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الرياضيات_ المرحلة الأولى

Integration of hyperbolic trigonometric functions

$\int \sinh x\, dx = \cosh x + C$	$\int \cosh x\, dx = \sinh x + C$
$\int \text{sech}^2 x\, dx = \tanh x + C$	$\int \text{csch}^2 x\, dx = -\coth x + C$
$\int \tanh x\, \text{sech}\, x\, dx = -\text{sech}\, x + C$	$\int \text{csch}\, x \coth x\, dx = -\text{csch}\, x + C$

Example

$$1\text{-}\int \frac{4}{\sqrt{x}} csch\sqrt{x}\ \ coth\sqrt{x}dx = 4\int \frac{1}{\sqrt{x}} csch\sqrt{x}\ \ coth\sqrt{x}dx\ \ \times\frac{2}{2}$$

$$= 8\int csch\sqrt{x}\ \ coth\sqrt{x}\ \ \frac{dx}{2\sqrt{x}} = -\,8\,\mathrm{csch}\sqrt{x}\ + C$$

$$2\text{-}\int cosh^5(2x^4)\sinh(2x^4)\ \ x^3dx = \int cosh^5(2x^4)\sinh(2x^4)\ \ x^3dx\times\frac{8}{8}$$

$$= \frac{1}{8}\int cosh^5(2x^4)\sinh(2x^4)\ \ 8x^3dx\ = \frac{1}{8}\ \frac{cosh^6(2x^4)}{6} + C$$

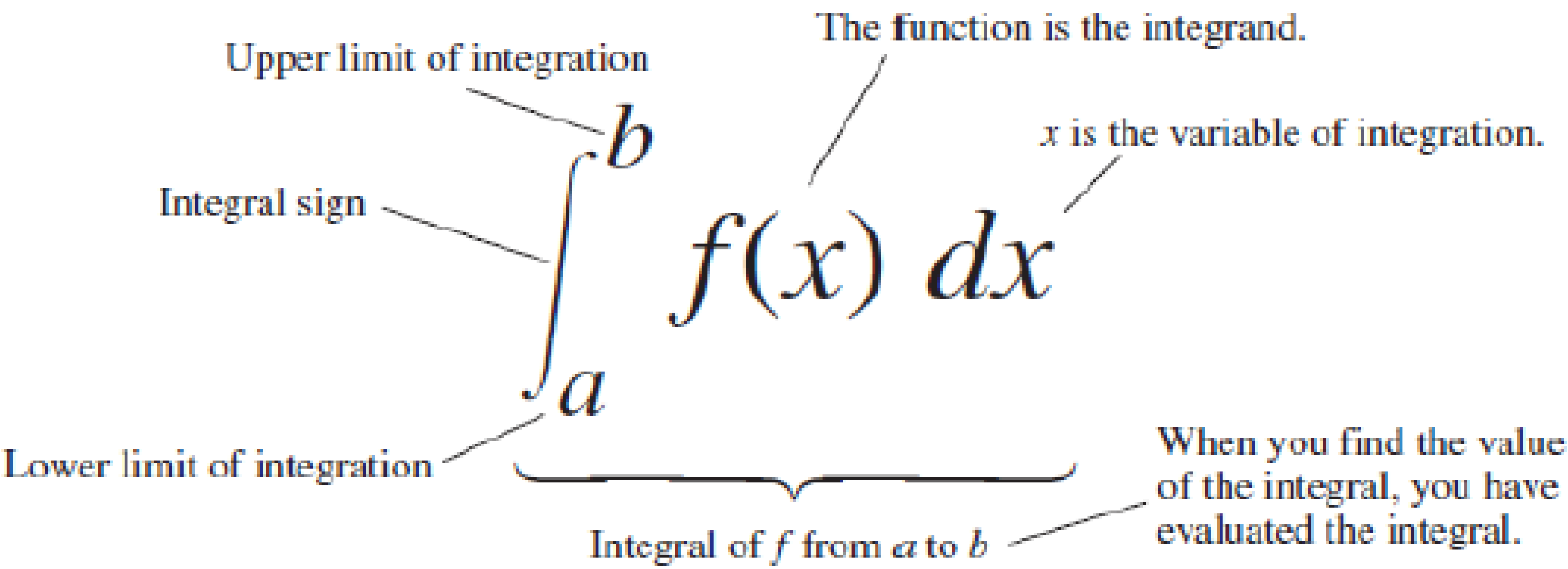
$$= \frac{cosh^6(2x^4)}{48} + C$$

Home Work

$\int \frac{dx}{\sqrt{x^2+a^2}} = \sinh^{-1}(\frac{x}{a}) + C$	$\int \frac{dx}{\sqrt{x^2-a^2}} = \cosh^{-1}(\frac{x}{a}) + C$
$\int \frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}(\frac{x}{a}) + C = \frac{1}{a}\coth^{-1}(\frac{x}{a}) + C$	
$\int \frac{dx}{x\sqrt{a^2-x^2}} = -\frac{1}{a}\operatorname{sech}^{-1}(\frac{x}{a}) + C$	$\int \frac{dx}{x\sqrt{x^2+a^2}} = -\frac{1}{a}\operatorname{csch}^{-1}(\frac{x}{a}) + C$

1. $\int sech^2\left(x - \frac{1}{2}\right) dx$
2. $\int \sinh\left(\frac{x}{7}\right) dx$
3. $\int 6\cosh\left(\frac{x}{2} - \ln x\right)\left(\frac{1}{2} - \frac{1}{x}\right) dx$
4. $\int \frac{\cosh(lnx)dx}{x}$
5. $\int_1^4 \frac{\cosh\sqrt{x}dx}{\sqrt{x}}$
6. $\int_0^{\ln 10} 4sinh^2\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right) dx$

Definite Integration



$$\int_a^b f(x)dx = [\, F(x) \,]_a^b = F(b) - F(a)$$

Where $f(x)\,dx = F'(x)$

Rules Satisfied by definite integrals

Rules satisfied by definite integrals

Order of Integration:	$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$	A Definition
Zero Width Interval:	$\int_a^a f(x)\,dx = 0$	Also a Definition
Constant Multiple:	$\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$	Any Number k
	$\int_a^b -f(x)\,dx = -\int_a^b f(x)\,dx$	$k = -1$
Sum and Difference:	$\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$	
Additivity:	$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$	

Example

Calculate the following integration

1) $\int_{2}^{6} \frac{dx}{x+2}$

2) $\int_{\pi/2}^{3\pi/2} \cos x \, dx$

3) $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{dx}{1+x^2}$

4) $\int_{0}^{\sqrt{3}/2} \frac{dx}{\sqrt{1-x^2}}$

5) $\int_{-2}^{4} e^{-\frac{x}{2}} \, dx$

Solution:

$$1)\ \int_2^6 \frac{dx}{x+2} = ln(x+2)\Big|_2^6 = ln(6+2) - ln(2+2) = ln8 - ln4 = 3ln2 - 2ln2 = ln2$$

$$2)\ \int_{\pi/2}^{3\pi/2} cos\,x\,dx = sin\,x\Big|_{\pi/2}^{3\pi/2} = sin(\frac{3}{2}\pi) - sin(\frac{\pi}{2}) = -1 - 1 = -2$$

$$3)\ \int_{-\sqrt{3}}^{\sqrt{3}} \frac{dx}{1+x^2} = tan^{-1}\Big|_{-\sqrt{3}}^{\sqrt{3}} = tan^{-1}\sqrt{3} - tan^{-1}(-\sqrt{3}) = \frac{\pi}{3} - (-\frac{\pi}{3}) = \frac{2}{3}\pi$$

Cont'd

$$4)\ \int_0^{\sqrt{3}/2} \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x \Big|_0^{\sqrt{3}/2} = \sin^{-1}\frac{\sqrt{3}}{2} - \sin^{-1} 0 = \frac{\pi}{3} - 0 = \frac{\pi}{3}$$

$$5)\ \int_{-2}^{4} e^{-\frac{x}{2}}\, dx = -2e^{-\frac{x}{2}}\Big|_{-2}^{4} = -2(e^{-2} - e) = 2(e - e^{-2})$$

Example: Determine

$$\int_0^{\frac{\pi}{2}} 3\sin 2x\,dx = \left[(3)\left(-\frac{1}{2}\right)\cos 2x\right]_0^{\frac{\pi}{2}} = \left[-\frac{3}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$= \left\{-\frac{3}{2}\cos 2\left(\frac{\pi}{2}\right)\right\} - \left\{-\frac{3}{2}\cos 2(0)\right\}$$

$$= \left\{-\frac{3}{2}\cos\pi\right\} - \left\{-\frac{3}{2}\cos 0\right\} = \left\{-\frac{3}{2}(-1)\right\} - \left\{-\frac{3}{2}(1)\right\}$$

$$= \frac{3}{2} + \frac{3}{2} = \mathbf{3}$$

$$\int_1^4 \left(\frac{\theta+2}{\sqrt{\theta}}\right)d\theta = \int_1^4 \left(\frac{\theta}{\theta^{\frac{1}{2}}} + \frac{2}{\theta^{\frac{1}{2}}}\right)d\theta = \int_1^4 \left(\theta^{\frac{1}{2}} + 2\theta^{\frac{-1}{2}}\right)d\theta$$

$$= \left[\frac{\theta^{\left(\frac{1}{2}\right)+1}}{\frac{1}{2}+1} + \frac{2\theta^{\left(\frac{-1}{2}\right)+1}}{-\frac{1}{2}+1}\right]_1^4 = \left[\frac{\theta^{\frac{3}{2}}}{\frac{3}{2}} + \frac{2\theta^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4$$

$$= \left[\frac{2}{3}\sqrt{\theta^3} + 4\sqrt{\theta}\right]_1^4 = \left\{\frac{2}{3}\sqrt{(4)^3} + 4\sqrt{4}\right\} - \left\{\frac{2}{3}\sqrt{(1)^3} + 4\sqrt{1}\right\}$$

$$= \left\{\frac{16}{3} + 8\right\} - \left\{\frac{2}{3} + 4\right\} = 5\frac{1}{3} + 8 - \frac{2}{3} - 4 = \mathbf{8\frac{2}{3}}$$

Cont’d

Example: Find $\int_1^9 \sqrt{5x+4}\ dx$

Solution : $\int_1^9 (5x+4)^{\frac{1}{2}}\ dx$,{ f(x) = 5x+4 , f '(x) = 5 }

$$\int_1^9 (5x+4)^{\frac{1}{2}}\ dx \times \frac{5}{5} = \frac{1}{5}\int_1^9 (5x+4)^{\frac{1}{2}}\ 5dx$$

$$= \frac{1}{5}\left[\frac{(5x+4)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\right]_1^9 = \frac{1}{5}\left[\frac{(5x+4)^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^9 = \frac{2}{15}\{(5*9+4)^{\frac{3}{2}} - (5*1+4)^{\frac{3}{2}}\}$$

$$= \frac{2}{15}\{\sqrt{49^3} - \sqrt{9^3}\} = \{343-27\} = \frac{2}{15}\{316\} = \frac{632}{15}$$

Home Work

1. (a) $\int_1^4 5x^2\,dx$ (b) $\int_{-1}^{1} -\frac{3}{4}t^2\,dt$

$\left[\text{(a) } 105 \quad \text{(b) } -\frac{1}{2}\right]$

2. (a) $\int_{-1}^{2} (3-x^2)\,dx$ (b) $\int_1^3 (x^2-4x+3)\,dx$

$\left[\text{(a) } 6 \quad \text{(b) } -1\frac{1}{3}\right]$

3. (a) $\int_0^{\pi} \frac{3}{2}\cos\theta\,d\theta$ (b) $\int_0^{\frac{\pi}{2}} 4\cos\theta\,d\theta$

[(a) 0 (b) 4]

4. (a) $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} 2\sin 2\theta\,d\theta$ (b) $\int_0^2 3\sin t\,dt$

[(a) 1 (b) 4.248]

5. (a) $\int_0^1 5\cos 3x\,dx$ (b) $\int_0^{\frac{\pi}{6}} 3\sec^2 2x\,dx$

[(a) 0.2352 (b) 2.598]

6. (a) $\int_1^2 \csc^2 4t\,dt$

(b) $\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} (3\sin 2x - 2\cos 3x)\,dx$

[(a) 0.2572 (b) 2.638]

7. (a) $\int_0^1 3e^{3t}\,dt$ (b) $\int_{-1}^{2} \frac{2}{3e^{2x}}\,dx$

[(a) 19.09 (b) 2.457]

8. (a) $\int_2^3 \frac{2}{3x}\,dx$ (b) $\int_1^3 \frac{2x^2+1}{x}\,dx$

[(a) 0.2703 (b) 9.099]

Thank You