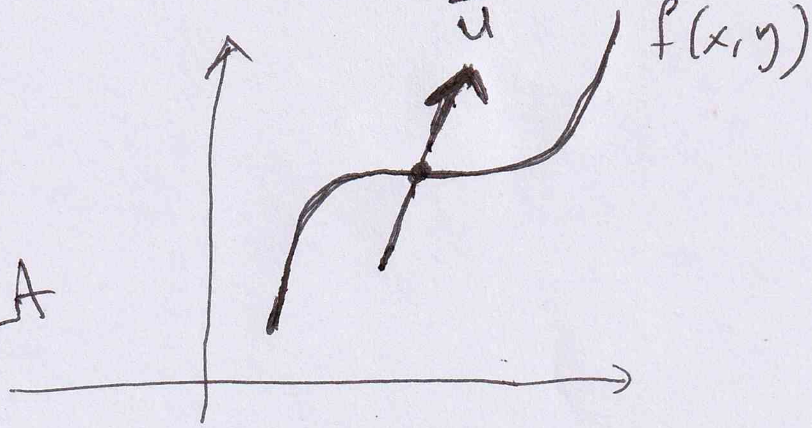
Directional Derivatives ①

The rate of change of $f(x,y)$ in the direction of the unit vector $\vec{u} = \langle a, b \rangle$.



gradient

$$\boxed{D_u f = \frac{\nabla f \cdot \vec{u}}{|\vec{u}|}}$$

Ex1/ $f(x,y) = x^2 + xy$, $\vec{u} = i + 2j$, $(1,1)$

$\langle 1, 2 \rangle$

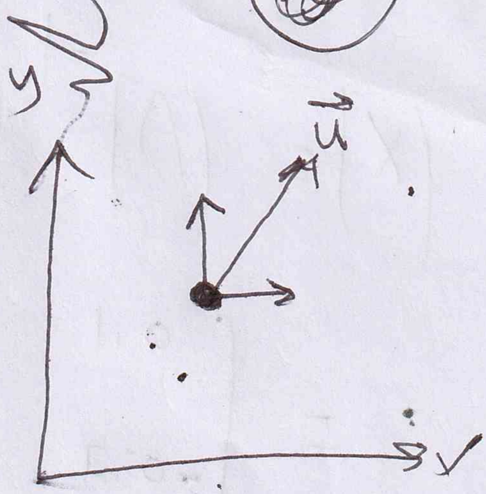
Ex2/ $f(x,y) = xe^y + ye^x$, $\vec{u} = i - j$, $(2,1)$

Ex/ what is the directional derivative of

$f(x,y) = \boxed{x^2 + xy}$

in the direction $\boxed{i + 2j}$

at the point $(1,1)$



Sol/ $\vec{u} = i + 2j$

① تحديد متجه القيمة (الوحدة) unit vector

② ايجاد مطلق Unit vector

$$|\vec{u}| = \sqrt{(1)^2 + (2)^2}$$

$$= \sqrt{5}$$

③ ايجاد المشتقة لـ x مع تثبيت y و لـ y مع تثبيت x

$$\frac{\partial f}{\partial x} = \boxed{2x + y}$$

$$\frac{\partial f}{\partial y} = \boxed{x}$$

④ اعوض قيم النقطة المعطاة بالسؤال

$$\frac{\partial f}{\partial x} = 2 + 1 = \boxed{3}$$

$$\frac{\partial f}{\partial y} = \boxed{1}$$

⑤ اطبق المعادلة الرئيسية

$$D_{\vec{u}} f = \frac{\nabla f \cdot \vec{u}}{|\vec{u}|} = \frac{(3,1) \cdot (1,2)}{\sqrt{5}} = \frac{3+2}{\sqrt{5}} = \frac{5}{\sqrt{5}}$$

$$= \frac{\cancel{\sqrt{5}}\sqrt{5}}{\cancel{\sqrt{5}}} = \boxed{\sqrt{5}} \text{ Ans}$$

EX2 جد المشتقة الاتجاهية للدالة

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$$\boxed{y = e^x}$$

$$f(x,y) = x e^{y} + y e^{x}$$

عند النقطة $(2,1)$ وباتجاه المتجه $\vec{v} = \hat{i} - \hat{j}$

$y = e^x$

$y = 3^x$

Sol

$$\frac{\partial f}{\partial x} = e^{y} + y e^{x} \rightarrow \boxed{e + e^2}$$

$$\frac{\partial f}{\partial y} = x e^{y} + e^{x} \rightarrow \boxed{2e + e^2}$$

$$\vec{v} = \hat{i} + \hat{j}$$

$$|\vec{v}| = \sqrt{1^2 + 1^2} = \sqrt{2} \qquad \vec{u} = \frac{\vec{u}}{|\vec{u}|} = \frac{1}{\sqrt{2}} i - \frac{1}{\sqrt{2}}$$

$$D_{\vec{u}} f(2,1) = \frac{1}{\sqrt{2}}(e + e^2) - \frac{1}{\sqrt{2}}(2e + e^2)$$

$$= \frac{1}{\sqrt{2}}\left(e + e^2 - 2e - e^2\right)$$

$$= \frac{1}{\sqrt{2}}(-e) = \boxed{\frac{-e}{\sqrt{2}}}$$

Ans

Ex3 / Find the directional derivative (2)
of $\boxed{z = e\,x^2 \cos(y) - 2\ln(x)\,y^2}$ in the
direction of the vector $\vec{v} = (1, 4)$.

Sol) $D_{\vec{u}} z = \nabla f \cdot \dfrac{\vec{u}}{|\vec{u}|}$

(١) تحديد متجه الوحدة
unit vector

$\vec{v} = i + 4j$

(٢) ايجاد مطلق المتجه
unit vector

$|\vec{v}| = \sqrt{(1)^2 + (4)^2} = \sqrt{17}$

$$\boxed{u = \left(\frac{1}{\sqrt{17}}, \frac{4}{\sqrt{17}}\right)}$$

(٣) نجد المشتقة لـ x
〃 〃 لـ y

$$\frac{\partial z}{\partial x} = \boxed{2e\,x\cos(y) - \frac{2y^2}{x}}$$

$$\frac{\partial z}{\partial y} = \boxed{-e\,x^2\sin(y) - 4\ln(x)\,y}$$

$$D_u z = (\cdot , \cdot) \cdot (\cdot , \cdot)$$

$$= \left(2e\,x\cos(y) - \frac{2y^2}{x}\right) \cdot \left(\frac{1}{\sqrt{17}}\right) + \left(-e\,x^2\sin(y) - 4\ln(x)\,y\right) \cdot \left(\frac{4}{\sqrt{17}}\right)$$

Ans

Ex 4 / Find the directional derivative of the function $\boxed{f(x,y,z) = 3xy + z^2}$ at the point $(1,-2,2)$ in the direction from that toward the origin.

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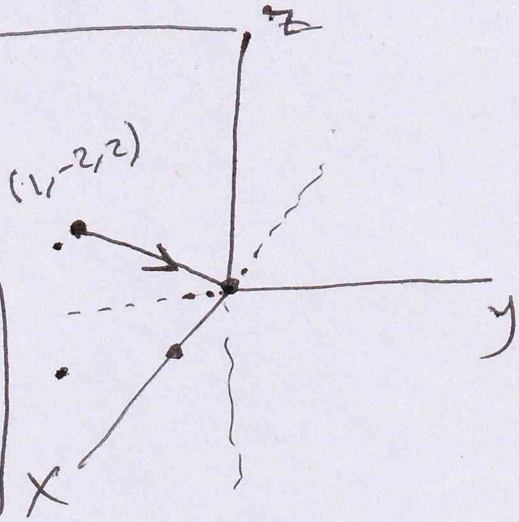

Sol /

$u = \langle -1, 2, -2 \rangle$

ملاحظة/ عند السير من نقطة محددة الى مركز الرسم (٠،٠،٠) نحتاج لتغير الاشارات لتلك النقطة من اجل الحصول على unit vector

$|\vec{u}| = \sqrt{(-1)^2 + (2)^2 + (-2)^2} = \sqrt{9} = \boxed{3}$

$$\boxed{\langle \vec{u} \rangle = \left(\frac{-1}{3}, \frac{2}{3}, \frac{-2}{3} \right)}$$

$$\nabla f(x,y,z) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right)$$

$$= (3y, 3x, 2z)$$

$$= \boxed{(-6, 3, 4)}$$

$$D_u f = \left(-6 \cdot \frac{-1}{3} \right) + \left(\frac{3 \times 2}{3} \right) + \left(\frac{4 \times -2}{3} \right) = 2 + 2 + \frac{-8}{3}$$

$$= \boxed{\frac{4}{3}} \text{ Ans}$$

Ex5 Let $\boxed{w = 2x^2 y^4 z}$. Determine the (4) rate of change of w in the direction of the vector $\vec{u} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$ at $(1,2,3)$.

Sol

$$\frac{\partial w}{\partial x} = \boxed{4xy^4z}, \quad 4(1)(2)^4(3) = 192$$

$$\frac{\partial w}{\partial y} = \boxed{8x^2y^3z}, \quad 8(1)^2(2)^3(3) = 192$$

$$\frac{\partial w}{\partial z} = \boxed{2x^2y^4}, \quad 2(1)^2(2)^4 = 32$$

$$D_u f = (192, 192, 32) \cdot \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}\right)$$

$$= \frac{192}{\sqrt{14}} + \frac{192 * 2}{\sqrt{14}} + \frac{32 * 3}{\sqrt{14}}$$

$$= \boxed{\frac{672}{\sqrt{14}}}$$ ANS

Ex6/ Let $\boxed{f(x,y,z) = x\,y\,e^{x^2+z^2-5}}$ 5

calculate the gradient of (f) at the point $(1,3,-2)$ and find the directional derivative at the same point of the vector

$$v = (3,-1,4).$$

Sol/ $\boxed{v = 3i - j + 4k}$

$$|\vec{v}| = \sqrt{(3)^2+(-1)^2+(4)^2} = \sqrt{26}$$

$$u = \left(\frac{3}{\sqrt{26}}, \frac{-1}{\sqrt{26}}, \frac{4}{\sqrt{26}}\right)$$

$$\frac{\partial f}{\partial x} = \qquad \text{H.W}$$

$$\frac{\partial f}{\partial y} = \boxed{x\,e^{x^2+z^2-5}}$$

$$\frac{\partial f}{\partial z} = \boxed{2xyz\,e^{x^2+z^2-5}}$$

$$D_u f =$$