

## Theory of structure

### Stability and determinacy of structures

#### Beams

- ❖ Total equation of equilibrium of beam

$$\sum F_X = 0$$

$$\sum F_Y = 0$$

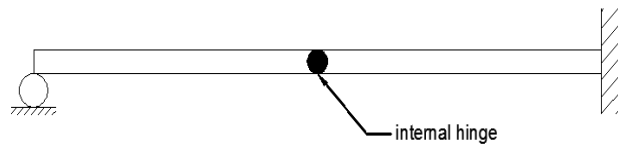
$$\sum M = 0$$

- ❖ Equation of condition

Internal hinge:-

$$\sum M = 0$$

$$C=1$$

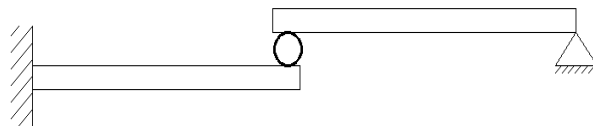


Roller:-

$$\sum M = 0$$

$$\sum F_X = 0$$

$$C=2$$



Let  $r$  = No. of reaction

1- If  $r < c+3$ , unstable

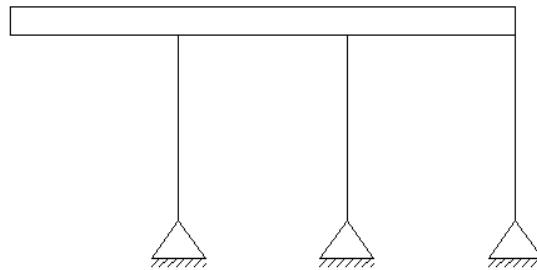
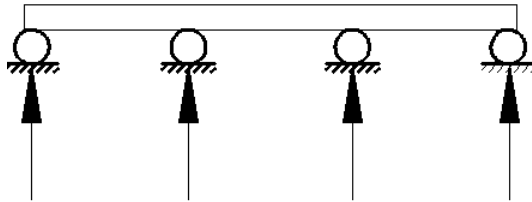
2-  $r = c+3$ , determine if stable

3-  $r > c+3$ , indeterminate if stable

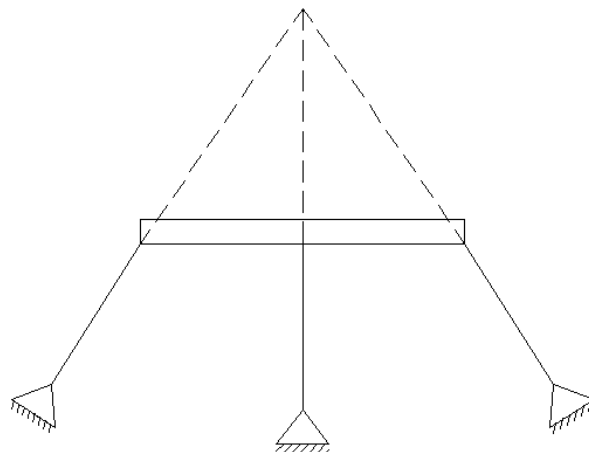
Let  $(m)$  degree of indeterminate

$$m = r - (c+3)$$

- ❖ the structure is said to be unstable if one of the following facts counter
- 1-  $r < c+3$
  - 2- The reaction element constitutes a parallel force system.



- 3- The reaction element constitutes a concurrent force system.



4- Internal geometric instability:-

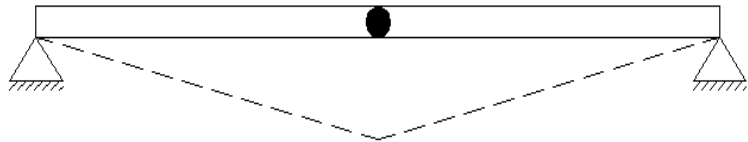
Example

$$r = 4$$

$$c = 1$$

$$r = c + 3$$

$$4 = 4$$



The beam is unstable because the Internal geometric instability

Example

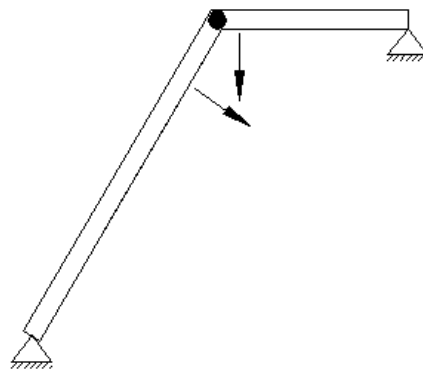
$$r = 4$$

$$c = m - 1, \quad c = 1$$

$$r = c + 3$$

$$4 = 4$$

The beam is unstable



Example

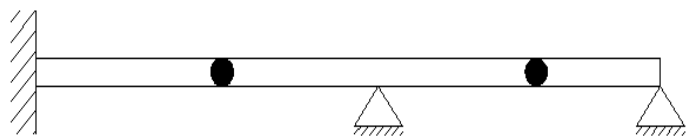
$$r = 7$$

$$c = 2$$

$$r > c + 3$$

$$7 > 5$$

The beam is indeterminate 2<sup>nd</sup> degree if stable



Example

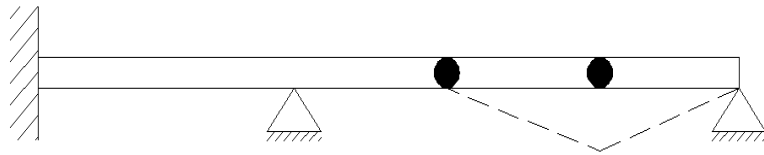
$r = 7$

$c = 2$

$r > c + 3$

$7 > 5$

The beam is unstable

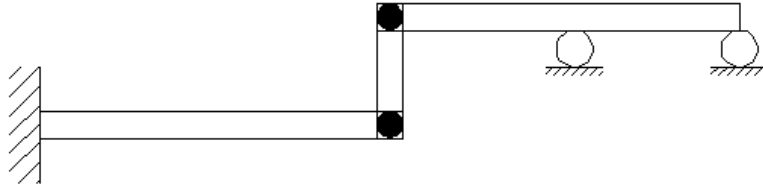


Examples:-

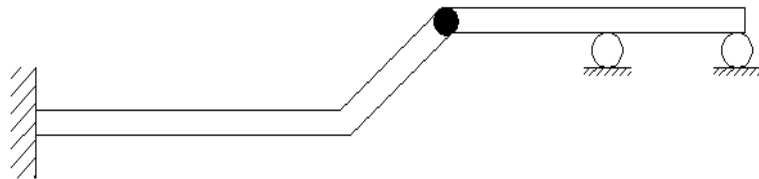
Beam	r	c	c+3	state	Stability & determinate.
	3	0	3	$r = c + 3$	Stable & deter.
	4	0	3	$r > c + 3$	Stable & indeter. First degree
	6	1	4	$r > c + 3$	Stable & indeter. Second degree
	6	2	5	$r > c + 3$	unstable
	3	0	3	$r = c + 3$	unstable

### Home Works

H.W1: Find the stability and determinacy of beam.



H.W2: Find the stability and determinacy of beam.



### Stability and Determinacy of Trusses

$b + r = \text{unknown}$

$j = \text{equations}$

- 1-  $b + r < 2j$ , the truss is unstable
- 2-  $b + r = 2j$ , the truss is determinate if stable
- 3-  $b + r > 2j$ , the truss is indeterminate if stable

Let (m) equal to the degree of indeterminate

$$m = (b + r) - 2j$$

$b = \text{No. of bars}$

$r = \text{No. of reactions}$

$j = \text{No. of joints}$

**Examples:** - Find the stability and determinacy of trusses below.

Ex1

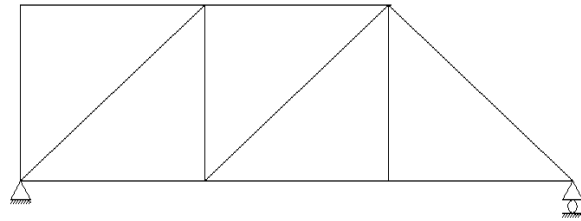
$$r = 3, b = 11, j = 7$$

$$b+r = 14$$

$$2j = 14$$

$$b+r = 2j$$

The truss is stable & determinate



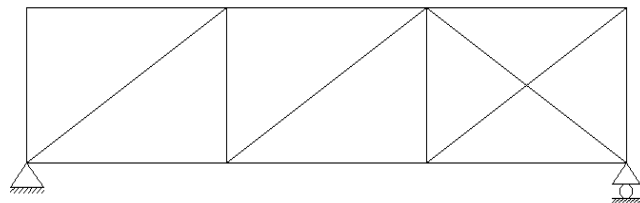
Ex2

$$r = 3, b = 14, j = 8$$

$$b+r = 17$$

$$2j = 16$$

$b+r > 2j$ , the truss is stable & indeterminate 1<sup>st</sup> degree



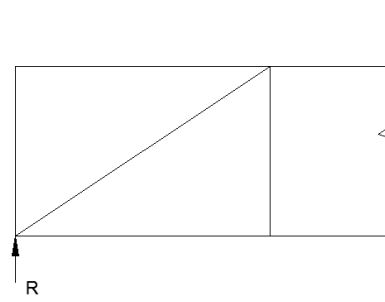
Ex3

$$r = 3$$

$$b = 13$$

$$2j = 16$$

$b+r = 2j$ , the truss is unstable because of  $\sum F_y \neq 0$ , in this section



Ex4

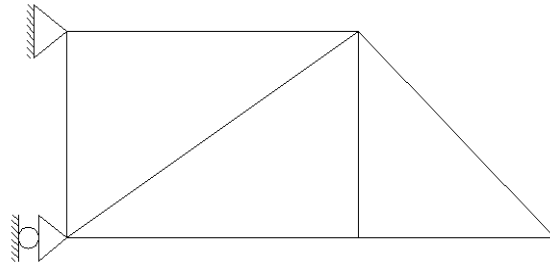
$$r = 3, b = 7, j = 5$$

$$b + r = 10$$

$$2j = 10$$

$$b + r = 2j$$

The truss is stable & determinate



Ex5

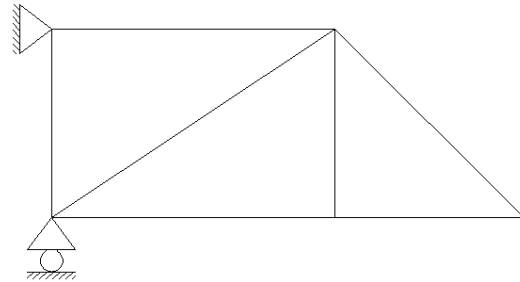
$$r = 3, b = 7, j = 5$$

$$b + r = 10$$

$$2j = 10$$

$$b + r = 2j$$

The truss is unstable



Ex6

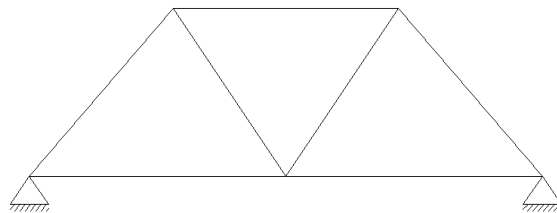
$$r = 4, b = 7, j = 5$$

$$b + r = 11$$

$$2j = 10$$

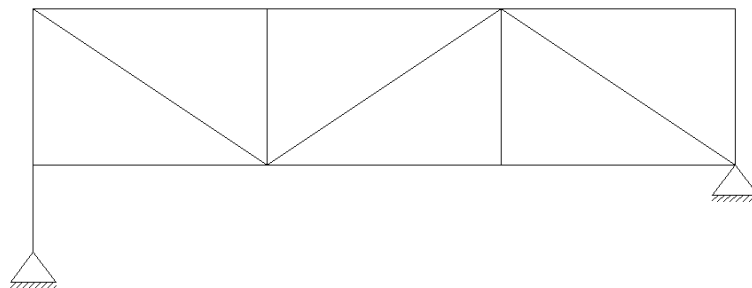
$$b + r > 2j$$

The truss is stable & indeterminate 1<sup>st</sup> degree

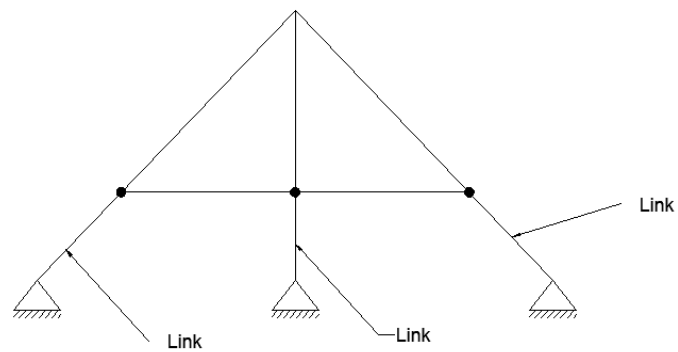


## Home works

H.W1



## H.W2



## Stability and Determinacy of Frames

### 1- Open frames

$r < C+3$ , unstable

$r = C+3$ , determinate if stable

$r > C+3$ , indeterminate if stable

Ex1:- Find the stability and determinacy of frame below

$$C_1 = m-1, C_1 = 2-1 = 1$$

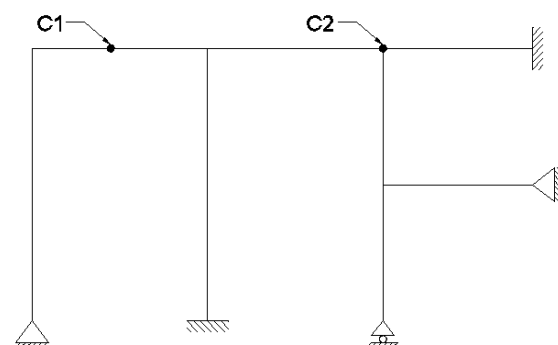
$$C_2 = m-1, C_2 = 3-1 = 2$$

$$C = C_1 + C_2, C = 3$$

$$r = 11$$

$$C+3 = 6$$

$r > C+3$ , the frame is stable & indeterminate 5<sup>th</sup> degree.



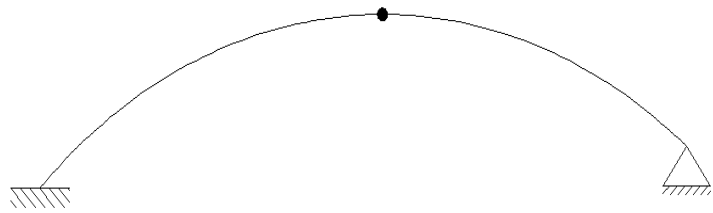


Ex2:-

$$r = 5$$

$$C = 2 - 1 = 1$$

$r > C + 3$ , the frame is stable & indeterminate 1<sup>st</sup> degree.



Ex3:-

$$C_1 = m - 1, C_1 = 4 - 1 = 3$$

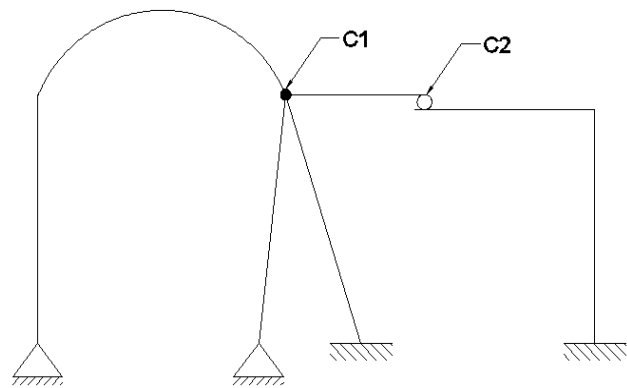
$$C_2 = 2$$

$$C = C_1 + C_2, C = 5$$

$$r = 10$$

$$C + 3 = 8$$

$r > C + 3$ , the frame is stable & indeterminate 2<sup>nd</sup> degree.

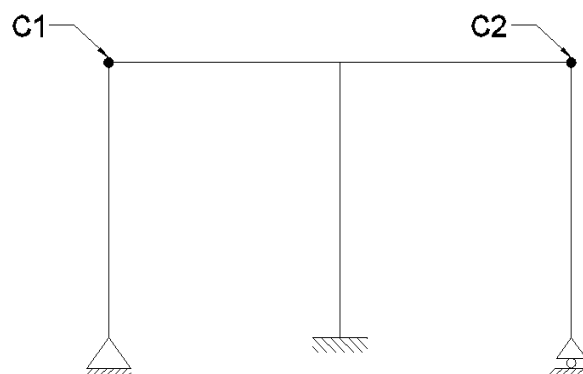


Ex4:-

$$r = 6$$

$$C = 2$$

$$r > C + 3$$



The frame is unstable because of internal geometric instability

## 2- Closed Frames:-

$$3b+r < 3j+c, \text{ unstable}$$

$$3b+r = 3j+c, \text{ determinate if stable}$$

$$3b+r > 3j+c, \text{ indeterminate if stable}$$

Where,

$$3b+r = \text{unknown}$$

$$3j+c = \text{equations}$$

$$b = \text{No. of members}$$

$$r = \text{No. of reactions}$$

$$j = \text{No. of joints}$$

Ex1:-

$$b = 10$$

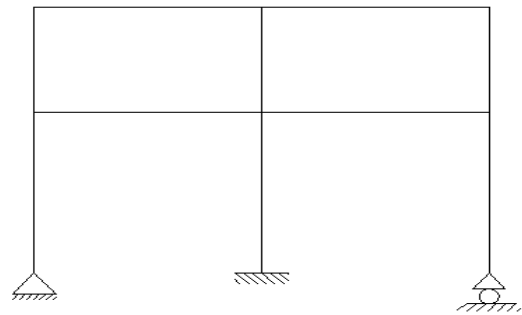
$$r = 9$$

$$j = 9$$

$$3b+r = 39$$

$$3j+c = 27$$

$$3b+r > 3j+c, \text{ stable \& indeterminate } 12^{\text{th}} \text{ degree}$$



Ex2:-

$$b = 10$$

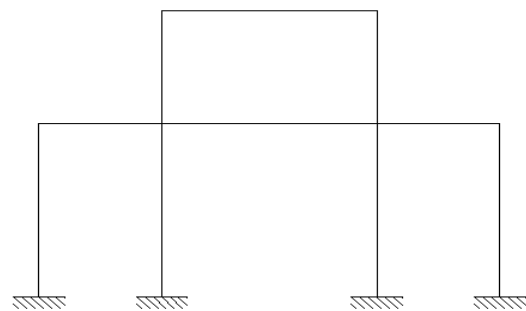
$$r = 12$$

$$j = 10$$

$$3b+r = 42$$

$$3j+c = 30$$

$$3b+r > 3j+c, \text{ stable \& indeterminate } 12^{\text{th}} \text{ degree}$$



Ex3:-

$$b = 4$$

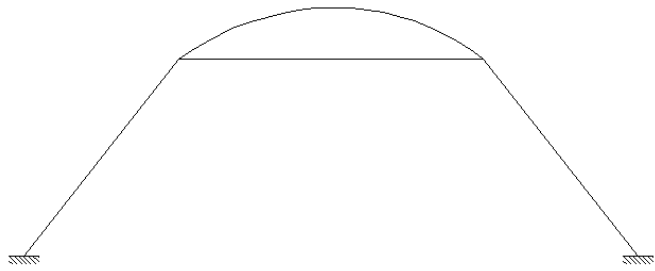
$$r = 6$$

$$j = 4$$

$$c = 0$$

$$3b+r = 18$$

$$3j+c = 12$$

$$3b+r > 3j+c, \text{ stable \& indeterminate } 6^{\text{th}} \text{ degree}$$


Ex4:-

$$b = 9$$

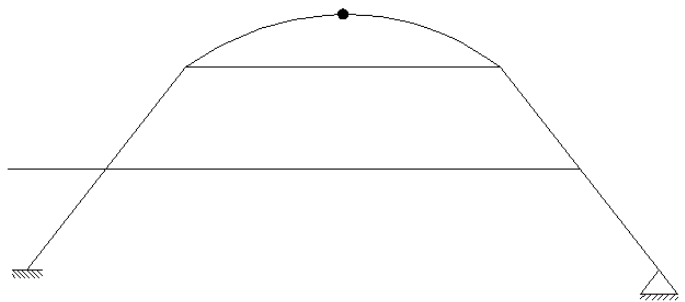
$$r = 5$$

$$j = 7$$

$$c = m-1 \Rightarrow c = 1$$

$$3b+r = 31$$

$$3j+c = 22$$

$$3b+r > 3j+c, \text{ stable \& indeterminate } 9^{\text{th}} \text{ degree}$$


Ex5:-

$$b = 10$$

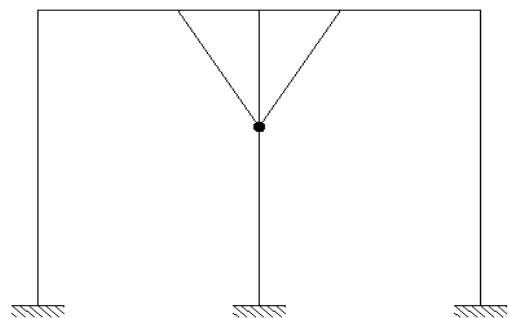
$$r = 9$$

$$j = 9$$

$$c = m-1 \Rightarrow c = 4-1 \Rightarrow c = 3$$

$$3b+r = 39$$

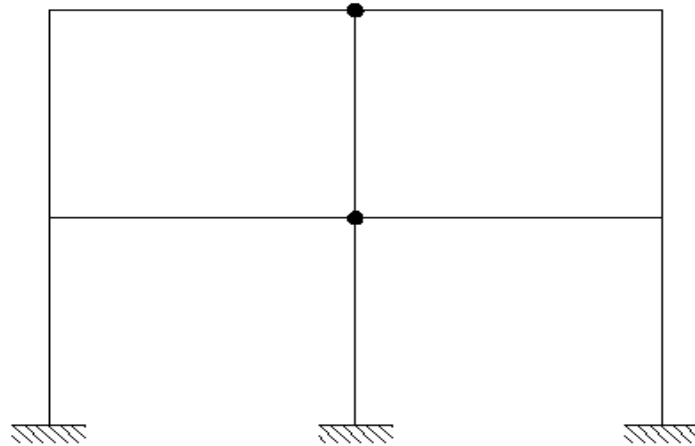
$$3j+c = 30$$

$$3b+r > 3j+c, \text{ stable \& indeterminate } 9^{\text{th}} \text{ degree}$$


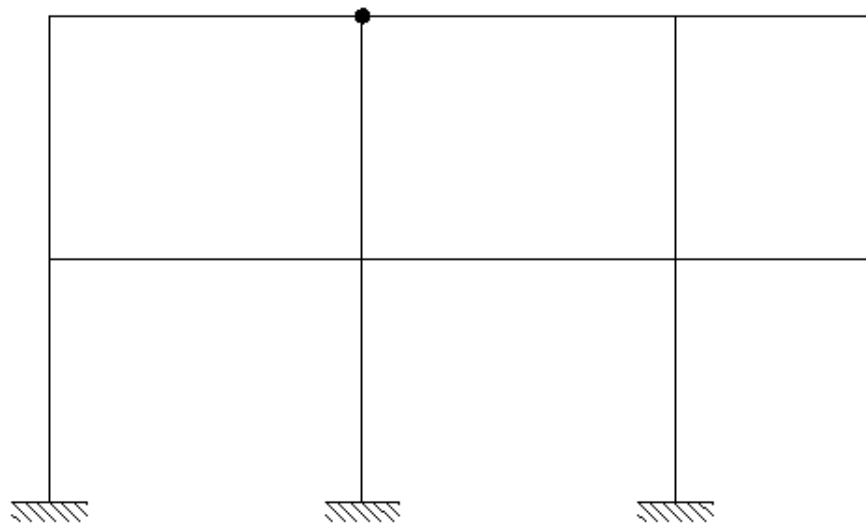
ملاحظة:- اذا جاء ال internal hinge في بداية او نهاية الضلع فيحسب منه (c & j) اما اذا جاء في داخل الضلع فيحسب منه c فقط

Home work:

H.W1: Find the stability and determinacy of frame below



H.W2: Find the stability and determinacy of frame below



## Stability and Determinacy of Composite Structure

Unknowns	Equations
1- Each truss member give one unknown	1- each member carry moment give ( 3 equations)
2- reactions	2- each joint connect truss members only give (2 equations)
3- each joint connect member carry moment give unknown in these equation (2*(m-1))	

Ex1:- Find the stability and determinacy of composite structure as shown below.

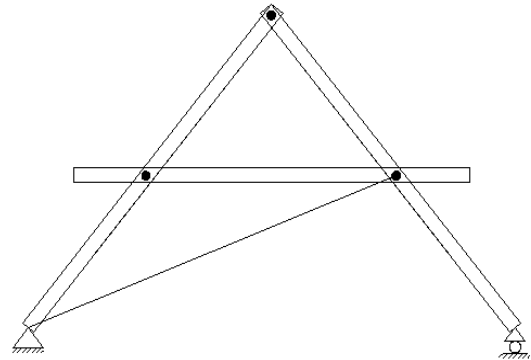
Solution:

Equations

$$(3*3) + 0 = 9$$

Unknowns

$$1 + 3 + (3*(2(2-1))) = 10$$



Unknowns > Equations, Stable & indeterminate 1<sup>st</sup> degree

Ex2:- Find the stability and determinacy of composite structure as shown below.

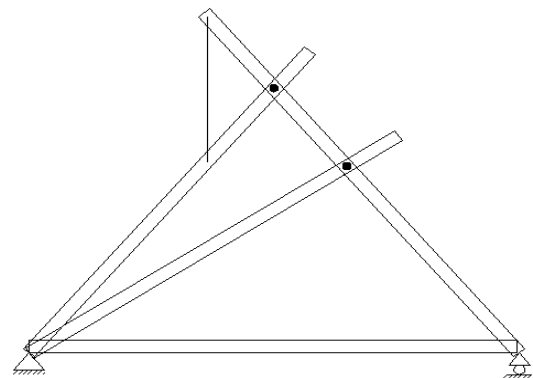
Solution:

Equations

$$(4*3) + 0 = 12$$

Unknowns

$$1 + 3 + (3*(2(2-1))) + (2(3-1)) = 14$$



Unknowns > Equations, Stable & indeterminate 2<sup>nd</sup> degree

Ex3:- Find the stability and determinacy of composite structure as shown below.

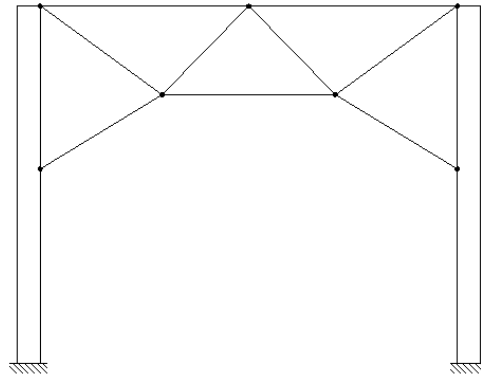
Solution:

Equations

$$(2*3) + (3*2) = 12$$

Unknowns

$$9+6+ 0 = 15$$



Unknowns > Equations, Stable & indeterminate 2<sup>nd</sup> degree

Ex4:- Find the stability and determinacy of composite structure as shown below.

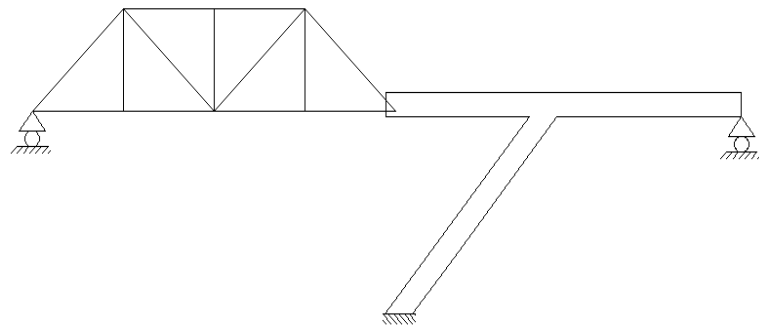
Solution:

Equations

$$(1*3) + (7*2) = 17$$

Unknowns

$$13+5+ 0 = 18$$



Unknowns > Equations, Stable & indeterminate 1<sup>st</sup> degree

H.w: Find the stability and determinacy of composite structure as shown below.

