Inverse Matrix

1

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$ 2x2 Matrix

By using the formula

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ General

Ex/ Find the inverse of the 2x2 matrix below:

$$A = \begin{bmatrix} 5 & 2 \\ -7 & -3 \end{bmatrix} \Longleftrightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

sol

$$\det A = (-15) - (-14) = \boxed{-1}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} -3 & -2 \\ 7 & 5 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 3 & 2 \\ -7 & -5 \end{bmatrix}$$ Ans

3x3 matrix, find the inverse (2)

$$A^{-1} = \frac{1}{\det A} \text{ Adjoint of } A$$

By using the Determinants and Cofactors

Ex

$$A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$|A| = 3 \begin{vmatrix} -2 & 1 \\ -5 & 2 \end{vmatrix} - 3 \begin{vmatrix} -2 & 1 \\ -4 & 2 \end{vmatrix} + (-1) \begin{vmatrix} -2 & -2 \\ -4 & -5 \end{vmatrix}$$

$$= 3(1) - 3(0) - 1(2) = 3 - 0 - 2$$

$$= \boxed{1}$$

ملاحظة لإيجاد المحدد لمصفوفة ٣×٣

نضرب العمود الثاني في سالب

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$$

$$+(3)\begin{vmatrix} -2 & 1 \\ -5 & 2 \end{vmatrix} - (3)\begin{vmatrix} -2 & 1 \\ -4 & 2 \end{vmatrix} + (-1)\begin{vmatrix} -2 & -2 \\ -4 & -5 \end{vmatrix}$$

3

Cofactor matrix $A = \begin{pmatrix} a_{11} & -a_{12} & a_{13} \\ -a_{21} & a_{22} & -a_{23} \\ a_{31} & -a_{32} & a_{33} \end{pmatrix}$

$a_{11} = \begin{vmatrix} -2 & 1 \\ -5 & 2 \end{vmatrix} = \boxed{1}$, $a_{12} = \begin{vmatrix} -2 & 1 \\ -4 & 2 \end{vmatrix} = \boxed{0}$ (-4) (-4)

$a_{13} = \begin{vmatrix} -2 & -2 \\ -4 & -5 \end{vmatrix} = \boxed{2}$, $a_{21} = \begin{vmatrix} 3 & -1 \\ -5 & 2 \end{vmatrix} = \boxed{1}$

$a_{22} = \begin{vmatrix} 3 & -1 \\ -4 & 2 \end{vmatrix} = \boxed{2}$, $a_{23} = \begin{vmatrix} 3 & 3 \\ -4 & -5 \end{vmatrix} = \boxed{-3}$

$a_{31} = \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = \boxed{1}$, $a_{32} = \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = \boxed{1}$

$a_{33} = \begin{vmatrix} 3 & 3 \\ -2 & -2 \end{vmatrix} = \boxed{0}$

The cofactor of $A = \begin{pmatrix} 1 & -0 & 2 \\ -1 & 2 & +3 \\ 1 & -1 & 0 \end{pmatrix}$

Now find the traspose of $A = A^T$

$$A^T = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{pmatrix}$$ Ans

$$I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$AA^{-1} = I$$

$$\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 3+0-2 & -3+6+(-3) & 3+(-3)+(0) \\ -2+(0)+2 & 2-4+3 & -2+2+0 \\ -4-0+4 & 4-10+6 & -4+5+0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Using Augumented Matrix

1 Aug. matrix أوجد الصيغة التالية للـ

$$\left[A \mid I_n \right]$$

Identity Matrix

2 نغير في الصفوف للحصول على الصيغة التالية

$$\left[I_n \mid A^{-1} \right]$$

Ex/

$$A=\begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$$

Sol

$$\left(A \mid I_n\right)$$

$$\begin{array}{r} R_1+R_2 \\ 2R_1+3R_2 \\ -2R_2+R_3 \end{array}\left(\begin{array}{ccc|ccc} 3 & 3 & -1 & 1 & 0 & 0 \\ -2 & -2 & 1 & 0 & 1 & 0 \\ -4 & -5 & 2 & 0 & 0 & 1 \end{array}\right)$$

$$\begin{array}{r} R_1+R_3 \\ \\ \\ \end{array}\begin{pmatrix} 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 2 & 3 & 0 \\ 0 & -1 & 0 & 0 & -2 & 1 \end{pmatrix}$$

$$\begin{array}{r} \\ -R_2 \\ \\ \end{array}\begin{pmatrix} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & -1 & 0 & 0 & -2 & 1 \\ 0 & 0 & 1 & 2 & 3 & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & -1 & 1 \\ 0 & 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & 2 & 3 & 0 \end{array}\right)$$

$$\left(I_n \mid A^{-1}\right)$$

$$A = \begin{pmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{pmatrix}$$

تحقق

$$A A^{-1} = A^{-1} A = I_n$$

Rule to check