Ministry of Higher Education and Scientific Research

Al-Mustaqbal University College

Department of Technical Computer Engineering

Electrical control fundamentals

3rd Stage

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$x = x_1 + x_2 + x_3$

$x = \frac{f}{K} + \frac{f}{BD} + \frac{f}{MD^2}$ where $D = \frac{d}{dt}$

$x = (\frac{1}{K} + \frac{1}{BD} + \frac{1}{MD^2})f$

$$f = \frac{1}{\frac{1}{K} + \frac{1}{BD} + \frac{1}{MD^2}}x \quad 12$$

Take Laplace transform for equation 12.

$$F(s) = \frac{1}{\frac{1}{K} + \frac{1}{BS} + \frac{1}{MS^2}}X(s) \quad 13$$

The force ($F(s)$) is the input, and $X(s)$ is the output. The transfer function and the block diagram are:

$$\text{Transfer Function} = \frac{X(s)}{F(s)} = \frac{1}{MS^2} + \frac{1}{BS} + \frac{1}{K} \quad 14$$

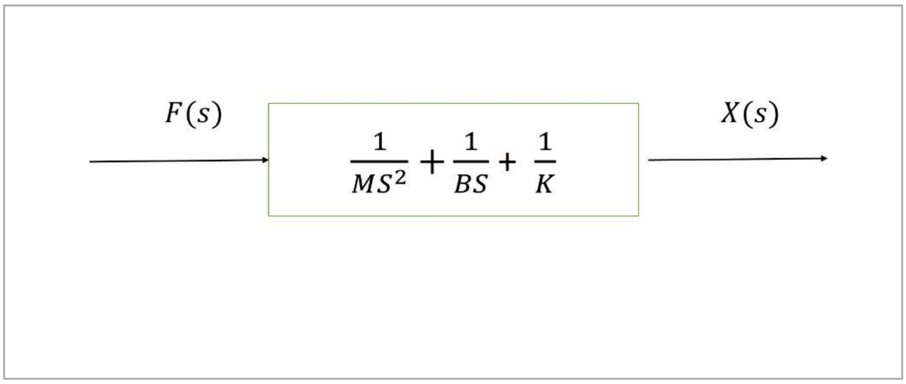


Figure 21. Block Diagram Representation of Mass-Spring-Damper Parallel Combination.

Analogies

The equation that describes the operation of a series Mechanical Components is

$$f(t) = M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx. \qquad 15$$

The equation of the series Electrical Components is

$$v(t) = Ri(t) + L\frac{d}{dt}i(t) + \frac{1}{c}\int i(t)dt \qquad 16$$

The current $i(t)$ represents the rate of flow of electric charge (q):

$$i = \frac{dq}{dt} \qquad 17$$

Substitute (iii) in (ii)

$$v(t) = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q \qquad 18$$

By comparing (18) with (15), the analogies are obtained:

- The applied force [$f(t)$] is analogous to the applied voltage [$v(t)$].
- The mass [M] is analogous to the [L].
- The coefficient of viscous friction [B] is analogous to the resistance [R].
- The spring deflection constant [K] is analogous to the reciprocal capacitance [$\frac{1}{C}$].
- The displacement [x] is analogous to the electric charge[q].

❖ The quantities $M\frac{d^2x}{dt^2}$, $B\frac{dx}{dt}$, Kx, and $f(t)$ are <u>forces.</u>

❖ The quantities $L\frac{d^2q}{dt^2}$, $R\frac{dq}{dt}$, $\frac{1}{c}q$, $v(t)$ and are <u>voltages</u>.

Therefore, this analogy is called **Force- Voltage Analogy.**

The equation of parallel electrical components is

$$\frac{v(t)}{R} + \frac{1}{L}\int v(t)\, dt + C\,\frac{dv(t)}{dt} = i(t) \qquad 19$$

The voltage $v(t)$ is related to flux linkages ψ associated with inductance L.

$$v(t) = \frac{d\psi}{dt} \qquad 20$$

Substitute (20) in (19).

$$C\,\frac{d^2\psi}{dt^2} + \frac{1}{R}\frac{d\psi}{dt} + \frac{1}{L}\,\psi = i(t) \qquad 21$$

By comparing between (21) and (15), the analogies are obtained:

- The applied force [$f(t)$] is analogous to the injected current [$i(t)$].
- The mass [M] is analogous to the [C].
- The coefficient of viscous friction [B] is analogous to the reciprocal of resistance [$\frac{1}{R}$].
- The spring deflection constant [K] is analogous to the reciprocal inductance [$\frac{1}{L}$].
- The displacement [x] is analogous to the flux linkages[ψ].

- The quantities $M\frac{d^2x}{dt^2}$, $B\frac{dx}{dt}$, Kx, and $f(t)$ are <u>forces.</u>
- The quantities $C\frac{d^2\psi}{dt^2}$, $\frac{1}{R}\frac{d\psi}{dt}$, $\frac{1}{L}\,\psi$, $i(t)$ and are <u>currents</u>.

Therefore, this analogy is called **Force- Current Analogy.**

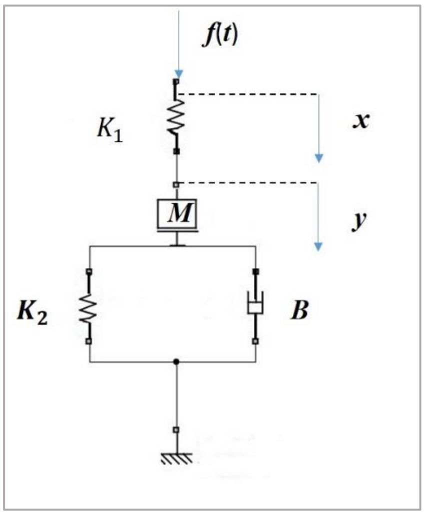
The concept of analogous systems is very useful in practice because one type of systems may be easier to handle experimentally than another type. For example, instead of building and studying a mechanical system, we may build and study its electrical and analogy because electrical and electronic systems are generally easier to deal with experimentally. However, the analogies between two systems break down when the region of operation are extended too far.

Table 2. Force-Voltage and Force- Current analogies *(Raven, 1961)*.

	Force-Voltage Analogy					**Force-Current Analogy**				
Translational Mechanical Components	$f(t)$	M	B	K	x	$f(t)$	M	B	K	x
Electrical Components	$v(t)$	L	R	$\frac{1}{C}$	q	$i(t)$	C	$\frac{1}{R}$	$\frac{1}{L}$	ψ

Example

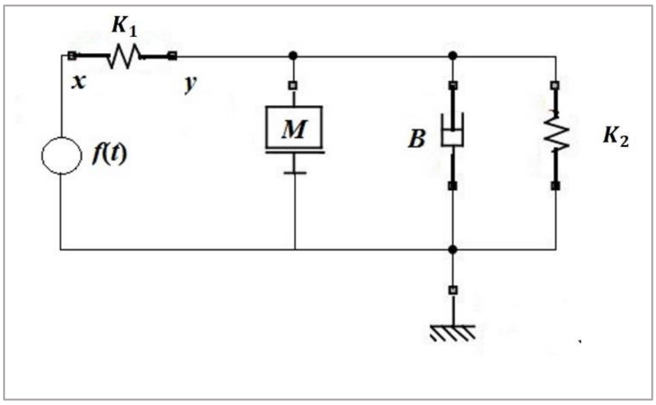
For mass- spring- damper combination shown in the figure below,



a) Find the transfer function relating the applied force $f(t)$ and the displacement (x).
b) Draw its analogous circuit.

Solution

a) The mechanical system circuit can be drawn like this:



$K_1(x\text{-}y) = f(t)$ 1

$K_2y + B\frac{dy}{dt} + M\frac{d^2y}{dt^2} + K_1(y\text{-}x) = 0$ 2

Take Laplace transform for (1) and (2).

$K_1X(s) - K_1Y(s) = F(\text{s})$ 3

$(MS^2 + BS + K_1 + K_2)\ Y(s) = K_1\ X(s)$ 4

From equation (4),

$Y(s) = \frac{K_1}{MS^2+BS+K_1+K_2}X(s)$ 5

Substitute (5) in (3).

$K_1X(s) - K_1\frac{K_1}{MS^2+BS+K_1+K_2}X(s) = F(\text{s})$

$F(\text{s}) = \frac{K_1(MS^2+BS+K_2)}{MS^2+BS+K_1+K_2}X(s)$

The applied force F(s) is the input, and the displacement X(s) is the output, so the transfer function of this system is

$$\frac{X(s)}{F(s)} = \frac{MS^2 + BS + K_1 + K_2}{K_1(MS^2 + BS + K_2)}$$

b)

Convert equations (3) and (4) to comparable electrical analogous.

$$\frac{1}{C_1}\int i_1 dt - \frac{1}{C_1}\int i_2 dt = v(t)$$

The equation above can be rewritten like

$$\frac{1}{C_1}\int (i_1 - i_2) dt = v(t) \qquad 6$$

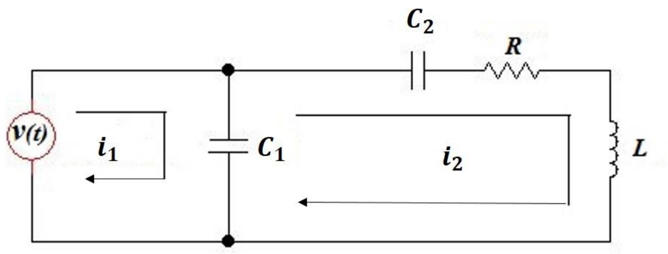
And

$$L\frac{di_2}{dt} + Ri_2 + \frac{1}{C_1}\int i_2 dt + \frac{1}{C_2}\int i_2 dt - \frac{1}{C_1}\int i_1 dt = 0$$

The equation above can be rewritten like

$$L\frac{di_2}{dt} + Ri_2 + \frac{1}{C_2}\int i_2 dt + \frac{1}{C_1}\int (i_2 - i_1) dt = 0 \qquad 7$$

From equations (6) and (7), the electrical analogous circuit based on a force- voltage analogy is shown in the figure below:



Where,

$v(t) = f(t)$, $C_1 = \frac{1}{K_1}$, $C_2 = \frac{1}{K_2}$, $R = B$, and $L = M$.