Ministry of Higher Education and Scientific Research

Al-Mustaqbal University College

Department of Technical Computer Engineering

Electrical control fundamentals

3rd Stage

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Where,

$f_s(t)$ = Spring force required to deflect the spring a distance.

$x(t)$ = Deflection distance.

K = Spring Constant

Slope of the curve of the load (F_s) versus deflection (X)

$v(t)$ = velocity

The input to the spring is usually the force (F_s), and the output is the deflection $X(s)$. Therefore, the Block Diagram of it is

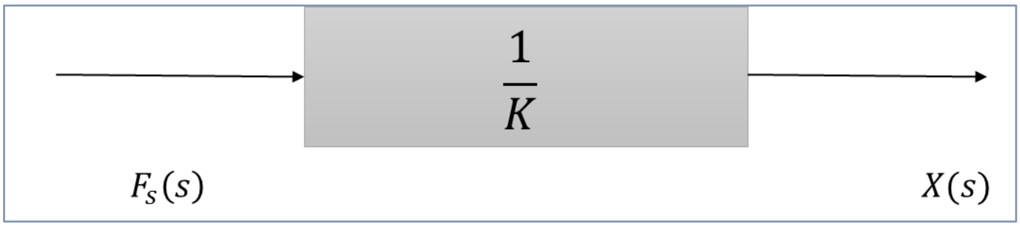

Figure 13. Block Diagram of Linear Spring.

b. Viscous Damper B ($\frac{N.S}{m}$)

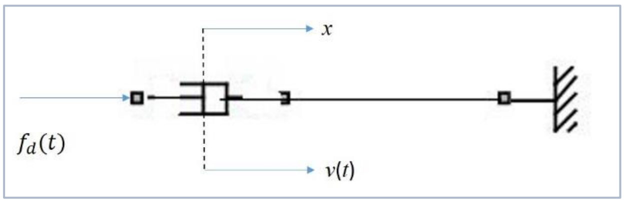

Figure 14. Viscous Damper.

$f_d(t) = Bv(t)$

$$= B\frac{dx}{dt}$$

Take the Laplace transform

$F_d(s) = BSX(s)$

Where,

B = Viscous Damper Coefficient

$v(t)$ = velocity

The input is the force (F_d), and the output is the displacement (X). The Block Diagram Representation is

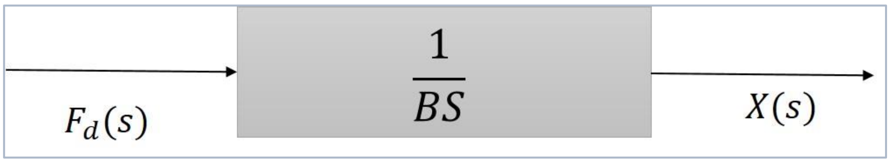

Figure 15. The Block Diagram of Viscous Damper.

c. Mass M (kg)

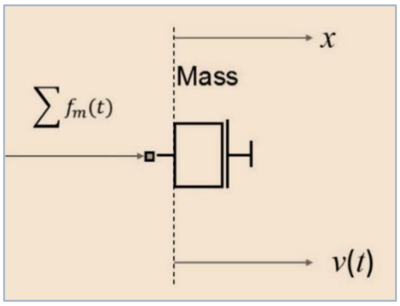

Figure 16. Mass

By using the Newton's Second Law, "the algebraic sum of forces acting on a rigid body in a given direction is equal to the product of the mass of the body and its acceleration in the same direction, i.e:

$\sum f_m(t) = Ma$

Where,

a is the acceleration in the considered direction.

$a = \frac{d^2x}{dt^2}$,

$\therefore \sum f_m(t) = M\frac{d^2x}{dt^2}$

Take the Laplace transform

$\sum F_m(s) = MS^2X(s)$

The Block Diagram Representation is

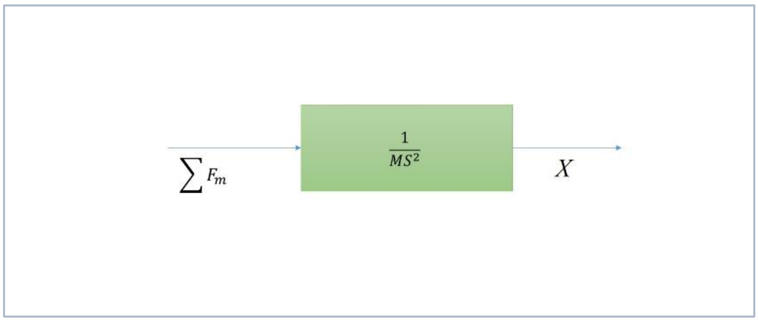

Figure 17. Block Diagram of a Mass.

d. Mass- Spring- Damper Combination

Series Combination

A series arrangement of the translational mechanical components is shown in figure below (Raven, 1961):

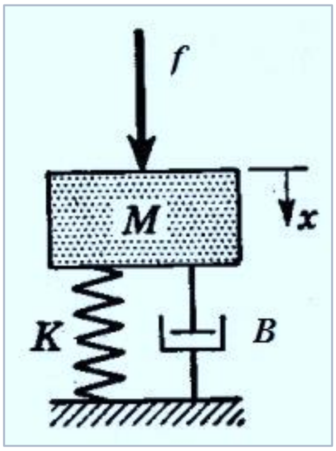

Figure 18. Series Combination (Raven, 1961).

For mass- spring- damper Combination shown, the spring and damper forces opposed to the motion caused by the applied force (F). The summation of the external forces acting on the mass is:

$$\sum F_m = F + Mg - B\frac{dX}{dt} - KX \quad 1$$

$$M\frac{d^2X}{dt^2} = F + Mg - B\frac{dX}{dt} - KX \quad 2$$

$$F = M\frac{d^2X}{dt^2} - Mg + B\frac{dX}{dt} + KX \quad 3$$

Where,

g is the gravitational acceleration,

Mg is the weight of the mass.

It is convenient to make measurements with respect to some reference position X_r. Therefore

$x = X - X_r$ 4

Consequently, the force equation at the reference position is:

$F_r = B\frac{dX_r}{dt} + KX_r - Mg$ 5

$\because X_r$ is constant

$\therefore B\frac{dX_r}{dt} = 0.$

Therefore, equation 5 is written

$F_r = KX_r - Mg$ 6

F_r is the required force to maintain the mass equilibrium at the reference position X_r.

Subtract equation 6 from equation 3.

$F - F_r = M\frac{d^2X}{dt^2} - Mg + B\frac{dX}{dt} + KX - (KX_r - Mg),$

$F - F_r = M\frac{d^2X}{dt^2} + B\frac{dX}{dt} + K(X - X_r).$ 7

Substitute equation 4 in 7.

$F - F_r = M\frac{d^2(x + X_r)}{dt^2} + B\frac{d(x + X_r)}{dt} + Kx.$ 8

$\because X_r$ is constant

$\therefore \frac{d^2(x+X_r)}{dt^2} = \frac{d^2x}{dt^2}$ and

$\frac{d(x+X_r)}{dt} = \frac{dx}{dt}$.

Let $f = F - F_r$, so the equation 8 can be written as:

$f = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx.$ 9

For the series mechanical components, the force f is equal to the summation of the force acting on each individual component, and each component undergoes the same displacement x.

Take the Laplace transform for the equation 9.

$F(s) = MS^2X(s) + BS\,X(s) + K\,X(s)$

$F(s) = (MS^2 + BS + K)\,X(s)$ 10

The input to the system is the force F, and the output is the deflection X. Therefore, the transfer function is

$\frac{X(s)}{F(s)} = \frac{1}{MS^2 + BS + K}$ 11

The Block-Diagram representation for the equation 11 is (Raven, 1961)

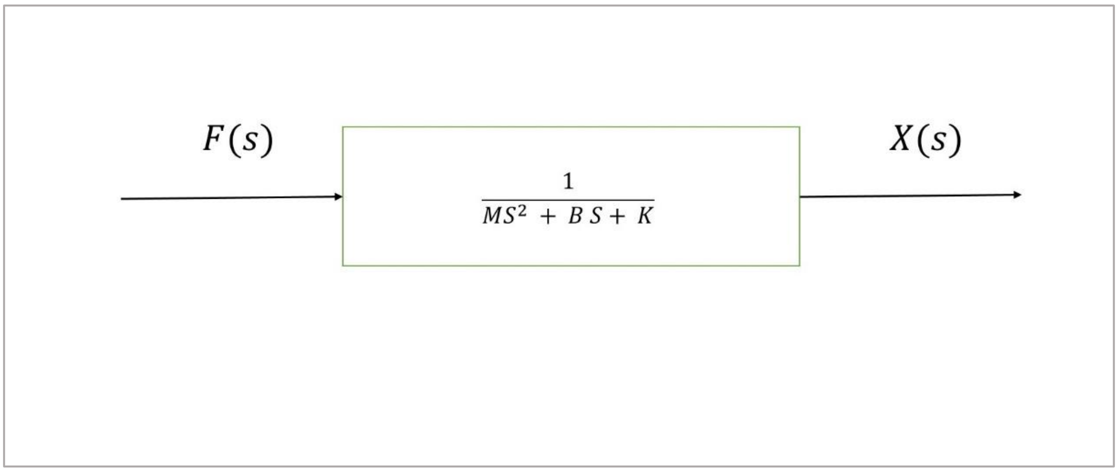

Figure 19. Block Diagram Representation of Mass- Spring- Damper Series Combination (Raven, 1961).

<u>Parallel Combination</u>

A parallel arrangement of the translational mechanical components is shown in figure below:

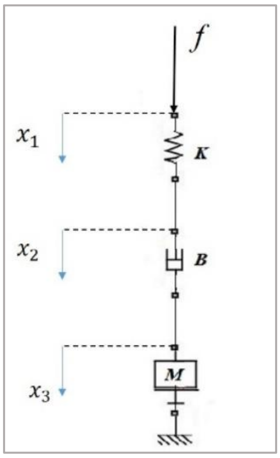

Figure 20. Mass- Spring- Damper Parallel Combination.

For parallel components, the same force *f* is transmitted through each component, but the total displacement *x* is the summation of the individual displacement of each component.