Cramer's Rule

1

Ex/

$$x + y - z = 6 \quad (1)$$

$$3x - 2y + z = -5 \quad (2)$$

$$x + 3y - 2z = 14$$

Sol/

$$2 + 3 + -6 = -1$$

$$A = \begin{vmatrix} 1 & 1 & -1 \\ 3 & -2 & 1 \\ 1 & 3 & -2 \end{vmatrix} \begin{matrix} 1 & 1 \\ 3 & -2 \\ 1 & 3 \end{matrix} = -4 + 1 = \boxed{-3}$$

$$4 + 1 + -9 = -4$$

$$28 + 18 + 10 = 56$$

$$A_x = \begin{vmatrix} 6 & 1 & -1 \\ -5 & -2 & 1 \\ 14 & 3 & -2 \end{vmatrix} \begin{matrix} 6 & 1 \\ -5 & -2 \\ 14 & 3 \end{matrix} = 53 - 56 = \boxed{-3}$$

$$24 + 14 + 15 = 53$$

$$x = \frac{A_x}{A} = \frac{-3}{-3} = \boxed{1}$$

Inverse Matrix

2

$$(A)^{-1} = \frac{1}{\det A}\left[\text{Adjoint of } A\right]$$

$$A = \begin{bmatrix} d & w \\ b & y \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} y & -w \\ -b & d \end{bmatrix}$$

Ex/ $x_1 - 2x_2 + x_3 = 3$

$2x_1 + x_2 - x_3 = 5$

$3x_1 - x_2 + 2x_3 = 12$

3

Sol/

$$[A] = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & -1 & 2 \end{bmatrix}, \quad [X] = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \quad [B] = \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$$

$$[A][X] = [B]$$

Inverse

$$[X] = \frac{[B]}{[A]} = [A]^{-1}[B]$$

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(1)

$$3 + 1 - 8 = -4$$

$$|A| = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & -1 & 2 \end{vmatrix} \begin{matrix} 1 & -2 \\ 2 & 1 \\ 3 & -1 \end{matrix} = 6 + 4 = \boxed{10}$$

$$2 + 6 - 2 = 6$$

$$[A]^{-1} = \frac{1}{\det A} \begin{pmatrix} \text{Adjoint} \\ \text{of} \\ A \end{pmatrix}$$

Cofactor of A

$$= \begin{pmatrix} a_{11} & -a_{12} & a_{13} \\ -a_{21} & a_{22} & -a_{23} \\ a_{31} & -a_{32} & a_{33} \end{pmatrix}$$

$a_{11} = \begin{vmatrix} 1 & -1 \\ -1 & 2 \end{vmatrix} = \boxed{1}$, $a_{12} = \begin{vmatrix} 2 & -1 \\ 3 & 2 \end{vmatrix} = \boxed{7}$, $a_{13} = \begin{vmatrix} 2 & 1 \\ 3 & -1 \end{vmatrix} = -5$

$a_{21} = \begin{vmatrix} -2 & 1 \\ -1 & 2 \end{vmatrix} = \boxed{-3}$, $a_{22} = \begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} = \boxed{-1}$, $a_{23} = \begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} = \boxed{5}$

$a_{31} = \begin{vmatrix} -2 & 1 \\ 1 & -1 \end{vmatrix} = \boxed{1}$, $a_{32} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = \boxed{-3}$, $a_{33} = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = \boxed{5}$

$$\text{Cofactor of } A = \begin{pmatrix} 1 & -7 & -5 \\ +3 & -1 & -5 \\ 1 & +3 & 5 \end{pmatrix}$$

$$A^T = \begin{pmatrix} 1 & 3 & 1 \\ -7 & -1 & 3 \\ -5 & -5 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{10} \begin{pmatrix} 1 & 3 & 1 \\ -7 & -1 & 3 \\ -5 & -5 & 5 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} 0.1 & 0.3 & 0.1 \\ -0.7 & -0.1 & 0.3 \\ -0.5 & -0.5 & 0.5 \end{pmatrix} \checkmark$$

$$(X) = (A)^{-1} (B)$$

$$[X] = [A]^{-1}[B]$$

$$= \begin{bmatrix} 0.1 & 0.3 & 0.1 \\ -0.7 & -0.1 & 0.3 \\ -0.5 & -0.5 & 0.5 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 12 \end{bmatrix}$$

$$3 \times \cancel{3} \qquad \cancel{3} \times 1$$

$$\rightarrow 3 \times 1$$

$$[X] = \begin{bmatrix} (0.1 \times 3) + (0.3 \times 5) + (0.1 \times 12) \\ (-0.7 \times 3) + (-0.1 \times 5) + (0.3 \times 12) \\ (-0.5 \times 3) + (-0.5 \times 5) + (0.5 \times 12) \end{bmatrix}$$

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$$

$$\therefore \boxed{\begin{aligned} X_1 &= 3 \\ X_2 &= 1 \\ X_3 &= 2 \end{aligned}}$$

Ans