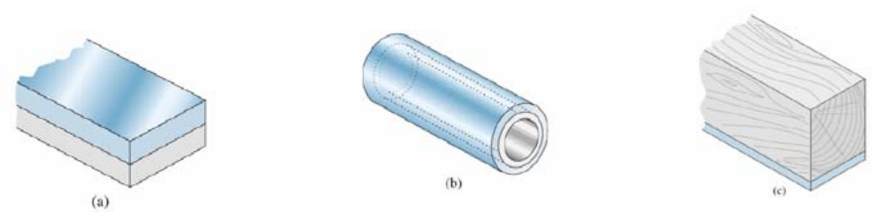
in this chapter we continue the study of the bending of beam for several specialized topics

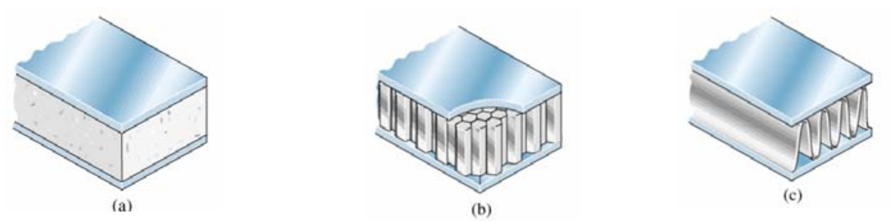
composite beams, elastoplastic bending, nonlinear bending

beams with inclined loads, unsymmetric beams, shear stress in thin-walled beams, shear center (these topics will discuss in Machines of Materials II)

6-2 Composite Beams

beams are built of more than one material, e.g. bimetallic beam, plastic coated steel pipes, wood beam reinforced with a steel plate, sandwich beam, reinforced concrete beam etc.





composite beam can be analyzed by the same bending theory

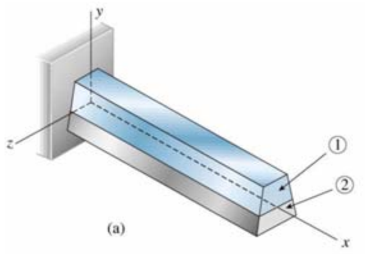
ε_x vary linearly from top to bottom, but the position of the N. A. is not at the centroid of the cross sectional area

$$\varepsilon_x = -\frac{y}{\rho} = -\kappa y$$

normal stress σ_x can be obtained from ε_x, assume that the materials behave in a linear elastic manner

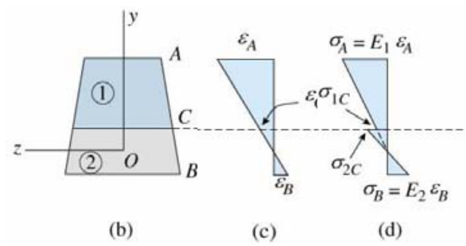
$$\sigma_x = E\,\varepsilon_x$$

denoting E_1 and E_2 are the moduli of elasticity for materials 1 and 2, and assume $E_2 > E_1$, then



$$\sigma_{x1} = -E_1\,\kappa\,y \qquad \sigma_{x2} = -E_2\,\kappa\,y$$

$\Sigma F_x = 0$ force equilibrium in x-axis



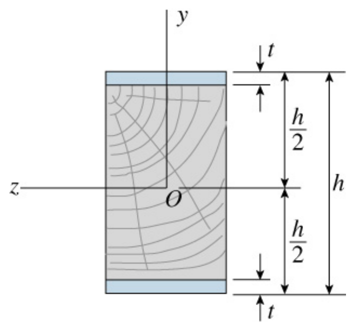
$$\int_1 \sigma_{x1}\,dA + \int_2 \sigma_{x2}\,dA = 0$$

$$-\int_1 E_1\,\kappa\,y\,dA + \int_2 E_2\,\kappa\,y\,dA = 0$$

$$E_1 \int_1 y\,dA + E_2 \int_2 y\,dA = 0$$

this equation can be used to located the N. A. of the cross section for beam of two materials, the integrals represent the 1st moment of two parts w. r. t. the N. A.

if the cross section of a beam is doubly symmetric, the N. A. is located at the midheight of the cross section



moment equilibrium

$$M = -\int_A \sigma_x\,y\,dA = -\int_1 \sigma_{x1}\,y\,dA + -\int_2 \sigma_{x2}\,y\,dA$$

$$= \kappa\,E_1 \int_1 y^2\,dA + \kappa\,E_2 \int_2 y^2\,dA$$

$$= \kappa\,(E_1 I_1 + E_2 I_2)$$

then

$$\kappa = \frac{1}{\rho} = \frac{M}{E_1 I_1 + E_2 I_2}$$

where I_1 and I_2 are moments of inertia about the N. A. of the area of materials 1 and 2, respectively (note that $I = I_1 + I_2$)

this equation is known as moment-curvature relationship

$E_1 I_1 + E_2 I_2$ is the flexural rigidity of the composite beam

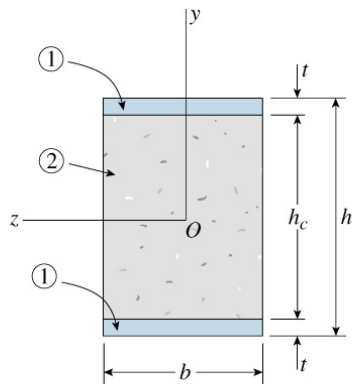
the normal stresses in the beam are obtained

$$\sigma_{x1} = - \frac{M y E_1}{E_1 I_1 + E_2 I_2} \qquad \sigma_{x2} = - \frac{M y E_2}{E_1 I_1 + E_2 I_2}$$

for $E_1 = E_2$, the above equation reduces to the flexural formula

Approximate Theory for bending of Sandwich Beams

consider a doubly symmetric sandwich beam, if the material of the faces has a much larger modulus than the material of the core



assume the modulus of elasticity E_2 of the core is zero, then

$$\sigma_{x1} = - \frac{M y}{I_1} \qquad \sigma_{x2} = 0$$

Where

$$I_1 = \frac{b}{12}(h^3 - h_c^3) \qquad h_c = h - 2t$$

the maximum normal stresses in the sandwich beam occur at the top and bottom

$$\sigma_{top} = - \frac{M h}{2 I_1} \qquad \sigma_{bottom} = \frac{M h}{2 I_1}$$

if the faces are thin compared to the thickness of the core ($t \ll h_c$), we assume that the core carries all of the shear stresses

$$\tau_{aver} = \frac{V}{b\,h_c} \qquad \gamma_{aver} = \frac{V}{b\,h_c\,G_c}$$

Limitations

both materials obey Hooke's law

materials are isotropic and homogeneous

for nonhomogeneous and nonlinear material, the above equation can not be applied, e.g. reinforced concrete beams are one of the most complex types of composite construction

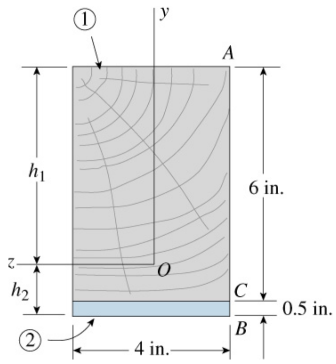
Example 6-1

a composite beam is constructed from wood beam and steel plate

$M = 6$ kN-m $\qquad E_1 = 10.5$ GPa

$E_2 = 210$ GPa

calculate σ_{max} and σ_{min} in wood and steel



firstly, we want to determine the N. A.

$$\int_1 y\,dA = y_1 A_1 = (h_1 - 75)\,(100 \times 150) = (h_1 - 75) \times 15{,}000$$

$$\int_2 y\,dA = y_2 A_2 = -\,(156 - h_1)\,(100 \times 012) = (h_1 - 156) \times 1{,}200$$

$$E_1 \int_1 y\,dA + E_2 \int_2 y\,dA = 0$$

$$10.5\,(h_1 - 75) \times 15{,}000 + 210\,(h_1 - 156) \times 1{,}200 = 0$$

$$h_1 = 124.8 \text{ mm}$$

$h_2 = 162 - h_1 = 37.2$ mm

moment of inertia

$$I_1 = \frac{1}{12} 100 \text{ x } 150^3 + 100 \text{ x } 150 (h_1 - 75)^2 = 65.33 \text{ x } 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} 100 \text{ x } 12^3 + 100 \text{ x } 12 (h_2 - 6)^2 = 1.18 \text{ x } 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 66.51 \text{ x } 10^6 \text{ mm}^4$$

maximum compressive stress in wood ($y = h_1 = 124.8$ mm)

$$\sigma_{1A} = - \frac{M h_1 E_1}{E_1 I_1 + E_2 I_2} = - \frac{(6 \text{ kN-m}) (124.8 \text{ mm}) (10.5\text{GPa})}{(10.5\text{GPa}) (65.33\text{x}10^6\text{mm}^4) + (210\text{GPa}) (1.18\text{x } 10^6\text{mm}^4)}$$

$= - 8.42$ MPa

maximum tensile stress in wood [$y = - (h_2 - 0.5) = - 25.2$ mm)

$$\sigma_{1C} = - \frac{M h_2 E_1}{E_1 I_1 + E_2 I_2} = - \frac{(6 \text{ kN-m}) (-25.2 \text{ mm}) (10.5\text{GPa})}{(10.5\text{GPa}) (65.33\text{x}10^6\text{mm}^4) + (210\text{GPa}) (1.18\text{x } 10^6\text{mm}^4)}$$

$= 1.7$ MPa

minimum tensile stress in steel ($y = - 25.2$ mm)

$$\sigma_{2C} = - \frac{M y E_2}{E_1 I_1 + E_2 I_2} = - \frac{(6 \text{ kN-m}) (-25.2 \text{ mm}) (210\text{GPa})}{(10.5\text{GPa}) (65.33\text{x}10^6\text{mm}^4) + (210\text{GPa}) (1.18\text{x } 10^6\text{mm}^4)}$$

$= 34$ MPa

maximum tensile stress in steel ($y = - h_2 = - 37.2$ mm)

$$\sigma_{2B} = - \frac{M y E}{E_1 I_1 + E_2 I_2} = - \frac{(6 \text{ kN-m}) (-37.2 \text{ mm}) (10.5\text{GPa})}{(10.5\text{GPa}) (65.33\text{x}10^6\text{mm}^4) + (210\text{GPa}) (1.18\text{x } 10^6\text{mm}^4)}$$

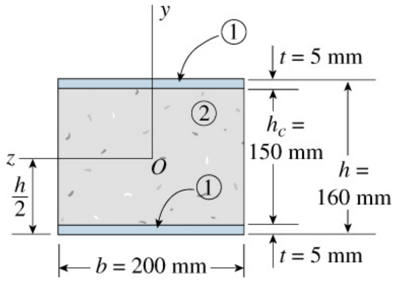
$= 50.2$MPa

note that

$$\frac{\sigma_{2C}}{\sigma_{1C}} = \frac{E_2}{E_1} = \frac{34}{17} = 20$$

Example 6-2

a sandwich beam having aluminum-alloy faces with plastic core, $M = 3$ kN-m



$E_1 = 72$ GPa $\quad E_2 = 800$ GPa

$t = 5$ mm $\quad h_c = 150$ mm

$h = 160$ mm $\quad b = 200$ mm

determine σ_{max} and σ_{min} in the faces and core

(a) using general theory

(b) using approximate theory

∵ the section is double symmetric, ∴ N. A. is located at midheight

$$I_1 = \frac{b}{12}(h^3 - h_c^3) = \frac{200}{12}(160^3 - 150^3) = 12.017 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{b}{12}h_c^3 = \frac{200}{12}150^3 = 56.25 \times 10^6 \text{ mm}^4$$

the flexural rigidity of the composite beam is

$$E_1 I_1 + E_2 I_2 = 72 \times 10^3 \times 12.017 \times 10^6 + 800 \times 56.25 \times 10^6$$

$$= 910{,}224 \times 10^6 \text{ N-mm}^2$$

$$= 910{,}224 \text{ N-m}^2$$

the σ_{max} for tension and compression in aluminum faces are

$$\sigma_{max} = \pm\frac{M(h/2)E_1}{E_1 I_1 + E_2 I_2} = \pm\frac{3 \times 10^6 \times 80 \times 72 \times 10^3}{910{,}224 \times 10^6} = \pm 19.0 \text{ MPa}$$

the σ_{max} for tension and compression in plastic core are

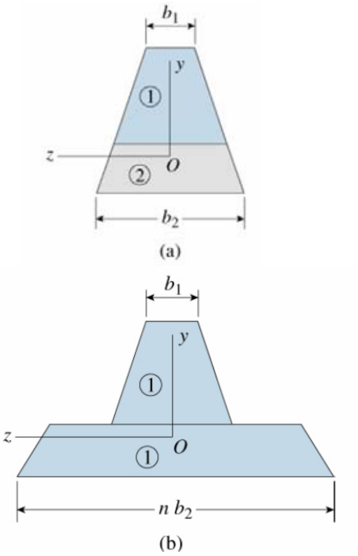
$$\sigma_{max} = \pm \frac{M\,(h_c/2)\,E_2}{E_1\,I_1 + E_2\,I_2} = \pm \frac{3 \text{ x } 10^6 \text{ x } 75 \text{ x } 800}{910{,}224 \text{ x } 10^6} = \pm\ 0.198 \text{ MPa}$$

from the approximate theory for sandwich beam

$$\sigma_{max} = \pm \frac{M\,y}{I_1} = \pm \frac{M\,(h/2)}{I_1} = \pm \frac{3 \text{ x } 10^6 \text{ x } 80}{12.017 \text{ x } 10^6} = \pm\ 20.0 \text{ MPa}$$

this theory is conservative and gives slightly higher stresses

6.3 Transformed-Section Method



consider a composite beam, the N. A. of the cross section can be determined by the equation of equilibrium as state before

$$E_1 \int_1 y\,dA \quad + \quad E_2 \int_2 y\,dA \quad = \quad 0$$

denote n the modular ratio as

$$n \quad = \quad E_2 \,/\, E_1$$

then the equilibrium equation can be written

$$\int_1 y\,dA \quad + \quad \int_2 n\,y\,dA \quad = \quad 0$$

if each element of dA in material ***2*** x n, a new cross section is shown, the N. A. of the new area is the same of the composite beam

i.e. the new cross section consisting only one material, material ***1***, this section is called the transformed section

∵ only one material is considered, the based equation can be used

$$\sigma_x \quad = \quad -\,E_1\,\kappa\,y$$

and the moment-curvature relation for the transformed beam is

$$M = - \int_A \sigma_x\, y\, dA = - \int_1 \sigma_x\, y\, dA + - \int_2 \sigma_x\, y\, dA$$

$$= \kappa E_1 \int_1 y^2\, dA + \kappa E_1 \int_2 y^2\, dA$$ [dA in material ***2*** is equal $n\, dA$ of the original area]

$$= \kappa (E_1 I_1 + n E_1 I_2)$$

$$= \kappa (E_1 I_1 + E_2 I_2)$$

same result as before

for the transformed section, the bending stress is

$$\sigma_x = - \frac{M\, y}{I_T} \qquad I_T = I_1 + n I_2 = I_1 + \frac{E_2}{E_1} I_2$$

$$\sigma_{x1} = - \frac{M\, y\, E_1}{E_1 I_1 + E_2 I_2}$$ same as before

stress in material ***1*** can be calculated direct from the above equation, but in material ***2*** the stress in transformed section are not the same as in the original beam, it must be multiplied by n to obtain the stress in the transformed section, i.e.

$$\sigma_{x2} = - n \frac{M\, y}{I_T} = - \frac{M\, y\, n\, E_1}{E_1 I_1 + E_2 I_2} = - \frac{M\, y\, E_2}{E_1 I_1 + E_2 I_2}$$

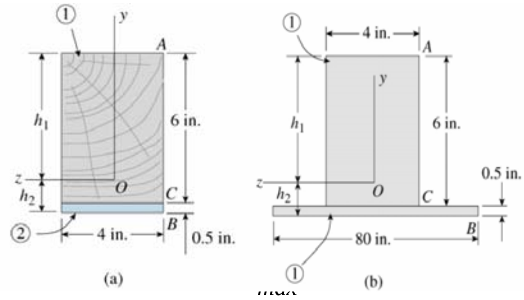
the result is same as before

it is possible to transform the original beam to a beam consisting material ***2***, use $n = E_1 / E_2$

the transformed-section method may be extended to composite beam of more than 2 materials

Example 6-3

a composite beam is formed of wood and steel plate as shown



$M = 6$ kN-m $\quad E_1 = 10.5$ N

using the transformed-section meth
and compression in wood, and σ_{max} and σ_{min} for tension in steel

[same problem as in example 6-1]

$$n = E_2 / E_1 = 30{,}000 / 1{,}500 = 20$$

the N. A. of the transformed section can be calculated

$$h_1 = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{75 \text{ x } 100 \text{ x } 150 + 156 \text{ x } 2{,}000 \text{ x } 12}{100 \text{ x } 150 + 2{,}000 \text{ x } 12} = \frac{4{,}869 \text{ x } 10^3}{39 \text{ x } 10^3}$$

$$= 124.8 \text{ mm}$$

$$h_2 = 162 - h_1 = 37.2 \text{ mm}$$

the moment of inertia of the transformed section is

$$I_T = \frac{1}{12} 100 \text{ x } 150^3 + 100 \text{ x } 150 \text{ x } (h_1 - 75)^2 + \frac{1}{12} 2{,}000 \text{ x} 12^3$$

$$+ 2{,}000 \text{ x } 12 \text{ x } (h_2 - 6)^2$$

$$= 65.3 \text{ x } 10^6 + 23.7 \text{ x } 10^6 = 89.0 \text{ x } 10^6 \text{ mm}^4$$

bending stresses in the wood (material ***1***)

$$\sigma_{1A} = -\frac{M\,y}{I_T} = -\frac{(6 \text{ x } 10^6 \text{ N-mm})(124.8 \text{ mm})}{89.0 \text{ x } 10^6 \text{ mm}^4} = -8.42 \text{ MPa}$$

$$\sigma_{1C} = -\frac{M\,y}{I_T} = -\frac{(6 \text{ x } 10^6 \text{ N-mm})(-25.2 \text{ mm})}{89.0 \text{ x } 10^6 \text{ mm}^4} = 1.7 \text{ MPa}$$

the bending stresses in steel (material ***2***)

$$\sigma_{2C} = -n\frac{M\,y}{I_T} = -20\frac{6 \text{ x } 10^6 \text{ x } (-37.2)}{89.0 \text{ x } 10^6} = 50.2 \text{ MPa}$$

$$\sigma_{2B} = -n\frac{M\,y}{I_T} = -20\frac{6 \text{ x } 10^6 \text{ x } (-25.2)}{89.0 \text{ x } 10^6} = 34 \text{ MPa}$$

the results are the same as in example 6-1

6-4 Doubly Symmetric Beams with Inclined Loads

6-5 Bending of Unsymmetric Beams

6-6 The Shear Center Concept

6-7 Shear Stresses in Beams of Thin-Walled Open Cross Section

6-8 Shear Centers of Thin-Walled Open Sections

6-9 Elastoplastic Bending

6.10 Nonlinear Bending