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الرياضيات_ المرحلة الأولى

Lecture 7

- Integration of Trigononmetric Functions.
- Integration of Inverse Trigononmetric Functions.

Integration of Trigononmetric Functions.

$\int \sin x\,dx = -\cos x + C$	$\int \cos x\,dx = \sin x + C$
$\int \sec^2 x\,dx = \tan x + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int \sec x \tan x\,dx = \sec x + C$	$\int \csc x \cot x\,dx = -\csc x + C$

Example

1. $\int \sin(3x^2)\, x\, dx = \int \sin(3x^2)\ \ x\, dx * \frac{6}{6} = \frac{1}{6}\int \sin(3x^2)\ \ 6x\, dx$

$= \frac{-1}{6}\, cos\ (3x^2)\ + C$

2. $\int \frac{\csc^2(\sqrt{x})}{\sqrt{x}} dx\ \ = \int \frac{\csc^2(\sqrt{x})}{\sqrt{x}} dx\ \ * \frac{2}{2} = 2\int \frac{\csc^2(\sqrt{x})}{2\sqrt{x}} dx$

$= -2 \cot(\sqrt{x}\ ) + C$

3. $\int cos^4(x)\sin(x)\, dx = \int cos^4(x)\sin(x)\, dx\, \frac{-1}{-1}$

$= -\int cos^4(x) * -\sin(x)\, dx = -\frac{cos^{4+1}(x)}{4+1} + C = -\frac{cos^5(x)}{5} + C$

Cont'd

4. $\int sec^4(2x)\tan(2x)\,dx = \int sec^3(2x)\ \sec(2x)\tan(2x)\,dx * \frac{2}{2}$

$$= \frac{1}{2}\int sec^3(2x)\ \sec(2x)\tan(2x)\,2dx = \frac{sec^4(2x)}{4} + C$$

5. $\int sin^2(x)dx = \int \frac{1}{2}\,(1 - \cos(2x))dx = \frac{1}{2}\{\int 1\,dx - \int \cos(2x)\,dx\} = \frac{1}{2}$

$$\{x - \frac{1}{2}\sin(2x)\} + C = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

Example: Determine

1) $\int \cos(3\theta-1)d\theta$

2) $\int x\cdot \sin(2x^2)\,dx$

3) $\int \cos^2(2y)\cdot \sin(2y)\,dy$

4) $\int \sec^3 x\cdot \tan x\ dx$

5) $\int \sqrt{2+\sin 3t}\cdot \cos 3t\ dt$

6) $\int \frac{d\theta}{\cos^2\theta}$

7) $\int \left(1-\sin^2 3t\right)\cdot \cos 3t\ dt$

8) $\int \tan^3(5x)\cdot \sec^2(5x)\ dx$

9) $\int \sin^4 x\cdot \cos^3 x\ dx$

10) $\int \frac{\cot^2\sqrt{x}}{\sqrt{x}}dx$

Solution:

1) $\frac{1}{3}\int 3\cos(3\theta-1)d\theta=\frac{1}{3}\sin(3\theta-1)+c$

2) $\frac{1}{4}\int 4x\cdot \sin(2x^2)dx=-\frac{1}{4}\cos(2x^2)+c$

3) $-\frac{1}{2}\int (\cos 2y)^2\cdot(-2\sin 2y\ dy)=-\frac{1}{2}\cdot\frac{(\cos 2y)^3}{3}+c=-\frac{1}{6}(\cos 2y)^3+c$

4) $\int \sec^2 x\cdot(\sec x\cdot \tan x\cdot dx)=\frac{\sec^3 x}{3}+c$

Cont'd

$$5)\ \frac{1}{3}\int(2+\sin 3t)^{1/2}(3\cos 3t\ \ dt)=\frac{1}{3}\cdot\frac{(2+\sin 3t)^{3/2}}{3/2}+c=\frac{2}{9}\sqrt{(2+\sin 3t)^{3}}+c$$

$$6)\ \int\frac{d\theta}{\cos^{2}\theta}=\int\sec^{2}\theta\cdot d\theta=\tan\theta+c$$

$$7)\ \int\left(1-\sin^{2}3t\right)\cdot\cos 3t\ \ dt=\frac{1}{3}\int 3\cos 3t\ \ dt-\frac{1}{3}\int(\sin 3t)^{2}\cdot 3\cos 3t$$

$$=\frac{1}{3}\sin 3t-\frac{1}{3}\cdot\frac{\sin^{3}3t}{3}+c=\frac{1}{3}\cdot\sin 3t-\frac{1}{9}\sin^{3}3t+c$$

$$8)\ \frac{1}{5}\int\tan^{3}5x\cdot\left(5\sec^{2}5x\ \ dx\right)=\frac{1}{5}\cdot\frac{\tan^{4}5x}{4}+c=\frac{1}{20}\tan^{4}5x+c$$

$$9)\ \int\sin^{4}x\cdot\cos^{3}x\ \ dx=\int\sin^{4}x\cdot\left(1-\sin^{2}x\right)\cdot\cos x\ \ dx$$

$$=\int\sin^{4}x\cdot\cos x\ \ dx-\int\sin^{6}x\cdot\cos x\ \ dx=\frac{\sin^{5}x}{5}-\frac{\sin^{7}x}{7}+c$$

Cont'd

$$10)\ \int \frac{\cot^2\sqrt{x}}{\sqrt{x}}\,dx = \int \frac{\csc^2\sqrt{x}-1}{\sqrt{x}}\,dx = 2\int \frac{\csc^2\sqrt{x}}{2\sqrt{x}} - \int x^{-\frac{1}{2}}\,dx$$

$$= 2\left(-\cot\sqrt{x}\right) - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + c = -2\cot\sqrt{x} - 2\sqrt{x} + c$$

Home Work

$\int cos^2(x)dx$	$\int 4\cos 3x\, dx$	$\int 7\sec^2 4t\, dt$
$\int \frac{4}{3}\sec 4t \tan 4t\, dt$	$\int e^x \cdot \sin e^x\, dx$	$\int \frac{\sin(\ln x)}{x}$ dx
$\int cot(2x+1)\cdot csc^2(2x+1)\, dx$		

Integration of Inverse Trigononmetric Functions.

$$\int \frac{du}{\sqrt{a^2 - u^2}} = sin^{-1}\frac{u}{a} + c = -cos^{-1}\frac{u}{a} + c \qquad ; \qquad \forall u^2 < a^2$$

$$\int \frac{du}{a^2 + u^2} = \frac{1}{a}tan^{-1}\frac{u}{a} + c = -\frac{1}{a}cot^{-1}\frac{u}{a} + c$$

$$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}sec^{-1}\left|\frac{u}{a}\right| + c = -\frac{1}{a}csc^{-1}\left|\frac{u}{a}\right| + c \qquad ; \qquad \forall u^2 > a^2$$

Example

$$1)\ \int \frac{dx}{\sqrt{9-x^2}} = \sin^{-1}\frac{x}{3} + c$$

$$2)\ \int \frac{x^2}{\sqrt{1-x^6}}\,dx = \frac{1}{3}\int \frac{1}{\sqrt{1-(x^3)^2}}\left(3x^2\,dx\right) = \frac{1}{3}\sin^{-1}x^3 + c$$

$$3)\ \int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx = \sin^{-1}(\tan x) + c$$

$$4)\ \int \frac{1}{5+x^2}\,dx = \int \frac{1}{(\sqrt{5})^2+x^2}\,dx$$

$$= \frac{1}{\sqrt{5}}\tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + c$$

$$5)\ \int \frac{1}{\sqrt{x^2\left(x^2-\frac{36}{25}\right)}}\,dx = \int \frac{1}{|x|\sqrt{x^2-\left(\frac{6}{5}\right)^2}}\,dx = \frac{1}{6/5}\sec^{-1}\left(\frac{x}{6/5}\right) + c$$

$$= \frac{5}{6}\sec^{-1}\left(\frac{5x}{6}\right) + c$$

Home Work

Find the following integration:

1. $\int \frac{1}{1+\theta^2}\, d\theta$

2. $\int \frac{dx}{\sqrt{16-x^2}}$

3. $\int \frac{1}{49+x^2}\, dx$

4. $\int \frac{dt}{0.25+t^2}$

5. $\int \frac{du}{\sqrt{u^2(u^2-1)}}$

6. $\int \frac{1}{|x|\sqrt{x^2-41}}\, dx$

7. $\int \frac{1}{\sqrt{\frac{81}{100}-x^2}}\, dx$

8. $\int \frac{1}{\pi^2+x^2}\, dx$

9. $\int \frac{1}{\sqrt{t^2(t^2-\frac{1}{4})}}\, dx$

10. $\int \frac{1}{|x|\sqrt{x^2-7}}\, dx$

Thank You