اسم التدريسي : زهراء هاشم كريم

اسم المادة : رياضيات

عنوان المحاضرة: some applications of integration

رقم المحاضرة: ٣

الايم يل الجامعي للتدريسي: zahraa.hashim@mustaqbal-college.edu.iq

AL-Mustaqbal university college
Class: 1st
Subject: mathmatic

Lecturer: zahraa hashim kareem

Lecture :3

E-mail : zahraa.hashim@mustaqbal-college.edu.iq

Some Application of Integration

1)Acceleration , Velocity , Distance

Time : t	*Acceleration : a(t)*
Velocity : $v(t) = \int a(t)dt$	*Distance :* $s(t) = \int v(t)dt$

Example: A body moves along a straight line according to v(t) = 2t -4 (m/s).

1. Find distance through [1,4].

2t -4 =0 , t = 2 ϵ [1,4] , $s(t) = |\int_1^2 (2t-4)dt| + |\int_2^4 (2t-4)dt| =$ $|[t^2-4t]_1^2| + |[t^2-4t]_2^4| = |-1| - |4| = 5m$

2. Find displacement through[1,4]

$s(t) = \int_1^4 (2t-4)dt = [t^2-4t]_1^4 = 3m$

3. Find distance through the tenth second.

$s(t) = \int_9^{10} (2t-4)dt = [t^2-4t]_9^{10} = \{10^2-4*10\} - \{9^2-4*9\} = 60 - 45 = 15$ m

Example: A body moves along a straight line according to the law a(t) = 4t+2 (m^2 /s). Its velocity at fourth second is 50 m/s. Determine:

1. Its velocity at any time.

$v(t) = \int a(t)dt = \int (4t+2)dt = 2t^2 + 2t + c$

$50 = 2(4)^2 + 2*4 + c$, c = 10 , $v(t) = 2t^2 + 2t + 10$

2. Distance through [3,7].

$s(t) = \int_3^7 (2t^2 + 2t + 10)dt = [\frac{2}{3}t^3 + t^2 + 10t]_3^7 = (\frac{2}{3}(7)^3 + (7)^2 + 10*7) - (\frac{2}{3}(3)^3 + (3)^2 + 10*3) = 193.3$ m

AL-Mustaqbal university college
Class: 1st
Subject: mathmatic

Lecturer: zahraa hashim kareem

Lecture :3

E-mail : zahraa.hashim@mustaqbal-college.edu.iq

3. Distance after 10 seconds

Note $(2t^2 + 2t + 10 \neq 0)$

s(t) = $\int_0^{10}(2t^2 + 2t + 10)dt = [\frac{2}{3}t^3 + t^2 + 10t]_0^{10} = (\frac{2}{3}(10)^3 + (10)^2 + 10*10) - (\frac{2}{3}(0)^3 + (0)^2 + 10*0) = 866.6$ m

Example: A body moves along a straight line according to the law $a(t) = 21t^2 - 10$ (m/s²). Determine its velocity after 2 seconds

s(t) = $\int_0^{2}(21t^2 - 10)dt = [7t^3 - 10t]_0^2 = \{7(2)^3 - 10*2\} - \{(7(0)^3 - 10*0\} = 36$ m/s

2) Length of curve

If *f(x)* is continuously differentiable on the interval *[a,b]*, the length of the curve $y = f(x)$ from $x = a$ to $x = b$ is

$$L = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

Example: Find the length the curve $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1$, $0 \leq x \leq 1$

Solution :

a = 0 , b = 1

$y' = \frac{4\sqrt{2}}{3} * \frac{3}{2}x^{1/2} = 2\sqrt{2x}$

$L = \int_0^1 \sqrt{1 + (2\sqrt{2x})^2}\, dx = \int_0^1 \sqrt{1 + 8x}\, dx \quad * \frac{8}{8}$

$= \frac{1}{8} * \frac{2}{3}(1+8x)^{3/2}]_0^1 = 13/6$

AL-Mustaqbal university college
Class: 1st
Subject: mathmatic

Lecturer: zahraa hashim kareem

Lecture :3

E-mail : zahraa.hashim@mustaqbal-college.edu.iq

Example: Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 4$

Solution :

$a = 0 \quad , b = 4$

$y' = \frac{3}{2} x^{1/2}$

$L = \int_0^4 \sqrt{1 + (\frac{3}{2}\, x^{1/2})^2}\, dx = \int_0^4 \sqrt{1 + \frac{9}{4}x}\, dx \quad * \frac{9/4}{9/4}$

$= \frac{1}{9/4} * \frac{2}{3} [(1+\frac{9}{4}x)^{3/2}]_0^4 \;= \frac{8}{27} (\sqrt{10^3} - 1)$

H.W.:

Find the length of the curve $y = \frac{1}{3} x^{3/2}$ from $x = 0$ to $x = 4$